1. Find the equation of the line that is parallel to the line $2x - 3y = 12$ and passes through the $x$-intercept of the line $y = -7x + 6$.

Parallel implies same slope:

$2x - 3y = 12$

$-3y = -2x + 12$

$y = \frac{-2x}{-3} + \frac{12}{-3}$

$y = \frac{2}{3}x - 4$

$m = \frac{2}{3}$

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{2}{3}(x - \frac{6}{7})$

$y = \frac{2}{3}x - \frac{4}{7}$

$y = -7x + 6$

Set $y=0$ to find $x$-int.

$0 = -7x + 6$

$-6 = -7x$

$x = \frac{6}{7}$

$\left(\frac{6}{7}, 0\right)$
2. Find the value of \( n \) that makes the following lines perpendicular.

\[ 4x - 9y = -7 \]
\[ 6x + ny = 10 \]

\[ m_1 = -\frac{4}{9} \]

\[ 6x + ny = 10 \]
\[ ny = -6x + 10 \]
\[ y = \frac{-6}{n}x + \frac{10}{n} \]

\[ m_2 = \frac{-6}{n} \]

\[ 4 \times \frac{4}{9} \times \frac{n}{6} = 9n \]
\[ 24 = 9n \]
\[ \frac{8}{3} = \frac{24}{9} = n \]
3. A keychain company incurs fixed costs of $1500 every month and it costs them $2 to make each keychain. When 300 keychains are sold, the company earns revenue of $1800.

(a) What is the selling price of a keychain?

\[ c = \text{cost to make one item} = 2 \]
\[ f = \text{fixed costs} = 1500 \]
\[ s = \text{selling price} \]
\[ r(x) = sx \]
\[ r(300) = 1800 \]

(b) What is the profit function for this company?

\[ p(x) = r(x) - c(x) = 6x - (2x + 1500) \]
\[ = 4x - 1500 \]

(c) What is the break-even point?

\[ c(x) = r(x) \]
\[ 2x + 1500 = 6x \]
\[ 1500 = 4x \]
\[ x = 375 \text{ keychains} \rightarrow \text{break-even quantity} \]
\[ r(375) = 375 \cdot 6 = 2250 \rightarrow \text{break-even revenue} \]

(375, 2250)
4. A wilderness store finds that when the price of a tent is $53, 176 people are willing to buy it, but the supplier is only willing to supply 152 tents. When the price increases by $8, only 112 people are willing to buy the tent, but the supplier is willing to supply 32 more tents than at the $53 price.

\[ (x, p) \rightarrow (\text{quantity}, \text{price}) \]

(a) Find the supply and demand equations.

**Demand**

\[
\begin{align*}
(176, 53) & \quad \text{m = 112 - 176} \\
(112, 61) & = -\frac{1}{8} \\
p - p_1 &= m(x - x_1) \\
p - 53 &= -\frac{1}{8}(x - 176) \\
p &= -\frac{1}{8}x + 75
\end{align*}
\]

(b) What is the market equilibrium (equilibrium point)?

\[ \text{intersection: (160, 55)} \]

\[ \text{supply} = \text{demand} \]

\[ \text{equilibrium quantity} \]

\[ \text{equil. price} \]

**Supply**

\[
\begin{align*}
(152, 53) & \quad 152 + 32 = 184 \\
(184, 61) & \quad \text{m = 61 - 53} \\
 & = \frac{8}{32} = \frac{1}{4} \\
p - 53 &= \frac{1}{4}(x - 152) \\
p &= \frac{1}{4}x + 15
\end{align*}
\]
5. Suppose data was gathered of how many McDonald’s (in thousands) there were in Texas from 2002-2006. The results are given in the table below.

<table>
<thead>
<tr>
<th>Year</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>McDonald’s</td>
<td>9</td>
<td>11</td>
<td>13</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>

(a) Find the least-squares line for this data where \( x \) is the number of years since 2002 and \( y \) is the number of McDonald’s in thousands:

\[
y = 0.3x + 10.2
\]

(b) Predict how many McDonald’s there will be in Texas in 2009.

\[
y = 0.3(7) + 10.2 = 12.3
\]

(c) In what year would you expect there to be 15000 McDonald’s in Texas?

\[
15 = 0.3x + 10.2 \quad y = 15
\]

\[
x = 16 \Rightarrow 2002 + 16 = 2018
\]

(d) How well does the line fit the data?

\[
 r = 0.3198
\]

\( |r| \) is not very close to 1 ⇒ line not very good.
6. For the following augmented matrices, state whether the matrix is in row-reduced form or not. If the matrix is\textbf{ NOT} in row-reduced form, state the \textbf{NEXT} row operation needed to get the matrix in row-reduced form.

