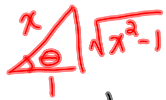
Section 8.3

1. $\int \frac{dx}{x^2\underbrace{\sqrt{x^2-1}}}=$ form $x^2 - a^2 \to x = a\sec\theta$

$x^2 - 1 \to \boxed{x = \sec\theta}$



$dx = \sec\theta\tan\theta\, d\theta$

$$\int \frac{\cancel{\sec\theta}\tan\theta\, d\theta}{\sec^{\cancel{2}}\theta\sqrt{\sec^2\theta - 1}}$$

$$\int \frac{\tan\theta\, d\theta}{\sec\theta\tan\theta} = \int \frac{d\theta}{\sec\theta} = \int \cos\theta\, d\theta$$

$$= \sin\theta + C$$

$$= \boxed{\frac{\sqrt{x^2-1}}{x} + C}$$

2. $\int_0^2 x^3\sqrt{x^2+4}\, dx=$ Form $x^2 + a^2 \to x = a\tan\theta$

$x^2 + 4 \to x = 2\tan\theta$ — $x = 2,\ \theta = \frac{\pi}{4}$; $x = 0,\ \theta = 0$

$dx = 2\sec^2\theta\, d\theta$

$$\int_0^{\frac{\pi}{4}} 8\tan^3\theta\sqrt{\underbrace{4\tan^2\theta + 4}_{\substack{4(\tan^2\theta+1)\\ 4\sec^2\theta}}}\ 2\sec^2\theta\, d\theta$$

$$\int_0^{\frac{\pi}{4}} 8\tan^3\theta \cdot 2\sec\theta \cdot 2\sec^2\theta\, d\theta$$

$$32\int_0^{\frac{\pi}{4}} \tan^3\theta\sec^3\theta\, d\theta$$

$u = \sec\theta$ — $\theta = \frac{\pi}{4},\ u = \sqrt{2}$; $\theta = 0,\ u = 1$

$du = \sec\theta\tan\theta\, d\theta$

$$32\int_0^{\frac{\pi}{4}} \underbrace{\tan^2\theta}_{\substack{\sec^2\theta - 1\\ \downarrow\\ u^2-1}}\ \underbrace{\sec^2\theta}_{u^2}\ \underbrace{\sec\theta\tan\theta\, d\theta}_{du}$$

$$32\int_1^{\sqrt{2}} (u^2-1)u^2\, du \to 32\int_1^{\sqrt{2}} (u^4 - u^2)\, du$$

$$32\left(\frac{u^5}{5} - \frac{u^3}{3}\right)\Bigg|_1^{\sqrt{2}} = \_\_\_$$

3. $\int \sqrt{-x^2+\underline{\underline{6x}}+7}\,dx=$

$-x^2+6x+7$ $\qquad \left(\frac{-6}{2}\right)^2$

$-(x^2-6x+9)+7+9$

$-(x-3)^2+16$

$\int \sqrt{16-(x-3)^2}\,dx$ $\qquad a^2-x^2 \to x=a\sin\theta$

$16-(x-3)^2 \to x-3=4\sin\theta$

$dx=4\cos\theta\,d\theta$

$\int \sqrt{\underbrace{16-16\sin^2\theta}_{\substack{16(1-\sin^2\theta)\\ 16\cos^2\theta}}}\;4\cos\theta\,d\theta$

$= \int 4\cos\theta\, 4\cos\theta\,d\theta$

$\to 16\int \cos^2\theta\,d\theta$

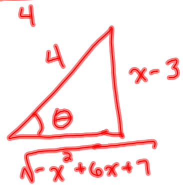
$16\int \frac{1}{2}(1+\cos(2\theta))\,d\theta$

$8\left[\theta+\frac{1}{\cancel{2}}\sin(2\theta)\right]+C$

$\cancel{2}\sin\theta\cos\theta$

$x-3=4\sin\theta$

$\frac{x-3}{4}=\sin\theta$



$8\left[\theta+\sin\theta\cos\theta\right]+C$

$8\left[\sin^{-1}\frac{x-3}{4}+\frac{x-3}{4}\,\frac{\sqrt{-x^2+6x+7}}{4}\right]+C$

Section 8.4

4. $\int_2^3 \frac{x^3+1}{x^2(x-1)}\,dx=$ $\int_2^3 \frac{x^3+1}{x^3-x^2}\,dx=\int_2^3\left(1+\frac{x^2+1}{x^3-x^2}\right)dx$

