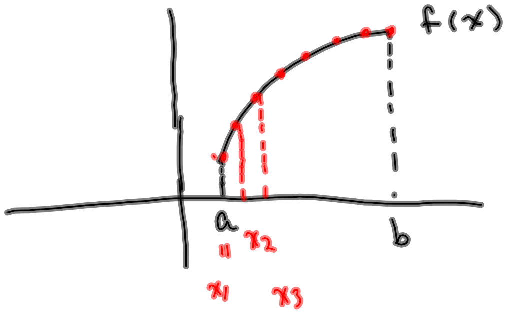
Section 7.5: Average Value

Definition Let $f(x)$ be a function defined on the interval $[a, b]$. Then the average value of $f(x)$ from $x = a$ to $x = b$ is

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

A short proof of this formula will be done below:

week in review tonight
covering 7.3- 7.4
5:45- 7:45 pm HELD 200



Partition $[a,b]$ into n subintervals, where

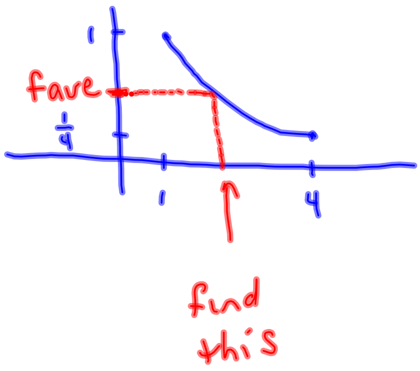
$\Delta x = \dfrac{b-a}{n} \rightarrow \boxed{\dfrac{\Delta x}{b-a} = \dfrac{1}{n}}$

$f_{ave} \approx \sum_{i=1}^{n} f(x_i^*)\,\frac{1}{n}$

$\approx \sum_{i=1}^{n} f(x_i^*)\,\frac{\Delta x}{b-a}$

$$\boxed{f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

1. Find the average value of $f(x) = \dfrac{1}{x}$, $\overset{a}{1} \le x \le \overset{b}{4}$.



$f_{ave} = \frac{1}{3}\int_1^4 \frac{1}{x}\,dx = \frac{1}{3}\ln x\Big|_1^4$

$= \frac{1}{3}\ln 4$

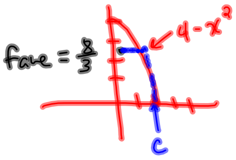
$\approx .46$

Mean Value Theorem for Integrals If f is continuous on the interval $[a, b]$, then there exists a number c, $a \le c \le b$, so that $f(c) = f_{ave}$, that is

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

2. Given $f(x) = 4 - x^2$, $0 \le x \le 2$:

   a.) Find the average value of $f(x)$.



$$f_{ave} = \frac{1}{2}\int_0^2 (4-x^2)\,dx$$

$$= \frac{1}{2}\left(4x - \frac{x^3}{3}\right)\Big|_0^2 = \frac{1}{2}\left(8 - \frac{8}{3}\right) = \frac{1}{2}\left(\frac{16}{3}\right) = \frac{8}{3}$$

   b.) Find all value(s) of c that satisfy the Mean Value Theorm for Integrals.

To find c, solve $f(c) = f_{ave}$, $0 \le c \le 2$

$$4 - c^2 = \frac{8}{3} \Rightarrow 4 - \frac{8}{3} = c^2$$

$$\frac{4}{3} = c^2 \Rightarrow \pm\sqrt{\frac{4}{3}} = c$$

$$c = \sqrt{\frac{4}{3}}$$

$$\boxed{c = \frac{2}{\sqrt{3}}}$$

3. In a certain city, the temperature (in degrees F) t hours after 9 am is modeled by the equation $T(t) = 35 + 16\sin\left(\frac{\pi t}{12}\right)$ Find the average temperature during the period from 9 am to 9 pm.

$t = 0 \Rightarrow$ 9:00 am

$t = 12 \Rightarrow$ 9:00 pm

interval $= [0, 12]$

degree one

$* \int \sin kx\,dx = -\frac{1}{k}\cos(k$

$$f_{ave} = \frac{1}{12}\int_0^{12}\left[35 + 16\sin\left(\frac{\pi t}{12}\right)\right]dt$$

$$= \frac{1}{12}\left[35t - 16\left(\frac{12}{\pi}\right)\cos\left(\frac{\pi t}{12}\right)\right]\Bigg|_0^{12}$$

$$= \frac{1}{\cancel{12}}\left[35(\cancel{12}) - 16\left(\frac{\cancel{12}}{\pi}\right)(-1) - \left(0 - 16\left(\frac{\cancel{12}}{\pi}\right)\right)\right]$$

$$= 35 + 2\left(\frac{16}{\pi}\right) = 35 + \frac{32}{\pi}\,^{\circ}$$

4. Find the value(s) b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.

Note: b must be positive

given $f_{ave} = 3$

$$\cancel{f_{ave}}^{\,3} = \frac{1}{b-0}\int_0^b (2 + 6x - 3x^2)\,dx$$

$$3 = \frac{1}{b}\left(2x + 3x^2 - x^3\right)\Big|_0^b$$

$$3 = \frac{1}{\cancel{b}}\left(2\cancel{b} + 3\cancel{b^2}^{\,b} - \cancel{b^3}^{\,b^2}\right)$$

$$3 = 2 + 3b - b^2$$

$$b^2 - 3b + 1 = 0$$

$$b = \frac{3 \pm \sqrt{9-4}}{2}$$

$$\boxed{b = \frac{3 \pm \sqrt{5}}{2}}$$

both positive
both solutions