Identities needed in this section: (more identities on last page)

- $\sin^2 x + \cos^2 x = 1$
- $\tan^2 x + 1 = \sec^2 x$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

TYPE I: Integrals of the form $\int \sin^m x \cos^n x \, dx$

m odd factor out sinx
n odd factor out cosx
both odd, do one of the above (not both)

Case 1: *EITHER* m or n (or both) is odd.

a.) $\int \sin^4 x \cos^3 x \, dx$

$\int \underbrace{\sin^4 x}_{u^4} \, \underbrace{\cos^2 x}_{1-\sin^2 x \,\to\, 1-u^2} \, \underbrace{\cos x \, dx}_{du}$

$u = \sin x$

$du = \cos x \, dx$

$\int u^4 (1-u^2) \, du = \int (u^4 - u^6) \, du$

$= \frac{u^5}{5} - \frac{u^7}{7} + C$

$= \boxed{\frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C}$

b.) $\int \sin^3 x \, dx$

$\int \underbrace{\sin^2 x}_{1-\cos^2 x \,\to\, 1-u^2} \, \underbrace{\sin x \, dx}_{-du}$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$\int (1-u^2)(-du) = \int (u^2 - 1) \, du$

$= \frac{u^3}{3} - u + C$

$= \boxed{\frac{\cos^3 x}{3} - \cos x + C}$

c.) $\int \frac{\cos^5 x}{\sin^7 x}\,dx = \int \frac{\cos^4 x}{\sin^7 x}\cos x\,dx$

$u = \sin x$
$du = \cos x\,dx$

$= \int \frac{(\cos^2 x)^2}{\sin^7 x}\cos x\,dx$

$= \int \frac{(1-\sin^2 x)^2}{\sin^7 x}\cos x\,dx \rightarrow \int \frac{(1-u^2)^2}{u^7}\,du$

$= \int \frac{1-2u^2+u^4}{u^7}\,du$

$= \int \left(u^{-7} - 2u^{-5} + u^{-3}\right) du$

$= \frac{u^{-6}}{-6} - \frac{2u^{-4}}{-4} + \frac{u^{-2}}{-2} + C$

$= -\frac{1}{6u^6} + \frac{1}{2u^4} - \frac{1}{2u^2} + C$

$= \boxed{\frac{-1}{6\sin^6 x} + \frac{1}{2\sin^4 x} - \frac{1}{2\sin^2 x} + C}$

Case 2: *BOTH m and n are even.*

a.) $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^2 x\,dx$

$\cos^2 x = \frac{1}{2}(1+\cos(2x))$

$\sin^2 x = \frac{1}{2}(1-\cos(2x))$

a.) $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^2 x\,dx$

$\int_0^{\frac{\pi}{2}} \frac{1}{2}(1-\cos 2x)\frac{1}{2}(1+\cos 2x)\,dx$

$\frac{1}{4}\int_0^{\frac{\pi}{2}} (1-\cos^2 2x)\,dx = \frac{1}{4}\int_0^{\frac{\pi}{2}} \sin^2 2x\,dx$

use
$\sin^2 x = \frac{1}{2}(1-\cos 2x)$
again

$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{1}{2}(1-\cos 4x)\,dx$

$= \frac{1}{8}\left(x - \frac{1}{4}\sin 4x\right)\Big|_0^{\frac{\pi}{2}}$

$= \frac{1}{8}\left(\frac{\pi}{2} - \cancel{\frac{1}{4}\sin(2\pi)} - 0\right) = \boxed{\frac{\pi}{16}}$

b.) $\int \frac{\cos^2(\ln x)}{x}\,dx$

$u = \ln x$
$du = \frac{1}{x}\,dx$

$\int \cos^2 u\,du$

$\int \frac{1}{2}(1+\cos(2u))\,du = \frac{1}{2}\left[u + \frac{1}{2}\sin(2u)\right] + C$

$= \frac{1}{2}\left[\ln x + \frac{1}{2}\sin(2\ln x)\right] + C$

TYPE II: Integrals of the form $\int \sec^m x \tan^n x \, dx$

Case 1: The power on *TANGENT* is odd. ← factor out $\sec x \tan x$

→ $u = \sec x$

a.) $\int \tan^3 x \sec^3 x \, dx$

$$\int \underbrace{\tan^2 x}_{\downarrow} \underbrace{\sec^2 x}_{u^2} \underbrace{\sec x \tan x \, dx}_{du}$$

$u = \sec x$

$du = \sec x \tan x \, dx$

$\sec^2 x - 1$

$\downarrow$

$u^2 - 1$

$$\int (u^2-1)u^2 \, du = \int (u^4 - u^2) \, du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= \boxed{\frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C}$$

b.) $\int \cot^5 x \csc^2 x \, dx$

$u = \cot x \quad du = -\csc^2 x \, dx$

$$-\int u^5 \, du = -\frac{u^6}{6} + C$$

$$= \boxed{\frac{-\cot^6 x}{6} + C}$$

Case 2: The power on *SECANT* is is even → factor out $\sec^2 x$

a.) $\int \tan^4 x \sec^4 x \, dx$ $u = \tan x$

$$\int \underbrace{\tan^4 x}_{u^4} \underbrace{\sec^2 x}_{\tan^2 x + 1 \to u^2+1} \underbrace{\sec^2 x \, dx}_{du}$$

$$= \int u^4 (u^2+1) \, du$$

$$= \int (u^6 + u^4) \, du = \frac{u^7}{7} + \frac{u^5}{5} + C$$

$$= \frac{\tan^7 x}{7} + \frac{\tan^5 x}{5} + C$$

TYPE III: Integrals of the form

- $\int \sin(Ax)\cos(Bx)\,dx$: Use the identity $\boxed{\sin A \cos B = \frac{1}{2}(\sin(A-B) + \sin(A+B))}$
- $\int \sin(Ax)\sin(Bx)\,dx$: Use the identity $\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$
- $\int \cos(Ax)\cos(Bx)\,dx$: Use the identity $\cos A \cos B = \frac{1}{2}(\cos(A-B) + \cos(A+B))$

don't need to know for exam!

a.) $\int \sin \underbrace{\cancel{3x}}_{A} \cos \underbrace{\cancel{x}}_{B} \, dx$

$$\int \frac{1}{2}\left(\sin(2x) + \sin(4x)\right) dx$$

$$\frac{1}{2}\left[-\frac{1}{2}\cos(2x) - \frac{1}{4}\cos(4x)\right] + C$$

Last Name, First Name

section number :

ICA #3

1. $\int \cos^4 x \, dx = \int (\cos^2 x)^2 dx = \int \left(\frac{1}{2}(1+\cos(2x))\right)^2 dx$

$$= \frac{1}{4}\int \left(1 + 2\cos 2x + \underbrace{\cos^2 2x}_{\text{use identity again}}\right) dx$$

2. $\int \sin^3 x \cos^2 x \, dx$