1. Solve the following systems of equations.

(a) $\begin{cases} x^2 y = 20 \\ x^2 + y = 9 \end{cases}$ $\longrightarrow$ $y = \dfrac{20}{x^2}$

$$x^2\left(x^2 + \frac{20}{x^2}\right) = (9)\,x^2$$

$$x^4 + 20 = 9x^2$$

$$x^4 - 9x^2 + 20 = 0$$

$$(x^2 - 4)(x^2 - 5) = 0$$

$$x^2 - 4 = 0 \qquad x^2 - 5 = 0$$

$$x = \pm 2 \qquad x = \pm\sqrt{5}$$

$x = 2$: $y = \dfrac{20}{2^2} = 5$: $(2,5)$

$x = -2$: $y = \dfrac{20}{(-2)^2} = 5$: $(-2,5)$

$x = \sqrt{5}$: $y = \dfrac{20}{(\sqrt{5})^2} = 4$: $(\sqrt{5},4)$

$x = -\sqrt{5}$: $y = \dfrac{20}{(-\sqrt{5})^2} = 4$: $(-\sqrt{5},4)$

$$\boxed{(2,5),\ (-2,5),\ (\sqrt{5},4),\ (-\sqrt{5},4)}$$

(b) $\begin{cases} (7x^2 + 8y = -5)(-4) \longrightarrow & \cancel{-28x^2} - 32y = 20 \\ (4x^2 + 3y = 16)(7) \longrightarrow & \cancel{28x^2} + 21y = 112 \end{cases}$

$$-11y = 132$$

$$y = -12$$

$$7x^2 + 8y = -5$$

$$7x^2 + 8(-12) = -5$$

$$7x^2 - 96 = -5$$

$$7x^2 = 91$$

$$x^2 = 13$$

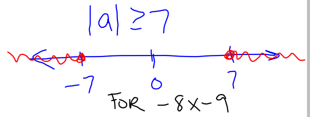
$$x = \pm\sqrt{13}$$

: $\boxed{(\sqrt{13}, -12), (-\sqrt{13}, -12)}$

Title: Dec 1-7:28 PM (2 of 22)

2. Solve the following equations.

(a) $|-8x-9| \geq 7$

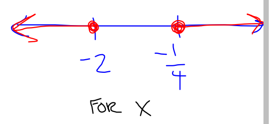


$-8x-9 \leq -7$ OR $-8x-9 \geq 7$

$-8x \leq 2$ OR $-8x \geq 16$

$x \geq \frac{-1}{4}$ OR $x \leq -2$

$$\boxed{(-\infty, -2] \cup [-\tfrac{1}{4}, \infty)}$$



(b) $\sqrt{4-3x} - x = 8$

$$(\sqrt{4-3x})^2 = (x+8)^2$$

$$4-3x = x^2 + 16x + 64$$

$$0 = x^2 + 19x + 60$$

$$0 = (x+15)(x+4)$$

$$x = -15$$
$$x = -4$$

Check: $x=-15$: $\sqrt{4-3(-15)} - (-15)$

$$= \sqrt{4+45} + 15$$

$$= \sqrt{49} + 15 = 7 + 15 = 22 \neq 8$$

$x=-15$ is an extraneous solution.

Check: $x=-4$: $\sqrt{4-3(-4)} - (-4)$

$$= \sqrt{4+12} + 4$$

$$= \sqrt{16} + 4 = 4 + 4 = 8 \checkmark$$

$$\boxed{x=-4}$$

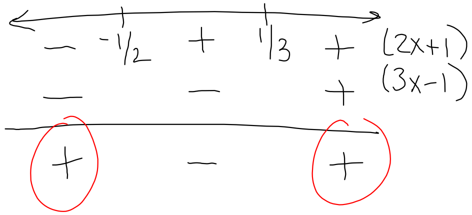
Title: Dec 1-7:28 PM (3 of 22)

3. Find the domains of the following functions.

(a) $\dfrac{\sqrt{6x^2+x-1}}{\sin^{-1}x}$

$6x^2+x-1 \geq 0$

$(2x+1)(3x-1) \geq 0$



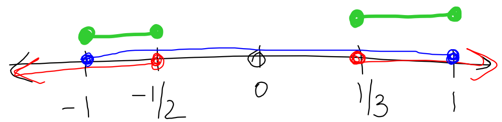
$x \leq -\tfrac{1}{2}$ OR $x \geq \tfrac{1}{3}$

$\sin^{-1}x$ cannot be 0.

