1. Solve the following equation for the variable h: $S = 2(lw + hw + hl)$

$$S = 2lw + 2hw + 2hl$$

$$S - 2lw = 2hw + 2hl$$

$$S - 2lw = h(2w + 2l)$$

$$\frac{S - 2lw}{2w + 2l} = h$$

$$\boxed{\frac{S - 2lw}{2(w + l)} = h}, \quad w \neq -l$$

2. Solve the following equations.

(a) $4x^2 + 24x + 1 = 0$

$x^2 + bx + \left(\frac{b}{2}\right)^2$

$4x^2 + 24x = -1$

$4(x^2 + 6x + 9) = -1 + 36$

$4(x+3)^2 = 35$

$(x+3)^2 = \frac{35}{4}$

$x + 3 = \pm\sqrt{\frac{35}{4}} = \pm\frac{\sqrt{35}}{2}$

$$x = -3 \pm \frac{\sqrt{35}}{2}$$

$\Rightarrow \frac{-6 \pm \sqrt{35}}{2}$

(b) $\sqrt{52-6x}+6=x$

$(\sqrt{52-6x})^2=(x-6)^2$

$52-6x=x^2-12x+36$

$0=x^2-6x-16$

$0=(x-8)(x+2)$

$x-8=0 \quad x+2=0$

$x=8 \quad x=-2$

$-1 \neq 1$

$(-1)^2=(1)^2$

$1=1$

False statements can be made true by squaring both sides. ⇒ Must check answers

Check: $x=8$: $\sqrt{52-6(8)}+6=2+6=8$ ✓

$x=-2$: $\sqrt{52-6(-2)}+6=8+6=14$ X

$x=8$ (-2 is extraneous)

(c) $|4x - 10| = 2$

$4x - 10 = -2$ or $4x - 10 = 2$

$4x = 8$ $\quad$ $4x = 12$

$x = 2$ $\quad$ $x = 3$

$x = 2, 3$

$(x+4)(x-2)\left(\frac{2}{x+4}-\frac{x}{x-2}\right)=(3)(x+4)(x-2)$

$2(x-2)-x(x+4)=3(x^2+2x-8)$

$2x-4-x^2-4x=3x^2+6x-24$

$0=4x^2+8x-20$

$0=4(x^2+2x-5)$

$x=\frac{-2\pm\sqrt{2^2-4(1)(-5)}}{2(1)}$

$x=\frac{-2\pm\sqrt{4+20}}{2}=\frac{-2\pm\sqrt{24}}{2}$

$x=\frac{-2\pm2\sqrt{6}}{2}=\frac{2(-1\pm\sqrt{6})}{2}$

$\boxed{x=-1\pm\sqrt{6}}$

$-1\neq 1$

$0(-1)=0(1)$

$0=0$

$\Downarrow$

False statements can be made true if you multiply by 0, which occurs when the denominators are 0.

(e) $x^{1/2} - 2x^{1/4} - 3 = 0$

$(x^{1/4})^2 - 2(x^{1/4}) - 3 = 0$ Let $w = x^{1/4}$

$\Downarrow$

$w^2 - 2w - 3 = 0$

$(w-3)(w+1) = 0$

$w = 3$

$(x^{1/4})^4 = (3)^4$

$x = 81$

$w = -1$

$(x^{1/4})^4 = (-1)^4$

$x = 1$

$x^{1/4} = -1$

$\sqrt[4]{x} = -1$

A fourth root cannot equal a negative #.

Check: $x = 81$: $81^{1/2} - 2(81)^{1/4} - 3 = 9 - 2(3) - 3$

$= 9 - 6 - 3 = 0$ ✓

$\boxed{x = 81}$

3. How many real solutions do the following quadratic equations have?

(a) $-2x^2 + 5x - 7 = 0$

$$\frac{-b \pm \sqrt{D}}{2a}$$

$D = b^2 - 4ac = 5^2 - 4(-2)(-7)$

$= 25 - 56$

$= -31$

NO real solutions, since $D < 0$

(b) $-3x^2 - 6x + 4 = 0$

$D = b^2 - 4ac = (-6)^2 - 4(-3)(4)$

$= 36 + 48$

$= 84$

2 real solutions, since $D > 0$

4. Kristina and Matt are putting together a puzzle. It would take Matt 5 hours more than Kristina to put the puzzle together by himself. Together, they can put the puzzle together in 6 hours. How long does it take Kristina to put it together by herself?

