Section 8.3: Integration by Trigonometric Substitution

Identifying the correct trig substitution: Look for the form under the square root. There are 3 cases:

- $\sqrt{a^2 - x^2}$: Substitute $x = a\sin\theta$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
- $\sqrt{a^2 + x^2}$: Substitute $x = a\tan\theta$,$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
- $\sqrt{x^2 - a^2}$: Substitute $x = a\sec\theta$, $0 \leq \theta \leq \pi$, $\theta \neq \dfrac{\pi}{2}$

Note: If in the form $\sqrt{ax^2 + bx + c}$, you must complete the square, and then do the correct trig substitution. Also, note that it does not necessarily need to be a square root, it can, for example in case 1, instead of $\sqrt{a^2 - x^2}$, it can be $(a^2 - x^2)^{3/2}$ as seen in example 7.

1. Integrate $\displaystyle\int \frac{dx}{x^2\sqrt{1 - x^2}}$

2. Integrate $\int \frac{x^3}{\sqrt{x^2+4}}\,dx$

3. Integrate $\int_0^2 x^3\sqrt{4-x^2}\,dx$

4. Integrate $\int \frac{dx}{x^2\sqrt{16x^2-9}}$

5. Integrate $\int \frac{dx}{\sqrt{x^2+4x+8}}$

6. Integrate $\int \sqrt{4x - x^2}\, dx$

7. $\displaystyle\int_0^3 \frac{x^3}{(x^2+9)^{3/2}}\,dx$