$\sin^{-1}x \neq 0$

Since $\sin 0 = 0$, we have to exclude $x=0$.

$\sin^{-1}x$ is only defined on $[-1,1]$. This is its domain.



$$\boxed{[-1,-\tfrac{1}{2}] \cup [\tfrac{1}{3},1]}$$

(b) $\dfrac{\log(-3x+8)}{\sqrt[3]{x^2+3x-28}}$

$-3x+8>0$

$-3x>-8$

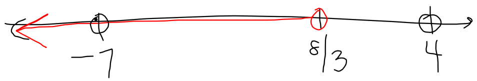
$x<\frac{8}{3}$

$\sqrt[3]{x^2+3x-28} \neq 0$

$x^2+3x-28 \neq 0$

$(x+7)(x-4) \neq 0$

$x \neq -7, 4$



$$\boxed{(-\infty,-7) \cup (-7,\tfrac{8}{3})}$$

4. Find the average rate of change of the function $f(x) = -3x^2 - x + 4$ from $x = -2$ to $x = 3$.

Avg rate of change from $x=a$ to $x=b$:

$$\frac{f(b) - f(a)}{b - a}$$

$$\frac{f(3) - f(-2)}{3 - (-2)} = \frac{\left[-3(3)^2 - 3 + 4\right] - \left[-3(-2)^2 - (-2) + 4\right]}{5}$$

$$= \frac{-26 - (-6)}{5} = \frac{-26+6}{5} = \frac{-20}{5}$$

$$= \boxed{-4}$$

5. Consider the quadratic function $f(x) = 3x^2 + 4x - 1$.

(a) What is the vertex of this parabola?

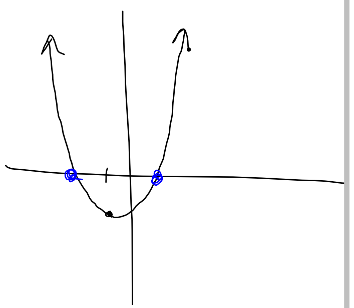
If $f(x) = ax^2 + bx + c$, the vertex is: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$$x = \frac{-4}{2(3)} = \frac{-4}{6} = \frac{-2}{3}$$

$$f\left(-\frac{2}{3}\right) = 3\left(-\frac{2}{3}\right)^2 + 4\left(-\frac{2}{3}\right) - 1 = 3\left(\frac{4}{9}\right) - \frac{8}{3} - 1$$

$$= \frac{4}{3} - \frac{8}{3} - 1 = \frac{4}{3} - \frac{8}{3} - \frac{3}{3} = \frac{-7}{3}$$

$$\boxed{\text{Vertex: } \left(-\frac{2}{3}, -\frac{7}{3}\right)}$$



(b) What are the x-intercepts of this function?

$$0 = 3x^2 + 4x - 1$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{-4 \pm \sqrt{16 + 12}}{6} = \frac{-4 \pm \sqrt{28}}{6} = \frac{-4 \pm 2\sqrt{7}}{6}$$

$$= \frac{-2 \pm \sqrt{7}}{3}$$

$$\text{x-int: } \boxed{\left(\frac{-2 \pm \sqrt{7}}{3}, 0\right)}$$

6. For the functions $f(x) = \dfrac{x}{x-3}$ and $g(x) = \dfrac{5}{x-1}$, find $(f \circ g)(x)$.

$$(f \circ g)(x) = f(g(x))$$

$$= f\left(\frac{5}{x-1}\right) = \frac{\left(\frac{5}{x-1}\right)}{\left(\frac{5}{x-1}\right) - 3} \cdot \frac{x-1}{x-1}$$

