Math 141 Key Topics: 9.1-9.2

Section 9.1

- A **Markov chain** is an experiment in which the probabilities associated with the outcomes at any stage of the experiment depend only on the outcomes of the preceding stage.
- The outcomes at any stage of a Markov chain are called the **states** of the experiment.
- The **transition matrix** for a Markov process is formed by finding the conditional probabilities associated with moving from one state to the next state. The entries of the matrix are defined by $a_{ij}=P(\text{state } i|\text{state } j)$.
- A **stochastic matrix** is a square matrix where all the entries are greater than or equal to 0 (≥ 0) and the sum of the entries in each column is 1.
- We can represent distributions in a Markov chain by using **distribution vectors**. A vector is just a matrix with 1 column.
- Given a transition matrix T and an initial-state distribution vector X_0, we can find the distribution vector at any stage of the Markov chain by using the formula:

$$X_m = T^m X_0$$

Section 9.2

- If the distribution vectors X_m are getting closer and closer to some vector as m gets bigger and bigger, then the vector that they are approaching is called the **steady-state distribution vector**. This is what we would expect the distribution to be in the long run for a Markov process.
- A Markov process will have a steady-state distribution if powers of the transition matrix are approaching some **steady-state matrix**. So as m gets bigger, T^m is getting closer and closer to the steady-state matrix.
- A stochastic matrix T is a **regular Markov chain** if the powers of T

$$T, T^2, T^3, \ldots$$

  approach a steady-state matrix in which all the entries are positive and within every row the entries are equal.
- **A stochastic matrix T is regular if some power of T has entries that are all positive.**
- If T is a regular stochastic matrix, then the steady-state distribution vector X can be found by solving the matrix equation

$$TX = X$$

  along with the condition that the sum of the entries of X must be 1.