PFD

$$\begin{array}{r|l} & \;1 \\ x^3-x^2 & \;x^3+1 \\ & -x^3+x^2 \\ \hline & \;x^2+1 \end{array}$$

$\frac{x^2+1}{\cancel{x^2(x-1)}}=\left(\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}\right)x^2(x-1)$

$x^2+1=Ax(x-1)+\cancel{B}^{-1}(x-1)+\cancel{C}^{2}x^2$

$x=0:\quad 1=B(-1)\to \boxed{B=-1}$

$x=1:\quad \boxed{2=C}$

$x=2:\quad 5=A(2)-1+8$

$5=2A+7\to 2A=-2$

$\boxed{A=-1}$

$\int_2^3\left(1+\frac{x^2+1}{x^3-x^2}\right)dx=\int_2^3\left(1+\frac{-1}{x}-\frac{1}{x^2}+\frac{2}{x-1}\right)dx$

$=\left(x-\ln|x|+\frac{1}{x}+2\ln|x-1|\right)\Big|_2^3$

$=$ ___

5. $\int \frac{x+1}{x^2-4}\,dx = \int \frac{x+1}{(x+2)(x-2)}\,dx$

PFD: $\frac{x+1}{(x+2)(x-2)} = \frac{A}{x+2} + \frac{B}{x-2}$

$x+1 = A(x-2) + B(x+2)$

$x = 2: \quad 3 = B(4) \rightarrow \boxed{B = \frac{3}{4}}$

$x = -2: \quad -1 = A(-4) \rightarrow \boxed{A = \frac{1}{4}}$

$$\int \left(\frac{\frac{1}{4}}{x+2} + \frac{\frac{3}{4}}{x-2} \right) dx = \boxed{\frac{1}{4}\ln|x+2| + \frac{3}{4}\ln|x-2| + C}$$

6. $\int \frac{2x^2-x+4}{x^3+4x}\,dx = \int \frac{2x^2-x+4}{x(x^2+4)}\,dx$

PFD: $\frac{2x^2-x+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$

$2x^2-x+4 = A(x^2+4) + (Bx+C)x$

$x=0: \quad 4 = A(4) \rightarrow \boxed{A=1}$

$2x^2-x+4 = x^2+4 + Bx^2 + Cx$

$2x^2 - x + 4 = (1+B)x^2 + Cx + 4$

$2 = 1+B \rightarrow \boxed{B=1}$

$\boxed{-1 = C}$

$$\int \left(\frac{1}{x} + \frac{x-1}{x^2+4} \right) dx = \int \left(\frac{1}{x} + \frac{x}{x^2+4} - \frac{1}{x^2+4} \right) dx$$

$\uparrow$ u-sub

$u = x^2+4$

$du = 2x\,dx$

$\frac{1}{2}\int \frac{du}{u}$

$\uparrow$ $\frac{1}{2}\arctan\frac{x}{2}$

$= \ln|x| + \frac{1}{2}\ln(x^2+4) \quad \frac{1}{2}\arctan\frac{x}{2} +$

Section 8.9

7. $\displaystyle\int_e^{\infty} \frac{dx}{x(\ln x)^2} = \lim_{t\to\infty} \int_e^t \frac{dx}{x(\ln x)^2}$

$u = \ln x$

$du = \dfrac{dx}{x}$

$\displaystyle\int \frac{du}{u^2} = -\frac{1}{u} = -\frac{1}{\ln x}$

$$= \lim_{t\to\infty} -\frac{1}{\ln x}\Big|_e^t$$

$$= \lim_{t\to\infty}\left(\cancel{-\frac{1}{\ln t}}^{\,0} + \frac{1}{\ln e}\right) = \boxed{1}$$

$\boxed{\text{converges to 1}}$

division by 0

8. $\displaystyle\int_1^{9} \frac{1}{\sqrt[3]{x-9}}\,dx = \lim_{t\to 9^-} \int_1^t \frac{dx}{\sqrt[3]{x-9}}$

$$= \lim_{t\to 9^-} \int_1^t (x-9)^{-\frac{1}{3}}\,dx$$

$$= \lim_{t\to 9^-} \frac{3}{2}(x-9)^{\frac{2}{3}}\Big|_1^t$$

$$= \lim_{t\to 9^-} \frac{3}{2}\left[\cancel{(t-9)^{\frac{2}{3}}}^{\,\to 0} - \underbrace{(-8)^{\frac{2}{3}}}_{\to 4}\right]$$

$$= \frac{3}{2}(-4) = \boxed{-6}$$

converges

9. $\int_{-1}^{2} \frac{1}{x^4}\,dx =$ $\int_{-1}^{0} \frac{1}{x^4}\,dx + \int_{0}^{2} \frac{1}{x^4}\,dx$

improper at 0!