$$= \frac{5}{5 - 3(x-1)}$$

$$= \frac{5}{5-3x+3} = \boxed{\frac{5}{8-3x}}$$

7. Find the inverse of the function $f(x) = \dfrac{x-2}{x+1}$.

$$y = \frac{x-2}{x+1}$$

$$x = \frac{y-2}{y+1}$$

$$x(y+1) = y-2$$

$$xy + x = y - 2$$

$$xy - y = -x - 2$$

$$y(x-1) = -x - 2$$

$$y = \frac{-x-2}{x-1}$$

$$\boxed{f^{-1}(x) = \frac{-x-2}{x-1}}$$

8. How would the graph of $g(x) = -\frac{1}{5}f(x-3) + 6$ be obtained from the graph of f?

$f(x)$

$f(x-3)$: Shift right 3

$\frac{1}{5}f(x-3)$: Vertically shrink by a factor of $\frac{1}{5}$.

$-\frac{1}{5}f(x-3)$: Reflect across x-axis.

$-\frac{1}{5}f(x-3)+6$: Shift up 6.

9. Find any asymptotes, intercepts, and holes for the rational function $r(x) = \dfrac{9x^2 - 25}{(3x^2 - 11x + 10)(x + 4)}$.

$$r(x) = \frac{\cancel{(3x-5)}(3x+5)}{\cancel{(3x-5)}(x-2)(x+4)}$$

$$= \frac{3x+5}{(x-2)(x+4)} \; ; \; x \neq \frac{5}{3}$$

$\boxed{\text{Hole at } x = \frac{5}{3}.}$

Vertical Asymptotes: $\boxed{x = 2, \; x = -4}$

Horizontal Asymptotes: Deg Num < Deg Den.

$\boxed{y = 0}$

x-int: $3x + 5 = 0$: $\boxed{\left(-\frac{5}{3}, 0\right)}$

$x = -5/3$

y-int: $f(0) = \dfrac{3(0)+5}{(0-2)(0+4)} = \dfrac{5}{-2(4)} = \dfrac{-5}{8}$

$\boxed{\left(0, -\frac{5}{8}\right)}$

10. Consider the polynomial $P(x) = -3x^3 + 2x^2 + 12x - 8$.

(a) Find the zeros of this polynomial.

$$\underbrace{-3x^3 + 2x^2}\underbrace{+12x - 8} = 0$$

$$x^2(-3x+2) - 4(-3x+2) = 0$$

$$(-3x+2)(x^2-4) = 0$$

$$(-3x+2)(x-2)(x+2) = 0$$

$$\boxed{\text{Zeros: } x = \frac{2}{3},\ 2,\ -2}$$

$-3x+2=0$

$-3x=-2$

$x=\frac{2}{3}$

(b) Find the quotient and remainder when $P(x)$ is divided by $x^2 - 2x + 3$.

$$\begin{array}{r} -3x - 4 \\ x^2-2x+3 \overline{) -3x^3 + 2x^2 + 12x - 8} \\ -(-3x^3 + 6x^2 - 9x) \\ \hline -4x^2 + 21x - 8 \\ -(-4x^2 + 8x - 12) \\ \hline 13x + 4 \end{array}$$

