9.1 Systems of Equations

A **system of equations** is a set of equations that all involve the same variables. A solution of a system is an assignment of values to each variable that makes EVERY equation in the system true.

There are 3 methods of solving systems of equations: Substitution, Elimination, and Graphing.

Substitution Method

1. Solve for one variable in one of the equations.
2. Substitute this expression into the other equation and solve for that variable.
3. Back-substitute back into the first equation.

Example: Solve this system of equations using substitution.
$x^2 + y = 9$
$x - y + 3 = 0$

Elimination Method

1. Adjust the coefficients: Multiply one or more of the equations by number(s) that will make the coefficient of a term in one equation the negative of the coefficient in the other equation.
2. Add the equations and solve.
3. Back-substitute.

Example: (9.1 #6) Solve this system of equations using the method of elimination.
$2x - 2y^2 = -6$
$6x^2 + 3y^2 = 12$

Example: Solve the following system of equations.
$xy = 12$
$2x^2 - y^2 - 2 = 0$

Graphical Method

1. Solve each equation for y so that you can put them in your calculator.
2. Graph the equations.
3. Find the points of intersection.

Example: (9.1 #41) Solve the following system graphically. Round solutions to 2 decimal places.
$x^2 + y^2 = 25$
$x^2 - y = 2x + 2$

10.1-10.3 Parabolas, Ellipses, and Hyperbolas

We've looked at parabolas before when talking about the graphs of quadratic functions.

Recall that the standard form of a parabola that opens up or down with vertex (h, k) is: $y = a(x - h)^2 + k$.
If $a > 0$, the parabola opens upward. If $a < 0$, the parabola opens downward.

We can also have parabolas that open left or right, even though these will not be functions.

The standard form a parabola that opens left or right with vertex (h, k) is $x = a(y - k)^2 + h$.
If $a > 0$, the parabola opens to the right. If $a < 0$, the parabola opens to the left.

Examples: Sketch the general shape of the following equations.

1. $y = -2(x+3)^2 + 4$

2. $y^2 + 10y = -6x - 13$

An **ellipse** is the set of all points where the sum of the distances from two fixed points F_1 and F_2 is a constant. These two fixed points are called the **foci** (plural of **focus**) of the ellipse.

An ellipse is essentially a circle that has been stretched or shrunk horizontally and/or vertically.

An ellipse will be more elongated vertically or horizontally. An ellipse has a vertical **major axis** if it is more elongated vertically and a horizontal **major axis** if it is more elongated horizontally.

The equation of an ellipse with center (h,k) is: $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$

If $a > b$, the major axis is horizontal. If $a < b$, the major axis is vertical.

Examples: Sketch graphs for the following equations.

1. $4x^2 + 25y^2 = 100$

2. $\dfrac{(x-2)^2}{25} + \dfrac{(y+3)^2}{4} = 1$

3. $9x^2 - 36x + 4(y-6)^2 = 0$

A **hyperbola** is the set of all points where the *difference* of the distances from two fixed points F_1 and F_2 is a constant. These two fixed points are called the **foci** of the hyperbola.

A hyperbola consists of two **branches**. The segment joining the two branches of the hyperbola is called the **transverse axis**. The transverse axis can be vertical or horizontal.

The equation of a hyperbola with center (h, k) and horizontal transverse axis is: $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

The equation of a hyperbola with center (h, k) and vertical transverse axis is: $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$

Examples: Sketch graphs for the following equations.

1. $9x^2 - 4y^2 = 36$

2. $9(x+2)^2 - 4(y-3)^2 = 36$.

3. $5y^2 + 20y - 6x^2 + 50 = 0$