

Section 9.3: Arc Length

There are three possible formulas which gives the length of a curve, depending on how the curve is described to you.

- If $y = f(x)$, $a \leq x \leq b$, then the length of the curve from $x = a$ to $x = b$ is

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

- If $x = g(y)$, $c \leq y \leq d$, then the length of the curve from $y = c$ to $y = d$ is

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

- If $x = f(t)$ and $y = g(t)$, $\alpha \leq t \leq \beta$, then the length of the curve from $t = \alpha$ to $t = \beta$ is

$$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

If we have time in class, a short proof will be supplied which you can write below.

1. Find the length of the curve $y = 2x^{3/2}$, $0 \leq x \leq 1$.

2. Find the length of the curve $y = \ln(\cos x)$, $0 \leq x \leq \frac{\pi}{4}$.

3. Find the length of the curve $x = \ln(1 - y^2)$, $0 \leq y \leq \frac{1}{2}$.

4. Find the length of the curve $x = 3t - t^3$, $y = 3t^2$, $0 \leq t \leq 2$.

5. A bomb is dropped from an aircraft flying at 200 m/s at an altitude of 4500 m. The parabolic trajectory of the bomb is described by $y = 4500 - \frac{x^2}{8000}$ until it hits the ground, where y is the distance traveled, in meters. Set up the integral that gives the distance traveled by the bomb from the time it is dropped until the time it hits the ground.