Section 9.4: Surface Area of Revolution

To compute a Surface Area of Revolution, we have 2 cases to consider, and each case has a subcase depending on whether the function is a function of x, y, or described parametrically. Think of surface as $\int 2\pi r\, ds$, where ds is the arc length function and r is the measured distance form the curve to the axis of revolution.

- Revolution around the x axis:
  - a.) If the curve $y = f(x)$, $a \leq x \leq b$ is revolved around the x axis, then the resulting surface area is given by

$$SA = 2\pi \int_a^b f(x)\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

  - b.) If the curve $x = g(y)$, $c \leq y \leq d$ is revolved around the x axis, then the resulting surface area is given by

$$SA = 2\pi \int_c^d y\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

  - c.) If the curve $x = f(t)$ and $y = g(t)$, $\alpha \leq t \leq \beta$, is revolved around the x axis, then the resulting surface area is

$$SA = 2\pi \int_\alpha^\beta g(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

- Revolution around the y axis:
  - a.) If the curve $y = f(x)$, $a \leq x \leq b$ is revolved around the y axis, then the resulting surface area is given by

$$SA = 2\pi \int_a^b x\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

  - b.) If the curve $x = g(y)$, $c \leq y \leq d$ is revolved around the y axis, then the resulting surface area is given by

$$SA = 2\pi \int_c^d g(y)\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

  - c.) If the curve $x = f(t)$ and $y = g(t)$, $\alpha \leq t \leq \beta$, is revolved around the y axis, then the resulting surface area is given by

$$SA = 2\pi \int_\alpha^\beta f(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

1. Find the surface area obtained by revolving the curve $y = x^3$, $0 \leq x \leq 2$ about the x-axis.

2. Find the surface area obtained by revolving the curve $y^2 = 4x + 4$, $2 \leq y \leq 6$ about the x-axis.

3. Find the surface area obtained by revolving the curve $x = 3t - t^3$, $y = 3t^2$, $0 \leq t \leq 1$ about the x-axis.

4. Find the surface area obtained by revolving the curve $y = 1 - x^2$, $0 \leq x \leq 1$ about the y-axis.

5. Set up the integral that gives the surface area obtained by revolving the curve $x = e^{2y}$, $0 \leq y \leq 1$ about the y-axis. Do not integrate.

6. Consider the surface area obtained by rotating curve $y = \sin x$, $0 \leq x \leq \frac{\pi}{2}$ about the y-axis. Set up both a dx and a dy integral that gives the resulting surface area. Do not evaluate either integral.

7. Find the surface area obtained by revolving the curve $x = e^t - t$, $y = 4e^{t/2}$, $0 \le t \le 1$ about the y-axis.