$$\lim_{t\to 0^-} \int_{-1}^{t} x^{-4}\,dx = \lim_{t\to 0^-} \left.\frac{x^{-3}}{-3}\right|_{-1}^{t}$$

$$= \lim_{t\to 0^-} \left.-\frac{1}{3x^3}\right|_{-1}^{t}$$

$$= \lim_{t\to 0^-} \left[-\frac{1}{3t^3} - \left(-\frac{1}{3(-1)}\right)\right]$$

$$= \boxed{+\infty}$$

integral diverges!

10. Use the comparison theorem to determine whether the following improper integrals converge or diverge:

a.) $\int_1^{\infty} \frac{1}{x+e^{2x}}\,dx \leq \int_1^{\infty} \frac{1}{e^{2x}}\,dx = \int_1^{\infty} e^{-2x}\,dx$

$$= -\frac{1}{2} e^{-2x} \Big|_1^{\infty}$$

$$= -\frac{1}{2}\left[\cancel{e^{-\infty}} - e^{-2}\right]$$

$$= \frac{1}{2e^2}$$

larger integral converges, so does smaller.

b.) $\int_5^{\infty} \frac{x}{x^{3/2}-x-1}\,dx \geq \int_5^{\infty} \frac{x}{x^{3/2}}\,dx$

$$= \int_5^{\infty} x^{-\frac{1}{2}}\,dx$$

$$= 2\sqrt{x}\,\Big|_5^{\infty}$$

$$= \infty$$

smaller diverges so must larger

Section 9.3

11. Find the length of the curve $x = \frac{4\sqrt{2}}{3}y^{3/2} - 1$ from $y = 0$ to $y = 1$.

$$L = \int_0^1 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$\frac{dx}{dy} = \frac{4\sqrt{2}}{3} \cdot \frac{3}{2} y^{\frac{1}{2}}$$

$$\frac{dx}{dy} = 2\sqrt{2}\, y^{\frac{1}{2}}$$

$$\left(\frac{dx}{dy}\right)^2 = 8y$$

$$L = \int_0^1 \sqrt{1 + 8y}\, dy$$

u-sub

$u = 1 + 8y$

12. Find the length of the curve $y = \frac{x^3}{6} + \frac{1}{2x}$ from the point $(1, 2/3)$ to the point $(3, 14/3)$.

$$y = \frac{1}{6}x^3 + \frac{1}{2}x^{-1}$$

$$\frac{dy}{dx} = \frac{1}{6} \cdot 3x^2 - \frac{1}{2}x^{-2}$$

$$\frac{dy}{dx} = \frac{x^2}{2} - \frac{1}{2x^2}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{x^4}{4} + 2\left(\frac{x^2}{2}\right)\left(-\frac{1}{2x^2}\right) + \frac{1}{4x^4}$$

$$= \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4}$$

$$L = \int_1^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_1^3 \sqrt{1 + \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4}}\, dx$$

$$L = \int_1^3 \sqrt{\frac{x^4}{4} + \frac{1}{2} + \frac{1}{4x^4}}\, dx$$

$$L = \int_1^3 \sqrt{\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2}\, dx$$

$$L = \int_1^3 \left(\frac{x^2}{2} + \frac{1}{2x^2}\right) dx = \_\_\_\_\_\_$$

13. Find the length of the curve $x = \cos t$, $y = \sin t$, $0 \le t \le \frac{\pi}{3}$.

$$L = \int_0^{\frac{\pi}{3}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$L = \int_0^{\frac{\pi}{3}} \sqrt{\sin^2 t + \cos^2 t}\, dt$$

$$L = \int_0^{\frac{\pi}{3}} dt = t \Big|_0^{\frac{\pi}{3}} = \boxed{\frac{\pi}{3}}$$

