

What if the force is not constant? In this case, finding the work requires integration. We will derive the procedure here:

3. When a particle is at a distance x meters from the origin, a force of $f(x) = 2x^2 + 1$ Newtons acts on the object, thus causing the object to move horizontally along the x -axis. How much work is done in moving the object from $x = 1$ to $x = 2$?

4. A spring has a natural length of 1 m. If a 25-N force is required to keep it stretched to a length of 3 m, how much work is done in stretching the spring from 2 m to 5 m? Here we will use **Hooke's Law** which states the force required to maintain a spring stretched x units beyond its natural length is given by $f(x) = kx$.

5. If the work required to stretch a spring 1 foot beyond its natural length is 12 foot pounds, how much work is needed to stretch it 9 inches beyond its natural length?

6. A rectangular tank 10 m long, 3 m wide and 2 m deep is filled with water. Find the work required to pump all the water to the top of the tank. How much work is done in pumping half the water to the top of the tank?

7. A tank is in the shape of an upright cylinder with radius 5 feet and height 20 feet. The tank is full of water to a depth of 12 feet. Find the work required to pump the water to the top of the tank.

8. A trough is in the shape of a half drum (half a cylinder lying on its side). The length of the tank is 6 feet and the radius is 1 foot. Assuming it is full of water, find the work done in pumping the water to the top of the tank.

9. A triangular trough has a length of 6 m, a distance of 2 m across the top and a height of 3 m. Assuming it is full of water, find the work done in pumping the water through a spout located at the top of the trough with length 1 m.

10. A spherical tank of radius 4 m is half full of a liquid that has density of 900 kg per cubic meter. Find the work done in pumping the water through a spout located at the top of the trough with length 1 m.

11. A 200 pound cable is 100 feet long and hangs vertically from the top of a tall building. How much work is required to pull the cable to the top of the building? How much work is done in pulling half the cable to the top of the building?