(a) \[
\begin{bmatrix}
1 & 0 & 4 & 5 \\
0 & 1 & -3 & 6 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[\text{YES ; NO solutions}\]

(b) \[
\begin{bmatrix}
1 & 9 & 0 & 6 \\
0 & 1 & 0 & 7 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[\text{NOT RR ; row changing}\]

(c) \[
\begin{bmatrix}
1 & 0 & -7 \\
0 & 1 & 8 \\
0 & 0 & 0
\end{bmatrix}
\]

\[\text{YES ; Exactly 1 soln.} (-7, 8)\]
\[
\begin{bmatrix}
1 & 0 & 1 & | & 6 \\
0 & 1 & 2 & | & 5
\end{bmatrix}
\]

NO; Change 1 into a 0

\[
\begin{bmatrix}
0 & 0 & 1 & | & 4 \\
0 & 3 & 9 & | & 0
\end{bmatrix}
\]

NO; Make 3 into a 1

Yes; Inf. Many Soln's.
7. Use the Guass-Jordan method to get the following matrix in row-reduced form.

\[
\begin{bmatrix}
4 & 8 & 32 \\
-3 & -5 & -18 \\
\end{bmatrix}
\]

\[
\begin{align*}
\text{R}_1 & \rightarrow \begin{bmatrix} 1 & 2 & 8 \\ -3 & -5 & -18 \end{bmatrix} \\
\text{R}_2 + 3\text{R}_1 & \rightarrow \begin{bmatrix} 1 & 2 & 8 \\ 0 & 1 & 6 \end{bmatrix} \\
\text{R}_1 - 2\text{R}_2 & \rightarrow \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 6 \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\text{R}_2 & \text{: -3 -5 -18} \\
\text{+3R}_1 & \text{: 3 6 24} \\
\text{R}_2 & \text{: 0 1 6}
\end{align*}
\]

\[
\begin{align*}
x & = -4 \\
y & = 6 \\
\end{align*}
\]
8. Kristina drinks both Coke and Pepsi. A 20 oz bottle of Coke contains 58 mg of Caffeine and 243 calories. A 20 oz bottle of Pepsi contains 63 mg of Caffeine and 250 calories. In a given week, Kristina drinks enough Cokes and Pepsis to take in 1326 mg of Caffeine and 5416 calories. How many Cokes and Pepsis did Kristina drink that week?

Let \( x = \# \text{Cokes she drank} \)
\( y = \# \text{Pepsis she drank} \)

\begin{align*}
\text{Caf} & \quad \text{Cal} \\
\hline
\text{Coke} \quad 58 & \quad 243 \\
\text{Pepsi} \quad 63 & \quad 250 \\
\end{align*}

\[
\begin{align*}
58x + 63y &= 1326 \\
243x + 250y &= 5416
\end{align*}
\]

\[
\begin{bmatrix}
58 & 63 & 1326 \\
243 & 250 & 5416
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & 12 \\
0 & 1 & 10
\end{bmatrix}
\]

\[
x = 12, \quad y = 10
\]

12 Cokes; 10 Pepsis
9. Solve the following systems of equations. If there are infinitely many solutions, make sure to parameterize the solution.

(a) \[10x - 8y = 6\]
\[-15x + 12y = 20\]  \[\rightarrow\]  \[
\begin{bmatrix}
10 & -8 \\
-15 & 12
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
6 \\
20
\end{bmatrix}
\]
\[\text{rref} \rightarrow \begin{bmatrix}
1 & -4/5 \\
0 & 1
\end{bmatrix}
\]
\[\rightarrow \text{NO solution}\]

(b) \[2x + y + z = 5\]
\[4x + 6z = 14 + 2y\]
\[-6x + 3y - 9z = -21\]
\[4x - 2y + 6z = 14\]  \[\rightarrow\]  \[
\begin{bmatrix}
2 & 1 & 1 \\
4 & -2 & 6 \\
-6 & 3 & -9 \\
4 & -2 & 6
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= 
\begin{bmatrix}
5 \\
14 \\
-21 \\
14
\end{bmatrix}
\]
\[\text{rref} \rightarrow \begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & -1 \\
0 & 0 & 0
\end{bmatrix}
\rightarrow \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= \begin{bmatrix}
3 \\
-1 \\
0
\end{bmatrix}
\]

\[\text{Inf. Many Soln's}\]
\[\text{Let } z = t, \text{ where } t \text{ is any #.}\]
\[x + t = 3 \rightarrow x = 3 - t\]
\[y - t = -1 \rightarrow y = -1 + t\]
\[(3 - t, -1 + t, t)\]
Section 2.4

- A matrix with \( m \) rows and \( n \) columns has size \( m \times n \). \( a_{ij} \) is the entry in the \( i \)th row and \( j \)th column of the matrix.