$\frac{-3x^3}{x^2} = -3x$

$\frac{-4x^2}{x^2} = -4$

$$\boxed{Q(x) = -3x - 4,\ R(x) = 13x + 4}$$

11. Perform the multiplication and write in standard form: $(8-\sqrt{-25})(-3+\sqrt{-4})$

$\sqrt{-25} = i\sqrt{25} = 5i$

$\sqrt{-4} = i\sqrt{4} = 2i$

$(8-5i)(-3+2i) = -24+16i+15i-10i^2$

$= -24+31i-10(-1)$

$= -24+31i+10$

$= \boxed{-14+31i}$

12. Solve the following equations.

(a) $3 \cdot 5^{x-2} = 8$

$$5^{x-2} = \frac{8}{3}$$

$$\frac{\log A}{\log B} \neq \log\left(\frac{A}{B}\right)$$

$$\log 5^{x-2} = \log \frac{8}{3}$$

$$(x-2) \cdot \log 5 = \log \frac{8}{3}$$

$$x - 2 = \frac{\log \frac{8}{3}}{\log 5}$$

$$\boxed{x = \frac{\log 8/3}{\log 5} + 2} = \frac{\log 8 - \log 3}{\log 5} + 2$$

(b) $\log_{100} x + \log_{100}(3x - 13) = \frac{1}{2}$

$\log A + \log B = \log(AB)$

$\log_{100} x(3x-13) = \frac{1}{2}$

$\log_a b = x$

$\Longleftrightarrow$

$a^x = b$

$\log_{100}(3x^2 - 13x) = \frac{1}{2}$

$100^{1/2} = 3x^2 - 13x$

$10 = 3x^2 - 13x$

$0 = 3x^2 - 13x - 10$

$0 = (3x+2)(x-5)$

$x = -2/3,\ 5$

Check: $x = -2/3$: $\log_{100}\left(-\frac{2}{3}\right) + \cdots\cdots$

Doesn't make sense.

$x = -2/3$ is extraneous.

$x = 5$: $\log_{100} 5 + \log_{100} 2 = \frac{1}{2}$

$\log_{100} 10 = \frac{1}{2}$ ✓

$\boxed{x = 5}$

13. Rewrite the following expression as a single logarithm:

$$\frac{1}{3}\log p^2 - \frac{3}{4}\log 16p^4 - \frac{2}{3}\log 8(p^3+27)$$

$\log A^c = c\log A$

$\log AB = \log A + \log B$

$\log \frac{A}{B} = \log A - \log B$

$$= \log(p^2)^{1/3} - \log(16p^4)^{3/4} - \log\left[8(p^3+27)\right]^{2/3}$$

$$= \log p^{2/3} - \log\left(\underline{\underline{16^{3/4}}}\,p^3\right) - \log\left[\underline{\underline{8^{2/3}}}(p^3+27)^{2/3}\right]$$

$16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$

$$= \log p^{2/3} - \log 8p^3 - \log 4(p^3+27)^{2/3}$$

$8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$

$$= \log\left(\frac{p^{2/3}}{8p^3\cdot 4(p^3+27)^{2/3}}\right)$$

$\frac{p^{2/3}}{p^3} = p^{2/3-3}$

$= p^{2/3-9/3}$

$= p^{-7/3}$

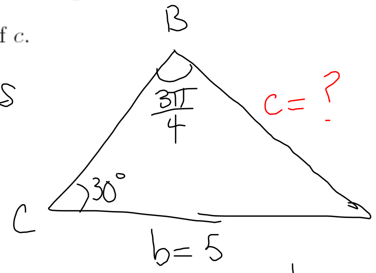
$$= \log\left(\frac{p^{-7/3}}{32(p^3+27)^{2/3}}\right)$$

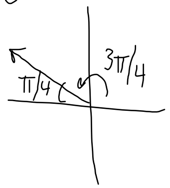
$$= \boxed{\log\left(\frac{1}{32p^{7/3}(p^3+27)^{2/3}}\right)}$$

14. Find the exact value of the indicated part of the triangle from the given information.

(a) Given: $C = 30°$, $B = \frac{3\pi}{4}$, $b = 5$; Find the exact value of c.

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$ Law of Sines





$$\frac{\sin \frac{3\pi}{4}}{5} = \frac{\sin 30°}{c}$$

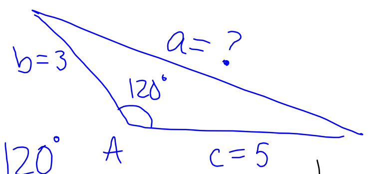
$$\frac{\frac{1}{\sqrt{2}}}{5} = \frac{\frac{1}{2}}{c}$$

$$\frac{1}{\sqrt{2}} c = \frac{5}{2}$$

$$\boxed{c = \frac{5\sqrt{2}}{2}} = \frac{5}{\sqrt{2}}$$

(b) Given: $A = 120°$, $b = 3$, $c = 5$; Find the exact value of a.

