Section 8.8: Approximate Integration

Approximate Integration: Suppose I'd like to know $\int_a^b f(x)\,dx$. There are three techniques of approximating an integral:

I. Midpoint Rule:

$$\int_a^b f(x)\,dx \approx \Delta x[f(\overline{x_1}) + f(\overline{x_2}) + f(\overline{x_3}) + ... + f(\overline{x_n})]$$

where $\Delta x = \dfrac{b-a}{n}$ and $\overline{x_i}$ is the midpoint of the ith subinterval.

II. Trapezoid Rule:

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + ... + f(x_n)]$$

where $\Delta x = \dfrac{b-a}{n}$ and x_i are the points of the partition.

III. Simpson's Rule:

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + ... + f(x_n)]$$

where $\Delta x = \dfrac{b-a}{n}$ and x_i are the points of the partition.

1. Use the Midpoint Rule with $n = 4$ to approximate $\int_0^1 e^{x^2}\,dx$

2. Use the Trapezoid Rule with $n = 4$ to approximate $\int_0^1 e^{x^2}\, dx$

3. Use the Simpson's Rule with $n = 4$ to approximate $\int_0^1 e^{x^2}\, dx$

Definition: The *error* in approximating $\int_a^b f(x)\,dx$ by M_n, T_n or S_n is definied as $\int_a^b f(x)\,dx -$ (The approximation).

4. Find the exact error in using T_4 to approximate $\int_0^1 e^x\,dx$.

Error Bound formulas:

- Error Bound for Midpoint Rule:

$|E_M| \leq \dfrac{K(b-a)^3}{24n^2}$, where $K = \max|f''(x)|$ for $a \leq x \leq b$

- Error Bound for Trapezoid Rule:

$|E_T| \leq \dfrac{K(b-a)^3}{12n^2}$, where $K = \max|f''(x)|$ for $a \leq x \leq b$

- Error Bound for Simpson's Rule:

$|E_S| \leq \dfrac{K(b-a)^5}{180n^4}$, where $K = \max|f^{(4)}(x)|$ for $a \leq x \leq b$

5. Use the appropriate error bound formula to approximate the error in using T_4 to approximate $\int_0^1 e^x \, dx$.

6. Suppose we used S_6 to approximate $\int_1^3 \ln x \, dx$. Find an upper bound on the error.

7. How large should we choose n so that M_n approximates $\int_1^3 \ln x \, dx$ to within $\frac{1}{100}$?