- In order to add or subtract matrices, they must have the same size. To multiply a matrix by a scalar, multiply each entry by this scalar.

- The transpose of a matrix \( A \), denoted \( A^T \), is found by making all the rows of \( A \) the columns of \( A^T \). (Just interchange rows and columns.)

\[
\begin{array}{ccc}
\text{A} & \text{B} \\
\hline
m \times n & \text{r} \times s \\
(\text{n} \times \text{r})
\end{array}
\]

Section 2.5

- In order for the matrix multiplication \( AB \) to make sense, the number of columns of \( A \) must equal the number of rows of \( B \). In other words, if \( A \) has size \( m \times n \) and \( B \) has size \( r \times s \), then \( n \) must equal \( r \). Then, the resulting product matrix will have size \( m \times s \).

- To multiply \( AB \) by hand, move across the rows of \( A \) as you move down the columns of \( B \).

- Matrix multiplication is not commutative: \( AB \neq BA \).

- An identity matrix \( I_n \) has 1’s along the diagonal and 0’s everywhere else.

- A system of equations can be written as a matrix equation \( AX = B \). ,

Section 2.6

- The inverse of a matrix \( A \), denoted \( A^{-1} \), is the matrix such that \( AA^{-1} = A^{-1}A = I_n \).

- Not all matrices have inverses. Those that have inverses are called nonsingular. Those that do NOT have inverses are called singular.

- The solution to the matrix equation \( AX = B \) is \( X = A^{-1}B \), if \( A \) has an inverse.
10. Solve for the variables in the following equation.

\[
3 \begin{bmatrix} 1 & -2 \\ -4 & 9 \end{bmatrix} + \begin{bmatrix} -5 & 2b \\ a & 6 \end{bmatrix} = \begin{bmatrix} -1 & c+1 \\ 20 & -2 \end{bmatrix} \]

\[
\begin{bmatrix} 9 \\ 4 \\ 1 \end{bmatrix} + \begin{bmatrix} -5 \\ a \\ 6 \end{bmatrix} = \begin{bmatrix} c+1 \\ 20 \\ 18d \end{bmatrix}
\]

\[
\begin{bmatrix} 3 \\ 12 \\ 8 \end{bmatrix} + \begin{bmatrix} -5 \\ a \\ 6 \end{bmatrix} = \begin{bmatrix} c+1 \\ 20 \\ 18d \end{bmatrix}
\]

\[
\begin{bmatrix} 22 \\ 6+2b \\ 12+a \end{bmatrix} = \begin{bmatrix} c+1 \\ -2 \\ 20 \end{bmatrix}
\]

\[
\begin{align*}
22 &= c+1 \\
6+2b &= -2 \\
b &= -4
\end{align*}
\]

\[
\begin{align*}
12+a &= 20 \\
a &= 8
\end{align*}
\]

\[
\begin{align*}
d &= \frac{1}{2}
\end{align*}
\]
11. John is throwing a Super Bowl party and wants to order pizzas. He finds that Papa John’s charges $6 for a small pizza, $9 for a medium pizza, and $12 for a large pizza. Pizza Hut charges $5 for a small pizza, $10 for a medium pizza, and $13 for a large pizza. This information is summarized in the following matrix.

\[
A = \begin{bmatrix}
6 & 5 \\
9 & 10 \\
12 & 13
\end{bmatrix}
\]

If John wants to order 5 small, 3 medium, and 6 large pizzas, find a matrix B so that when multiplied with A will give the total cost of his order if he orders from Papa John’s or if he orders from Pizza Hut. Then do the matrix multiplication.

2 more options: \(AB\) or \(BA\).

Case 1: \(B = [5, 5, 5] \quad AB\)

Case 2: \(B = \frac{5}{6} \quad BA\)

Case 3: \(B = \begin{bmatrix} 5 \\ 3 \\ 6 \end{bmatrix} \quad AB\)

Case 4: \(B = \begin{bmatrix} 5 \\ 3 \\ 6 \end{bmatrix} \quad BA\)

\[BA\] will have size \(1 \times 2\).

\[
B = \begin{bmatrix}
5 & 3 & 6 \\
S & M & L
\end{bmatrix}
\]

\[
A = \begin{bmatrix}
6 & 5 \\
9 & 10 \\
12 & 13
\end{bmatrix}
\]

\[
PJ \quad PH
\]

\[
\begin{bmatrix}
129 \\ 133
\end{bmatrix}
\]

Total Cost: $129 if he orders from PJ
$133 if he orders from PH
12. Given the following matrices and their sizes, determine whether the matrix operations are possible or not possible.

\[ A_{2\times 4} \quad B_{3\times 2} \quad C_{4\times 3} \quad D_{3\times 3} \quad E_{2\times 3} \]

(a) \(4AC + E\)  
\[ \begin{align*} 2\times 4 \quad 4\times 3 &+ 2\times 3 \\ 2\times 3 &+ 2\times 3 \end{align*} \]  
POSSIBLE

(b) \(BA + C\)  
\[ \begin{align*} 2\times 3 &+ 4\times 3 \\ 3\times 4 &+ 4\times 3 \end{align*} \]  
NOT POSS.

