Section 8.4: Integration by Partial Fractions

Suppose $f(x) = \dfrac{g(x)}{h(x)}$ where $g(x)$ and $h(x)$ are polynomials *and* the degree of $h(x)$ is BIGGER than the degree of $g(x)$ (if this is not true, you must first do long division). In order to integrate a partial fraction problem, you must first find the Partial Fraction Decomposition:

• **Case I**: $h(x)$ is a product of *linear* factors, *none* repeating, then the Partial Fraction Decomposition has the form:

$$\frac{x+1}{(x-2)(2x-11)} = \frac{A}{x-2} + \frac{B}{2x-11}$$

• **Case II**: $h(x)$ is a product of *linear* factors, *some* repeating, then the Partial Fraction Decomposition has the form:

$$\frac{x+1}{(x-1)(x-2)^2} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

• **Case III**: $h(x)$ contains *irreducible quadratic* factors, *none* repeating, then the Partial Fraction Decomposition has the form:

$$\frac{x+1}{(x-2)(x^2+1)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+1}$$

• **Case IV**: $h(x)$ contains *irreducible quadratic* factors, *some* repeating, then the Partial Fraction Decomposition has the form:

$$\frac{x+5}{(x-2)(x^2+4)^2} = \frac{A}{x-2} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}$$

Once you find A, B, etc., then you integrate the result.

1. Integrate $\displaystyle\int_0^1 \frac{2x}{(x+1)(2x+5)}\,dx$

2. Integrate $\int \frac{x^2+1}{x^2-x}\, dx$

3. Integrate $\displaystyle\int \frac{1}{(x-1)^2(x+4)}\,dx$

4. Integrate $\displaystyle\int \frac{3x^2 - 4x + 5}{(x-1)(x^2+1)}\,dx$

5. Integrate $\displaystyle\int \frac{x+6}{(x^2+1)(x^2+4)}\,dx$