

1.6 Modeling with Equations

Steps to Modeling Problems with Equations

1. Identify the variable you want to solve for.
2. Express all unknown quantities in terms of this variable.
3. Set up the model by finding the equation that relates all the expressions you found in Step 2.
4. Solve the equation, and check to make sure your answer makes sense.

Examples

- There are two trees in a field. The taller tree casts a 150 ft shadow. The shorter tree is 15 ft tall and the taller tree is 90 ft tall. How long is the shadow of the shorter tree?

- (1.6 #52) Suppose I have a 60% acid solution and I want to mix it with a 30% solution to produce 300 mL of a 50% solution for an experiment. How much 60% acid solution must I use?

1.7 Inequalities

Inequalities tell us when something is $>$, $<$, \geq , or \leq to something else. They give us a notion of order in math.

With inequalities, adding or subtracting the same thing from both sides does not change the inequality sign.

If we multiply by a positive number on both sides, the inequality does not change.

However, if we multiply by a negative number on both sides, or if we take reciprocals, the inequality sign reverses. (If it was \geq it becomes \leq , etc.)

Inequalities can be added. In other words, if $A \leq B$ and $C \leq D$, then $A + C \leq B + D$.

Solving linear inequalities

To solve a linear inequality, isolate the variable by using basic operations and remember when the sign reverses.

Examples

- $-1 < 4 - 3x \leq 7$

To solve a nonlinear inequality:

1. Move ALL terms to one side, find common denominator, and factor.
2. Determine the values for which each factor is zero. These values divide the real line into intervals.
3. Use a sign chart to determine the sign of the expression along the intervals.

- $x^2 - x \geq 20$

- (1.7 #59)

$$\frac{x+2}{x+3} < \frac{x-1}{x-2}$$

To solve an absolute value inequality, use the following properties:

1. $|x| \leq c$ means $-c \leq x \leq c$
2. $|x| \geq c$ means $x \geq c$ or $x \leq -c$

The same is true for strict inequalities ($<$ or $>$).

- $|5x - 2| < 6$

- $|x + 4| + 1 \geq 4$

1.8 Coordinate Geometry

Things to remember/review: Cartesian plane, coordinates, quadrants, how to graph regions in the coordinate plane.

Distance Formula:

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

The **midpoint** of the line segment from A to B is:

Consider the points $(-2, 3)$ and $(4, 7)$. Find the distance between these two points and the midpoint.

You can sketch a graph by plotting points, although this is not always the best way. See the textbooks for examples of this.

Sketch the region given by the set $\{(x, y) \mid y < 4 \text{ and } |x| \leq 5\}$

The x -intercepts of a graph are where a graph crosses the x -axis. Set _____ to find the x -intercepts.

The y -intercepts are where the graph crosses the y -axis. Set _____ to find the y -intercepts.

Find the x and y -intercepts of the following graphs.

$$x + y^2 = 4$$

$$y = 5x^2 + 34x - 7$$

The **equation of a circle** with center (h, k) and radius r is

This is called the **standard form** for the equation of a circle.

Example: Find the equation of the circle with center $(-1, 6)$ and passing through the point $(3, 3)$.

It is often necessary to complete the square to find the standard form for the equation of a circle.

Example (1.8, #89) Show that the equation represents a circle, and find the center and radius of the circle.

$$x^2 + y^2 - 4x + 10y + 13 = 0$$

Symmetry

- **Symmetry with respect to the x -axis.** The graph is unchanged when reflected across the x -axis. The equation is unchanged when we replace y by $-y$. Example: $x = y^2$
- **Symmetry with respect to the y -axis.** The graph is unchanged when reflected across the y -axis. The equation is unchanged when we replace x by $-x$. Example: $y = x^2$
- **Symmetry with respect to the origin.** The graph is unchanged when rotated 180 degrees about the origin. The equation is unchanged when x is replaced by $-x$ AND y is replaced by $-y$. Example: $y = x^3$

Check for symmetry in the following equations. (1.8, #56, 72, 75)

$$y = x^2 + 2$$

$$x = y^4 - y^2$$

$$y = x^3 + 10x$$

1.9 Solving Equations and Inequalities Graphically on a Calculator

We've already learned algebraic ways to solve equations and inequalities, but we can also solve them graphically.

To solve an equation graphically, you can either:

- Move all terms to one side, graph the equation and find the x -intercepts. OR
- Graph both sides of the equal sign (if one side is not 0), and then find where the graphs intersect.

When using the calculator, be careful that you set an appropriate window to be able to see what you need to find.

Examples: Solve the following equations graphically. Round answers to 4 decimal places.

- $\frac{2}{x+1} + \frac{1}{2(x+1)} = 7$

- $x^{1/5} = x^2 - x^{1/3} - 5$

To solve an inequality graphically, you can use either method from above, except your answer will be the intervals where the inequality is true.

Example: $\sqrt{0.5x^2 + 1} > x^3 - 4x^2 - 11x + 30$

1.10 Lines

If (x_1, y_1) and (x_2, y_2) are two points on a line, the **slope** of the line is defined to be:

Point-Slope Form

An equation of the line that passes through the point (x_1, y_1) with slope m is:

Find an equation of the line passing through the points $(2, -1)$ and $(-3, 4)$.

Slope-Intercept Form

An equation of the line with slope m and y -intercept b is:

The **General Equation of a Line** is:

An equation of the vertical line through (a, b) is _____

An equation of the horizontal line through (a, b) is _____

Two nonvertical lines are parallel if _____

Two lines are perpendicular if _____

Find an equation of the line with x -intercept -2 and parallel to the line $y = -4x - 9$.

Find an equation of the line passing through the point $(-1, 2)$ and perpendicular to the line $3x - 2y = 9$.

Example (1.10 #68) A small business buys a computer for \$4000. After 4 years the value of the computer is expected to be \$200. This business uses *linear* depreciation to assess the value of the computer V at any given time t .

1. Find the line that relates V in terms of t .
2. What do the slope and V -intercept of the graph represent?
3. When will the computer have no value?