Section 9.1: Differential Equations

Definition: We say a differential equation is *separable* if it can be written in the form

$$\frac{dy}{dx} = f(x)g(y)$$

Here, we say our *independant* variable is x and our *dependent* variable is y. Your goal is to try and solve the differential equation for the *dependent* variable, y, in this case.

To separate a separable differential equation, you must get it in the form

$$p(y)dy = q(x)dx$$

Once the differential equation is separated, integrate both sides.

1. Solve the differential equation:

   a.) $\dfrac{dy}{dx} = \dfrac{x + \sin x}{3y^2}$

b.) $\dfrac{dy}{dx} = \dfrac{1+x}{xy}$, $y(1) = -4$

c.) $\dfrac{du}{dt} = e^{u+2t}$, $u(0) = 1$

d.) $\dfrac{dx}{dt} = 1 + t - x - xt$

2. Find the equation of the curve that passes through the point $(2, 1)$ and whose slope at each point (x, y) is $x^3 y$

3. A tank contains 100L of pure water. Brine that contains .05 kg of salt per liter enters the tank at a rate of 5 liters per minute. Brine that contains .04 kg of salt per liter enters the tank at a rate of 10 liters per minute. The solution is kept mixed and drains from the tank at a rate of 15 liters per minute. How much salt is in the tank after t minutes?

4. Below is the direction field for the differential equation $y' = x^2 + y^2 - 1$. Sketch the solution curve that passes through the origin.