Section 9.4

14. Find the surface area of the region obtained by rotating the curve $y = x^2$, $0 \le x \le 2$, about the y axis.

$\hookrightarrow r = x$ ✓

$$SA = \int 2\pi r \text{ arclength } \begin{matrix} dx \\ dy \\ dt \end{matrix}$$

$$SA = \int_0^2 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \qquad y = x^2$$

$$\frac{dy}{dx} = 2x$$

$$SA = \int_0^2 2\pi x \sqrt{1 + 4x^2}\, dx$$

$u = 1 + 4x^2$

⋮

15. Find the surface area of the region obtained by rotating the curve $y = \sqrt{x}$, $1 \le x \le 4$ about the x axis.

$\hookrightarrow r = y$ ✓

$x = y^2$, $1 \le y \le 2$

$$SA = \int 2\pi r \,\text{arclength} \quad dx \; dy \; dt$$

$x = y^2$

$\frac{dx}{dy} = 2y$

$$= \int_1^2 2\pi y \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 2\pi y \sqrt{1+4y^2}\, dy = \_\_\_$$

u-sub

16. Find the surface area obtained by rotating the curve $x = t^3$, $y = t^2$, $0 \le t \le 1$, about the x-axis.

$\hookrightarrow r = y = t^2$

$$SA = \int_0^1 2\pi t^2 \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\, dt$$

$$= 2\pi \int_0^1 t^2 \sqrt{9t^4+4t^2}\, dt$$

$$= 2\pi \int_0^1 t^2 \sqrt{t^2(9t^2+4)}\, dt$$

$u = 9t^2+4$

$du = 18t\, dt$

$t=1,\ u=13$

$t=0,\ u=4$

$$= 2\pi \int_0^1 t^3 \sqrt{9t^2+4}\, dt$$

$$= \frac{2\pi}{18} \int_4^{13} t^2 \sqrt{u}\, du$$

$$= \frac{\pi}{9} \int_4^{13} \frac{u-4}{9} \sqrt{u}\, du$$

$$= \frac{\pi}{81} \int_4^{13} \left(u^{\frac{3}{2}} - 4u^{\frac{1}{2}}\right) du = \_\_\_$$

Section 10.1

17. Discuss the convergence or divergence of the following sequences:

a.) $a_n = \ln(3n+1) - \ln(4n^2)$

$$\lim_{n\to\infty} \ln\left(\frac{3n+1}{4n^2}\right) = \ln\left[\lim_{n\to\infty} \frac{3n+1}{4n^2}\right]$$

$$= \ln(\text{“}0\text{”})$$

$$= \boxed{-\infty}$$

$\boxed{\text{DIVERGES}}$

b.) $a_n = (-1)^n \frac{n}{n+1}$ $\rightarrow \lim_{n\to\infty} \frac{n}{n+1} = 1$

n even $\longrightarrow 1$

n odd $\rightarrow -1$

$\lim_{n\to\infty} (-1)^n \frac{n}{n+1}$ diverges by oscillation

c.) $a_n = (-1)^n \frac{n}{n^2+1}$ $\rightarrow 0$

$\boxed{\text{converges to } 0.}$

d.) $a_n = \sqrt{n^2 - 8n} - n$ $= \infty - \infty = 0$ No!

$$\lim_{n\to\infty} \frac{\left(\sqrt{n^2-8n} - n\right)\left(\sqrt{n^2-8n} + n\right)}{\sqrt{n^2-8n} + n}$$

$$\lim_{n\to\infty} \frac{n^2 - 8n - n^2}{\sqrt{n^2-8n} + n} = \lim_{n\to\infty} \frac{-8n}{\sqrt{n^2-8n} + n}$$

$$= \lim_{n\to\infty} \frac{-8n}{\sqrt{n^2} + n}$$

$$= \lim_{n\to\infty} \frac{-8n}{2n} = \boxed{-4}$$

18. Determine whether the sequence is bounded:

a.) $a_n = \left\{\frac{1}{n^2}\right\}_{n=1}^{\infty} = \left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots\right\}$

$N \leq a_n \leq M$

Bounded sequence

$0 < a_n \leq 1$

$\boxed{\text{is bounded}}$

b.) $a_n = \left\{\frac{n^2}{n+1}\right\}_{n=1}^{\infty} = \left\{\frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \ldots\right\}$

$\frac{1}{2} \leq a_n < \infty$

not bounded above,
so not bounded.

