Name:____________________________________ MATH 141, Section:________________

Homework Key: Chapter 2, Sections 1-3

1. Determine the value of k for which the following system of equations has no solution.

   $3x - 5y = 2$
   $4x + ky = 4$

   If a system of two lines has no solution, this means the lines are parallel and must have the same slope with different y-intercepts.

   The slope of the first line is: $\frac{3}{5}$
   The slope of the second line is: $-\frac{4}{k}$

   Setting these equal and solving for k yields: $\boxed{k = -\frac{20}{3}}$

   You can double check that if $k = -\frac{20}{3}$, you do get different y-intercepts.

Define variables and SET UP a system of equations to solve each of the following word problems. DO NOT SOLVE them.

2. (Tan 2.1 #16) Michael has a total of $2000 on deposit with two savings institutions. One pays interest at the rate of 6%/yr, whereas the other pays interest at a rate of 8%/yr. If Michael earned a total of $144 in interest during a single year, how much does he have on deposit in each institution?

   x =amount of money deposited at 6%
   y =amount of money deposited at 8%

   $x + y = 2000$
   $0.06x + 0.08y = 144$

3. (Tan 2.1 #24) A theater has a seating capacity of 900 and charges $4 for children, $6 for students, and $8 for adults. At a certain screening with full attendance, there were half as many adults as children and students combined. The receipts totaled $5600. How many children, students, and adults attended the show?

   x =number of children who attended the show
   y =number of students who attended the show
   z =number of adults who attended the show

   $x + y + z = 900$
   $z = \frac{1}{2}(x + y)$
   $4x + 6y + 8z = 5600$

4. John has money invested in 3 accounts that have annual interest rates of 6%, 8%, and 12%. The total return on his investment in one year (i.e. total interest) amounted to $2580. The total amount of money invested in the 8% and 12% accounts was 3 times the amount of money invested in the 6% account. Also, there was twice as much *interest* earned from the 12% account as from the 8% account. How much money did John invest in each account?

   x =amount of money invested in 6% account
   y =amount of money invested in 8% account
   z =amount of money invested in 12% account

   $0.06x + 0.08y + 0.12z = 2580$
   $y + z = 3x$
   $0.12z = 2(0.08y)$

5. (Tan 2.2 #26) Use the Gauss-Jordan Elimination Method to finish reducing the following augmented matrix into reduced row echelon form.

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & -5 \\ 0 & -3 & 3 & 2 \\ 0 & 4 & -1 & 3 \end{array}\right] \; -\frac{1}{3}R_2 \to R_2 \left[\begin{array}{ccc|c} 1 & 2 & 3 & -5 \\ 0 & 1 & -1 & -\frac{2}{3} \\ 0 & 4 & -1 & 3 \end{array}\right] \; -2R_2 + R_1 \to R_1 \left[\begin{array}{ccc|c} 1 & 0 & 5 & -\frac{11}{3} \\ 0 & 1 & -1 & -\frac{2}{3} \\ 0 & 4 & -1 & 3 \end{array}\right]$$

$$-4R_2+R_3 \to R_3 \left[\begin{array}{ccc|c} 1 & 0 & 5 & -\frac{11}{3} \\ 0 & 1 & -1 & -\frac{2}{3} \\ 0 & 0 & 3 & \frac{17}{3} \end{array}\right] \; \frac{1}{3}R_3 \to R_3 \left[\begin{array}{ccc|c} 1 & 0 & 5 & -\frac{11}{3} \\ 0 & 1 & -1 & -\frac{2}{3} \\ 0 & 0 & 1 & \frac{17}{9} \end{array}\right] -5R_3+R_1 \to R_1 \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{118}{9} \\ 0 & 1 & -1 & -\frac{2}{3} \\ 0 & 0 & 1 & \frac{17}{9} \end{array}\right]$$

$$R_3 + R_2 \to R_2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{118}{9} \\ 0 & 1 & 0 & \frac{11}{9} \\ 0 & 0 & 1 & \frac{17}{9} \end{array}\right]$$

6. Solve the following systems of equations. If there are infinitely many solutions, write a parameterized form of the solution. Note: You do not need to use Gauss-Jordan row operations.

