Section 7.5: Average Value

**Definition** Let \( f(x) \) be a function defined on the interval \([a, b]\). Then the average value of \( f(x) \) from \( x = a \) to \( x = b \) is

\[
\text{f}_{\text{ave}} = \frac{1}{b - a} \int_{a}^{b} f(x) \, dx
\]

A short proof of this formula will be done below:

1. Find the average value of \( f(x) = \frac{1}{x}, 1 \leq x \leq 4 \).
Mean Value Theorem for Integrals If $f$ is continuous on the interval $[a, b]$, then there exists a number $c$, $a \leq c \leq b$, so that $f(c) = \text{f}_{\text{ave}}$, that is

$$f(c) = \frac{1}{b-a} \int_{a}^{b} f(x) \, dx$$

2. Given $f(x) = 4 - x^2$, $0 \leq x \leq 2$:
   
   a.) Find the average value of $f(x)$.

   b.) Find all value(s) of $c$ that satisfy the Mean Value Theorem for Integrals.

3. In a certain city, the temperature (in degrees F) $t$ hours after 9 am is modeled by the equation $T(t) = 35 + 16 \sin \left( \frac{\pi t}{12} \right)$ Find the average temperature during the period from 9 am to 9 pm.
4. Find the value(s) \( b \) such that the average value of \( f(x) = 2 + 6x - 3x^2 \) on the interval \([0, b]\) is equal to 3.