19. Determine whether following sequences are increasing, decreasing, or not monotonic.

a.) $a_n = \frac{3}{n+5}$

decreasing sequence.

$\left(\frac{3}{n+5}\right)' = \frac{-3}{(n+5)^2} < 0 \rightarrow \frac{3}{n+5}$ decreases.

b.) $a_n = \cos\frac{n\pi}{2}$

neither inc nor dec.

oscillates between 0, 1, -1.

non monotonic.

20. Consider the recursive sequence defined by $a_1 = 2$, $a_{n+1} = 1 - \dfrac{1}{a_n}$. Find the first 5 terms of the sequence. Find the limit of the sequence, if it exists.

$a_1 = 2$

$a_{n+1} = 1 - \dfrac{1}{a_n}$

$a_1 = 2$

$a_2 = 1 - \dfrac{1}{a_1} = 1 - \dfrac{1}{2} = \dfrac{1}{2}$

$a_3 = 1 - \dfrac{1}{a_2} = 1 - \dfrac{1}{\frac{1}{2}} = -1$

$a_4 = 1 - \dfrac{1}{a_3} = 1 - \dfrac{1}{-1} = 2$

$a_5 = \dfrac{1}{2}$

sequence diverges by oscillation

21. Given the recursive sequence below is increasing and bounded, find the limit.

$a_1 = 2$, $a_{n+1} = 4 - \dfrac{3}{a_n}$.

$\lim_{n\to\infty} a_n = L$

$a_\infty = L$

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty}\left(4 - \frac{3}{a_n}\right)$$

$$a_\infty = 4 - \frac{3}{a_\infty}$$

$$L = 4 - \frac{3}{L}$$

$$L^2 = 4L - 3 \rightarrow L^2 - 4L + 3 = 0$$

$$(L-3)(L-1) = 0$$

$$\boxed{L = 3}$$

Section 10.2

22. Use the Test For Divergence to show the series diverges:

$$\sum_{n=1}^{\infty} \frac{n^2}{3(n+1)(n+2)}$$

TD: If $\lim_{n\to\infty} a_n \neq 0$

then $\sum_{n=1}^{\infty} a_n$ diverges

$$\lim_{n\to\infty} \frac{n^2}{3(n+1)(n+2)} = \frac{1}{3} \neq 0$$

$\boxed{\text{series diverges}}$

23. Explain why the Test for Divergence is inconclusive when applied to the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$.

Note: If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ may converge

T.D. $\lim_{n\to\infty} \sin\left(\frac{1}{n}\right) = \sin(0)$

$= 0 \to$ T.D. fails

24. If the n^{th} partial sum of the series $\sum_{n=1}^{\infty} a_n$ is $\boxed{s_n = \frac{n+1}{n+4}\text{, find:}}$

For $\sum_{n=1}^{\infty} a_n$, the n^{th} partial sum is

$$S_n = a_1 + a_2 + \dots + a_n$$

$$\boxed{\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n}$$

a.) s_{100}, that is $\sum_{n=1}^{100} a_n = ?$

$$S_{100} = \frac{101}{104}$$

b.) The sum of the series, that is $\sum_{n=1}^{\infty} a_n = ?$

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n+1}{n+4} = 1$$

c.) A general formula for a_n, then find a_6.

$$S_n = a_1 + a_2 + \dots + a_n \qquad S_n = \frac{n+1}{n+4}$$

$$S_{n-1} = a_1 + a_2 + \dots + a_{n-1}$$

$$a_n = S_n - S_{n-1}$$

$$\boxed{a_n = \frac{n+1}{n+4} - \frac{n}{n+3}}$$

$$a_6 = S_6 - S_5 = \frac{7}{10} - \frac{6}{9}$$

$\underline{\underline{\text{or}}}$ $a_6 = \frac{7}{10} - \frac{6}{9}$

25. Find the sum of the series:

a.) $\sum_{n=1}^{\infty}\left(\sin\frac{1}{n}-\sin\frac{1}{n+1}\right)$

① $S_n = a_1 + a_2 + a_3 + \cdots a_n$

$$= \underbrace{\sin 1 - \cancel{\sin\tfrac{1}{2}}}_{a_1} + \underbrace{\cancel{\sin\tfrac{1}{2}} - \cancel{\sin\tfrac{1}{3}}}_{a_2} + \underbrace{\cancel{\sin\tfrac{1}{3}} - \cancel{\sin\tfrac{1}{4}}}_{a_3} + \cdots + \underbrace{\cancel{\sin\tfrac{1}{n}} - \sin\tfrac{1}{n+1}}_{a_n}$$