(a)
$$\begin{aligned} 2x &= 3y - 4z \\ 2z &= -5y \\ x + y - 4z &= 5 \end{aligned} \quad \to \quad \begin{aligned} 2x - 3y + 4z &= 0 \\ 5y + 2z &= 0 \\ x + y - 4z &= 5 \end{aligned} \quad \to \left[\begin{array}{ccc|c} 2 & -3 & 4 & 0 \\ 0 & 5 & 2 & 0 \\ 1 & 1 & -4 & 5 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{13}{7} \\ 0 & 1 & 0 & \frac{2}{7} \\ 0 & 0 & 1 & -\frac{5}{7} \end{array}\right]$$

Solution: $\boxed{x = \frac{13}{7}, y = \frac{2}{7}, z = -\frac{5}{7} \quad \text{OR} \quad \left(\frac{13}{7}, \frac{2}{7}, -\frac{5}{7}\right)}$

(b)
$$\begin{aligned} x_1 + 2x_2 - 5x_3 &= 12 \\ 2x_1 + 4x_2 - 6x_3 &= 8 \\ x_1 - 7x_2 + x_3 &= 4 \\ -3x_1 - 6x_2 + 9x_3 &= -1 \end{aligned} \quad \to \left[\begin{array}{ccc|c} 1 & 2 & -5 & 12 \\ 2 & 4 & -6 & 8 \\ 1 & -7 & 1 & 4 \\ -3 & -6 & 9 & -1 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$\boxed{\text{No Solution}}$ since the last row yields the equation $0 = 1$, a contradiction.

(c)
$$\begin{aligned} x + 2y + 2z &= -2 \\ -2x - 3y - 2z &= 1 \\ 2x + 4y + 4z &= -4 \end{aligned} \quad \to \left[\begin{array}{ccc|c} 1 & 2 & 2 & -2 \\ -2 & -3 & -2 & 1 \\ 2 & 4 & 4 & -4 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 4 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Rewriting each row as an equation yields: $x - 2z = 4$, $y + 2z = -3$, and $0 = 0$

There is no contradiction and there is not a single value for each variable. Thus, there are infinitely many solutions and we need to parameterize the solution.

Let $z = t$, where t is any real number. Substituting this in and solving for the other variables, we get $x = 4 + 2t$ and $y = -3 - 2t$.

Solution: $\boxed{x = 4 + 2t, y = -3 - 2t, z = t \quad \text{OR} \quad (4 + 2t, -3 - 2t, t)}$

(d)
$$\begin{aligned} 3x - 4y &= 6 \\ x + 2y &= 2 \\ 5x - y &= 10 \end{aligned} \quad \to \left[\begin{array}{cc|c} 3 & -4 & 6 \\ 1 & 2 & 2 \\ 5 & -1 & 10 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Rewriting each row as an equation yields: $x = 2, y = 0$, and $0 = 0$.

There is no contradiction, but there IS a single value for each variable.

Solution: $\boxed{x = 2, y = 0 \quad \text{OR} \quad (2, 0)}$

(e) $\begin{array}{l} x+6y+4z=2 \\ x+3y+2z=1 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 6 & 4 & 2 \\ 1 & 3 & 2 & 1 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{2}{3} & \frac{1}{3} \end{array}\right]$

Rewriting each row as an equation yields: $x = 0$ and $y + \frac{2}{3}z = \frac{1}{3}$

There is no contradiction and there is not a single value for each variable. So there are infinitely many solutions.

Let $z = t$, where t is any real number.

Solution: $\boxed{x = 0, y = \frac{1}{3} - \frac{2}{3}t,\ z = t \quad \text{OR} \quad \left(0, \frac{1}{3} - \frac{2}{3}t, t\right)}$

7. Suppose the solution to a system of equations has parameterized form $(-2t + 7, 5t - 12, t)$.

   (a) What is the specific solution corresponding to $t = -2$?

   $(-2(-2) + 7, 5(-2) - 12, -2) = \boxed{(11, -22, -2)}$

   (b) Is $(5, -7, -1)$ a specific solution to the system?

   If this were a solution, then $t = -1$. However, the specific solution corresponding to $t = -1$ is $(9, -17, -1)$.

   Answer: $\boxed{\text{NO}}$