Law of Cosines



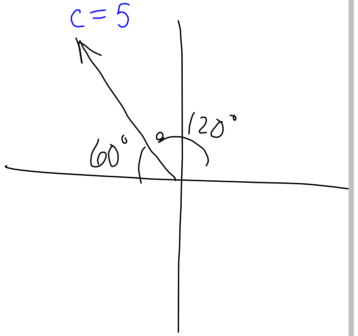
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 3^2 + 5^2 - 2(3)(5)\cos 120°$$

$$= 9 + 25 - 30\left(-\frac{1}{2}\right)$$

$$= 34 + 15$$

$$a^2 = 49$$



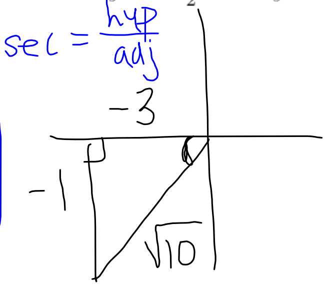
$$\boxed{a = 7}$$

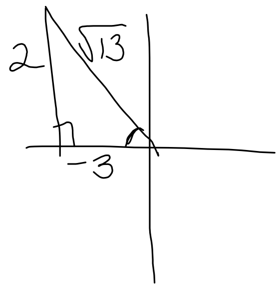
15. Given that $\tan x = \frac{1}{3}$ with x in Quadrant III and that $\cot y = -\frac{3}{2}$ with y in Quadrant II, find the exact values of the following:

(a) $\csc x + \sec y$

$\csc = \frac{hyp}{opp}$ $\quad \sec = \frac{hyp}{adj}$

$$= \frac{\sqrt{10}}{-1} + \frac{\sqrt{13}}{-3} = \boxed{-\sqrt{10} - \frac{\sqrt{13}}{3}}$$





(b) $\sin 2y$ $= 2\sin y \cos y$

$$= 2\left(\frac{2}{\sqrt{13}}\right)\left(\frac{-3}{\sqrt{13}}\right) = \boxed{\frac{-12}{13}}$$

(c) $\cos 2x + \cos(x - y)$

$$\underbrace{\cos^2 x - \sin^2 x}_{\cos 2x} + \underbrace{\cos x \cos y + \sin x \sin y}_{\cos(x-y)}$$

$$= \left(\frac{-3}{\sqrt{10}}\right)^2 - \left(\frac{-1}{\sqrt{10}}\right)^2 + \left(\frac{-3}{\sqrt{10}}\right)\left(\frac{-3}{\sqrt{13}}\right) + \left(\frac{-1}{\sqrt{10}}\right)\left(\frac{2}{\sqrt{13}}\right)$$

$$= \frac{9}{10} - \frac{1}{10} + \frac{9}{\sqrt{130}} - \frac{2}{\sqrt{130}}$$

$$= \frac{8}{10} + \frac{7}{\sqrt{130}} = \boxed{\frac{4}{5} + \frac{7}{\sqrt{130}}} = \frac{4}{5} + \frac{7\sqrt{130}}{130}$$

16. Find the exact value of $\sin(\underbrace{\cos^{-1}\frac{2}{3}}_{\theta_1} + \underbrace{\tan^{-1}\frac{4}{5}}_{\theta_2})$.