(c) \(AT - 7C\)  
\[ \begin{align*} 2\times 3 &- 4\times 3 \\ 4\times 3 &- 4\times 3 \end{align*} \]  
\[ \text{BED} \quad \rightarrow \quad \text{a 3x3 matrix} \]  
POSS.

(d) \(BET\)  
\[ \begin{align*} 3\times 2 &\quad 2\times 3 \quad 3\times 3 \end{align*} \]  
NOT POSS.

(e) \(DB^T\)  
\[ \begin{align*} 3\times 3 &\quad (3\times 2)^T \\ 3\times 3 &\quad 2\times 3 \end{align*} \]  
\[ \text{F} \]  
NOT POSS.
13. Suppose there are 3 major airlines that fly out of Houston: Southwest, Delta, and American. A Southwest flight holds 135 people, a Delta flight holds 160 people, and an American flight holds 148 people. On a given day, a total of 88,683 people flew out of Houston. There were 3 times as many Southwest flights as Delta flights, and the number of American flights was half the number of Southwest and Delta flights combined. How many flights out of Houston were there for each airline?

(a) Set up this problem.

Let \( x \) = # Southwest flights
\( y \) = # Delta flights
\( z \) = # American flights.

\[ 135x + 160y + 148z = 88,683 \]

\[ x = 3y \]

\[ z = \frac{1}{2}(x + y) = \frac{1}{2}x + \frac{1}{2}y \]

\[ z - \frac{1}{2}x - \frac{1}{2}y = 0 \]
(b) Write the system of equations as a matrix equation $AX = B$.

\[
\begin{bmatrix}
135 & 160 & 148 \\
1 & -3 & 0 \\
-\frac{1}{2} & 1 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= 
\begin{bmatrix}
88683 \\
0 \\
0
\end{bmatrix}
\Rightarrow
AX = B
\]

\[
A^{-1}AX = A^{-1}B
\]

\[
IX = A^{-1}B
\]

\[
X = A^{-1}B
\]

\[
\begin{bmatrix}
135 & 160 & 148 \\
1 & -3 & 0 \\
-\frac{1}{2} & 1 & 1
\end{bmatrix}^{-1}
\begin{bmatrix}
88683 \\
0 \\
0
\end{bmatrix}
= 
\begin{bmatrix}
309 \\
103 \\
206
\end{bmatrix}
\]

There were 309 SW flights; 103 D flights and 206 A flights.
Section 2.7

- Leontief Input-Output models involve 3 matrices:
  - A – This is the **input-output matrix**. It tells you how much input or consumption is required to produce 1 unit of output.
  - $\mathbf{X}$ – This is the **gross production matrix**. It gives the total output of the economy.
  - D – This is the consumer demand matrix.

- $\mathbf{AX} = \mathbf{D}$ (total output - internal consumption = consumer demand). The solution to this matrix equation is $\mathbf{X} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{D}$. 

14. (Note: This problem is from 2.7 which all instructors may not cover.) A simple economy consists of 3 industries: Agriculture, Transportation, and Service. The production of 1 unit of agricultural products requires the consumption of 0.2 units of agricultural products, 0.1 units of transportation, and 0.1 units of service goods. The production of 1 unit of transportation requires the consumption of 0.3 units of agricultural products, 0.4 units of transportation, and 0.3 units of service. Finally, the production of 1 unit of service goods requires the consumption of 0.2 units of agricultural products, 0.1 units of transportation, and 0.3 units of service.

(a) Find the Input-Output matrix $A$.

$$A = \begin{bmatrix} 0.2 & 0.3 & 0.2 \\ 0.1 & 0.4 & 0.1 \\ 0.1 & 0.3 & 0.3 \end{bmatrix}$$
(b) Find the **gross output** of goods needed to satisfy consumer demand for 790 units of agricultural products, 445 units of transportation, and 205 units of service goods.

\[ X = (I - A)^{-1} D \]

\[ X = A \begin{bmatrix} 1700 \\ 1200 \\ 1050 \end{bmatrix} \]

\[ T \begin{bmatrix} 790 \\ 445 \\ 205 \end{bmatrix} \]

(c) Find the value of goods consumed in the internal process of production in order to meet the above gross output.

\[ AX = X - D \]

\[ X - AX = D \]

\[ -AX = -X + D \]

\[ AX = X - D \]