$$\boxed{S_n = \sin(1) - \sin\frac{1}{n+1}}$$

② $$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\sin(1) - \cancel{\sin\frac{1}{n+1}}^{\,0}\right) = \boxed{\sin(1)}$$

b.) $\sum_{n=1}^{\infty}\frac{1}{n^2+2n} = \sum_{n=1}^{\infty}\frac{1}{n(n+2)}$

PFD: $\frac{1}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}$

$1 = A(n+2) + Bn$

$n=0: \ 1 = A(2)$ $\boxed{A = \frac{1}{2}}$

$n=-2: \ 1 = B(-2)$ $\boxed{B = -\frac{1}{2}}$

$$\sum_{n=1}^{\infty}\left(\frac{\frac{1}{2}}{n} - \frac{\frac{1}{2}}{n+2}\right)$$

$$\frac{1}{2}\left[\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+2}\right)\right]$$

$S_n = a_1 + a_2 + a_3 + a_4 + \cdots + a_{n-1} + a_n$

$$S_n = \underbrace{1 - \cancel{\tfrac{1}{3}}}_{a_1} + \underbrace{\tfrac{1}{2} - \cancel{\tfrac{1}{4}}}_{a_2} + \underbrace{\cancel{\tfrac{1}{3}} - \cancel{\tfrac{1}{5}}}_{a_3} + \underbrace{\cancel{\tfrac{1}{4}} - \cancel{\tfrac{1}{6}}}_{a_4} + \cdots \underbrace{\cancel{\tfrac{1}{n-1}} - \tfrac{1}{n+1}}_{a_{n-1}} + \underbrace{\cancel{\tfrac{1}{n}} - \tfrac{1}{n+2}}_{a_n}$$

$$\boxed{S_n = 1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right) = \frac{3}{2}$$

now multiply by $\frac{1}{2}$!

sum of series is $\frac{3}{4}$

c.) $\sum_{n=1}^{\infty} 2\left(\frac{5}{7}\right)^{n-1}$ $\quad\quad$ $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$ if $|r|<1$

$r = \frac{5}{7}$ $\quad$ $-1<r<1$ $\quad$ proceed to sum

$$\sum_{n=1}^{\infty} 2\left(\frac{5}{7}\right)^{n-1} = \frac{a}{1-r} = \boxed{\frac{2}{1-\frac{5}{7}}}$$

d.) $\sum_{n=1}^{\infty} \frac{3^{2n+1}}{10^n} = \sum_{n=1}^{\infty} \frac{3^{2n}\cdot 3}{10^n} = \sum_{n=1}^{\infty} \frac{9^n \cdot 3}{10^n} = \sum_{n=1}^{\infty} 3\left(\frac{9}{10}\right)^{\boxed{n}}$

$r = \frac{9}{10}$, $-1<r<1$ proceed to sum

$$\sum_{n=1}^{\infty} 3\left(\frac{9}{10}\right)\left(\frac{9}{10}\right)^{n-1} = \frac{a}{1-r}$$

$$= \boxed{\frac{\frac{27}{10}}{1-\frac{9}{10}}}$$

e.) $\sum_{n=0}^{\infty} \frac{2^{3n}}{(-5)^{n+1}} = \sum_{n=0}^{\infty} \frac{8^n}{(-5)(-5)^n}$

$= \sum_{n=0}^{\infty} -\frac{1}{5}\left(-\frac{8}{5}\right)^n$ $\quad$ $r = -\frac{8}{5}$ $\underline{\underline{\text{NOT}}}$ between ± 1

diverges

f.) $\sum_{n=0}^{\infty} \frac{(-1)^n + 3^n}{4^n} = \sum_{n=0}^{\infty} \frac{(-1)^n}{4^n} + \sum_{n=0}^{\infty} \frac{3^n}{4^n}$

$= \sum_{n=0}^{\infty} \left(-\frac{1}{4}\right)^n + \sum_{n=0}^{\infty} \left(\frac{3}{4}\right)^n$ $\quad$ $-1<r<1$ for both proceed

$$= \boxed{\frac{1}{1+\frac{1}{4}} + \frac{1}{1-\frac{3}{4}}}$$