$\theta_1 = \cos^{-1}\frac{2}{3}$

$\theta_2 = \tan^{-1}\frac{4}{5}$

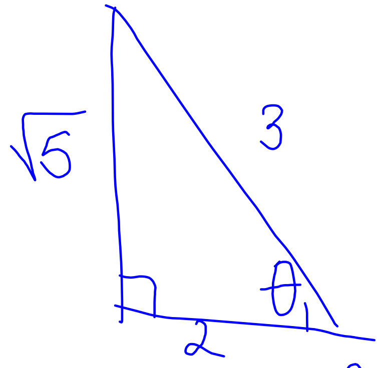
$$\sin(\theta_1 + \theta_2)$$

$$= \sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2$$

$$= \left(\frac{\sqrt{5}}{3}\right)\left(\frac{5}{\sqrt{41}}\right) + \left(\frac{2}{3}\right)\left(\frac{4}{\sqrt{41}}\right)$$

$$= \frac{5\sqrt{5}}{3\sqrt{41}} + \frac{8}{3\sqrt{41}}$$

$$= \boxed{\frac{5\sqrt{5}+8}{3\sqrt{41}}}$$



$\cos\theta_1 = \frac{2}{3}$

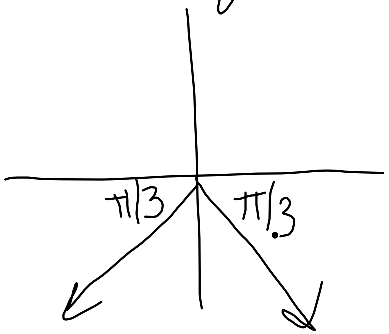
$\tan\theta_2 = \frac{4}{5}$

17. Find all solutions to the equation $(2\sin\frac{x}{4}+\sqrt{3})(\sqrt{2}\cos 6x-1)=0$.

$$2\sin\frac{x}{4}+\sqrt{3}=0$$

$$\sin\frac{x}{4}=\frac{-\sqrt{3}}{2}$$

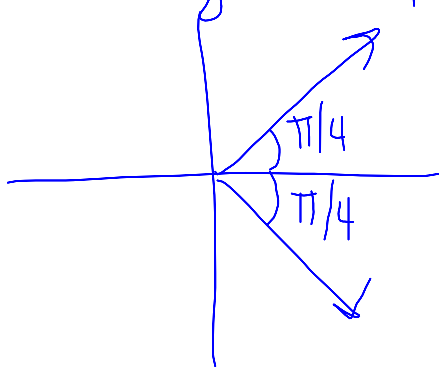
Ref Angle: $\frac{\pi}{3}$



$$\sqrt{2}\cos 6x-1=0$$

$$\cos 6x=\frac{1}{\sqrt{2}}$$

Ref Angle: $\frac{\pi}{4}$

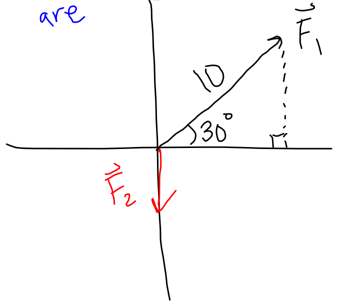


$$\frac{x}{4}=\frac{4\pi}{3}+2k\pi,\ \frac{5\pi}{3}+2k\pi$$

$$6x=\frac{\pi}{4}+2k\pi,\ \frac{7\pi}{4}+2k\pi$$

$$\boxed{x=\frac{16\pi}{3}+8k\pi,\ \frac{20\pi}{3}+8k\pi \qquad x=\frac{\pi}{24}+\frac{k\pi}{3},\ \frac{7\pi}{24}+\frac{k\pi}{3}}$$

18. Two forces are acting on an object. The first force has a magnitude of 10 pounds and is applied in the direction 30°. The second force is given by the vector $\mathbf{F}_2 = -3\mathbf{j}$. What is the total resulting force? What ~~is~~ are its magnitude and direction?



