

Section 9.2: First Order Linear Equations

Definition: A differential equation is linear if it is in the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

It is important that you recognize which variable is independent and which is dependent. If your equation contains $\frac{dy}{dx}$, then the independent variable is x ; the dependent variable is y .

• To solve a linear differential equation, you must first find the **integrating factor**, $I(x)$.

$$I(x) = e^{\int P(x) dx}$$

• Next, multiply both sides of the differential equation by $I(x)$:

$$I(x) \left(\frac{dy}{dx} + P(x)y \right) = I(x)Q(x)$$

which then becomes

$$(yI(x))' = I(x)Q(x)$$

Finally, integrate both sides and then solve for y .

1. Solve $\frac{dy}{dx} - 3y = e^x$

2. Solve $xy' + 2y = e^{x^2}$

3. Solve $x\frac{dy}{dx} + y = x \cos x$

4. Solve $y' + y = x + e^x$, $y(0) = 0$

5. $(1 + x^2)y' + 2xy = 3\sqrt{x}$, $y(0) = 2$

6. Solve $xy' - 3y = x^2$, $y(1) = 0$