Math 141 Key Topics: 8.1-8.3

Section 8.1

- A **random variable** is a rule that assigns a number to each outcome of an experiment.
- A random variable is called **finite discrete** if it takes on only finitely many values.
- A random variable is called **infinite discrete** it it takes on infinitely many values which can be arranged in some order or sequence.
- A random variable is called **continuous** if the values that it can take on make up an interval of numbers. (Examples are time, distance, or measurements.)
- A **histogram** is a visual representation of the probability distribution of a random variable. The sum of the areas (and heights) of all the rectangles in a histogram equals 1.

Section 8.2

- The **mean** or **average** of a set of numbers $x_1, x_2, \ldots, x_n$, denoted $\overline{x}$ is

$$\overline{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

- The **median** of a set of numbers is the number that is in the middle when they are arranged in increasing order if there are an odd number of entries. If there are an even number of entries, the median is the average of the two middle values.
- The **mode** of a set of numbers is the number that occurs the most frequently.
- If X is a random variable that takes on the values $x_1, x_2, \ldots, x_n$ with corresponding probabilities $p_1, p_2, \ldots, p_n$, then the **expected value** of X, denoted $E(X)$ is

$$E(X) = x_1p_1 + x_2p_2 + \cdot + x_np_n$$

  Visually, the expected value of a random variable is the point along the x-axis where the histogram of X would be balanced.
- A game is said to be **fair** if the expected value of the net winnings is 0.
- Converting probability to odds: If $P(E)$ is the probability of an event occurring, then
  - The odds in favor of E occurring are $\frac{P(E)}{P(E^c)}$.
  - The odds against E occurring are $\frac{P(E^c)}{P(E)}$.

  Note: If the odds of an event are $\frac{a}{b}$, we say the odds are a to b.
- Converting odds to probability: If the odds in favor of E occurring are a to b, then the probability of E occurring is

$$P(E) = \frac{a}{a+b}$$

Section 8.3

- The **variance** of a random variable, $Var(X)$, is a measure of the spread of the distribution about the expected value (or mean).

- The **standard deviation** of a random variable is another measure of spread. The standard deviation, σ, is defined to be

$$\sigma = \sqrt{Var(X)} \text{ which means that } \sigma^2 = Var(X)$$

Note: Not all instructors may cover the following topic.

- If X is a random variable with expected value μ and standard deviation σ, then **Chebychev's Inequality** says that

$$P(\mu - k\sigma \leq X \leq \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

What does this mean? Chebychev's Inequality allows us to estimate the probability that an outcome lies within k standard deviations of the mean (expected value). k standard deviations below the mean is $\mu - k\sigma$ and k standard deviations above the mean is $\mu + k\sigma$. The value $1 - \frac{1}{k^2}$ is the estimate used.