$$|\vec{F}_1| = 10 \; ; \; \theta = 30°$$

$$\vec{F}_1 = \langle 10\cos 30°, 10\sin 30° \rangle$$

$$= \langle 10\left(\frac{\sqrt{3}}{2}\right), 10\left(\frac{1}{2}\right) \rangle$$

$$\vec{F}_1 = \langle 5\sqrt{3}, 5 \rangle$$

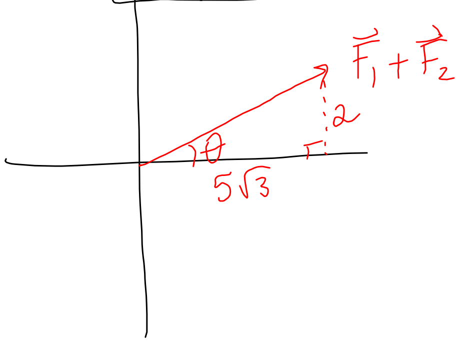
$$\vec{F}_2 = -3\vec{j} = \langle 0, -3 \rangle$$

$$\vec{F}_1 + \vec{F}_2 = \boxed{\langle 5\sqrt{3}, 2 \rangle}$$

$$|\vec{F}_1 + \vec{F}_2| = |\langle 5\sqrt{3}, 2 \rangle| = \sqrt{(5\sqrt{3})^2 + 2^2}$$

$$= \sqrt{75 + 4} = \sqrt{79}$$

$\boxed{\text{Magnitude: } \sqrt{79}}$

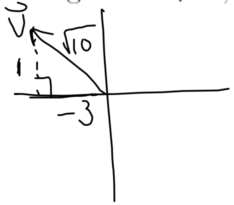


$$\tan\theta = \frac{2}{5\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{2}{5\sqrt{3}}\right) \approx \boxed{13.0039°}$$

Direction:

19. A vector **u** has initial point $(-3, 6)$ and terminal point $(-1, 7)$. A second vector **v** has magnitude $\sqrt{10}$, a vertical component of 1, and has a direction θ where $\tan\theta < 0$.



(a) Find $\mathbf{u} \cdot \mathbf{v}$

$$\vec{u} = \langle -1-(-3),\ 7-6 \rangle$$
$$= \langle 2, 1 \rangle$$

$$\vec{v} = \langle a, 1 \rangle$$
$$= \langle -3, 1 \rangle$$

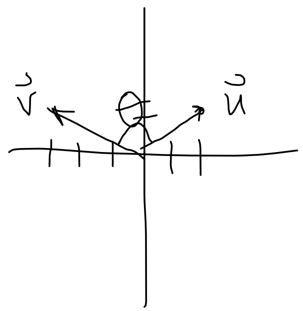
$$|\vec{v}| = \sqrt{10}$$
$$|\vec{v}| = \sqrt{a^2 + 1^2} = \sqrt{10}$$
$$a^2 + 1 = 10$$
$$a^2 = 9$$
$$a = -3$$

$$\vec{u} \cdot \vec{v} = 2(-3) + 1(1) = -6 + 1$$
$$= \boxed{-5}$$

(b) What is the angle between **u** and **v**?

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} = \frac{-5}{\sqrt{2^2+1^2} \cdot \sqrt{10}} = \frac{-5}{\sqrt{5} \cdot \sqrt{10}} = \frac{-5}{\sqrt{50}}$$

$$\cos\theta = \frac{-5}{5\sqrt{2}} = -\frac{1}{\sqrt{2}}$$



$$\boxed{\theta = \frac{3\pi}{4} \text{ OR } 135^\circ}$$

Ref Angle: $\frac{\pi}{4}$ OR 45°

FOR MRS. NITE'S CLASS:

20. Determine whether the following represent a parabola, ellipse, or hyperbola by writing the equation in standard form. What is the center of the conic?

(a) $\underbrace{x^2 - 6x} - \underbrace{4y^2 - 40y} - 95 = 0$

$$(x^2 - 6x + 9) - 4(y^2 + 10y + 25) = 95 + 9 - 100$$

$$(x-3)^2 - 4(y+5)^2 = 4$$

$$\frac{(x-3)^2}{4} - (y+5)^2 = 1$$

$$\frac{(x-h)^2}{a^2} \pm \frac{(y-k)^2}{b^2} = 1$$

Hyperbola: Center: $(3, -5)$

(b) $x^2 + 6(y-7)^2 = 36$

$$\frac{x^2}{36} + \frac{(y-7)^2}{6} = 1$$

Ellipse: Center: $(0, 7)$