Let x = amount of time it takes Kristina by herself.

It takes Matt $x+5$ hours

Together → 6 hours ⇒ They can do $\frac{1}{6}$ of puzzle in 1 hr.

Kristina can do $\frac{1}{x}$ of the puzzle in 1 hour.

Matt " " $\frac{1}{x+5}$ " " " " " "

$$6x(x+5)\left(\frac{1}{x}+\frac{1}{x+5}\right)=\left(\frac{1}{6}\right)6(x)(x+5)$$

$$6(x+5)+6x=x(x+5)$$

$$6x+30+6x=x^2+5x$$

$$0=x^2-7x-30$$

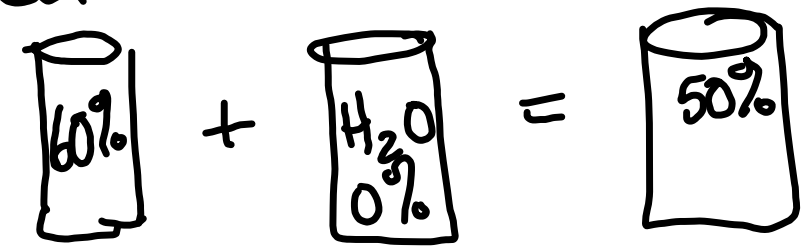
$$0=(x-10)(x+3)$$

$x=10$, ~~$x=-3$~~

10 hours

5. I have a 20 ounce bottle of lemonade that is 60% real lemon juice. How much water should I add to the bottle to reduce the concentration of real lemon juice to 50%?

Let x = ounces of water added.



Volume	20	+	x	$= 20 + x$
Concentration				

$$.6(20) + 0(x) = .5(20 + x)$$

$$12 = .5(20) + .5x$$

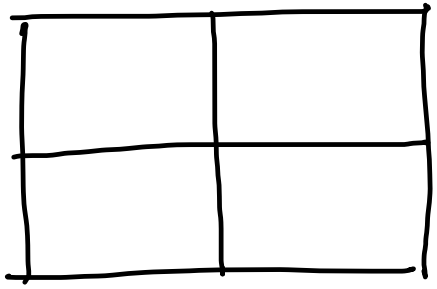
$$12 = 10 + .5x$$

$$2 = .5x$$

$$4 = x$$

4 ounces

6. A farmer plans to fence off a large rectangular lot and then further fence off 4 pens of equal size inside the lot. If the area of the lot is 96 ft^2 and the farmer uses 60 ft of fencing, find the dimensions of the lot.



$A = 96$ ft.^{2}

Let l = length of lot

$A = lw = 96$

$w = \frac{96}{l}$

$\frac{96}{l}$

l

$3(l) + 3\left(\frac{96}{l}\right) = 60$

$l\left(l + \frac{96}{l}\right) = (20)l$

$l^2 + 96 = 20l$

$l^2 - 20l + 96 = 0$

$(l-12)(l-8) = 0$

$l = 12$, $l = 8$

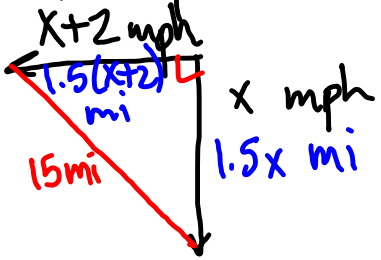
$w = \frac{96}{l} = \frac{96}{12} = 8$, $w = \frac{96}{l} = \frac{96}{8} = 12$

Length: 12 ft width: 8 ft.

12 ft x 8 ft

7. Two cars leave a parking lot at the same time. One heads due west and the other heads due south. The car heading west is going 2 mph faster than the car heading south. After 1.5 hours they will be 15 miles apart. How fast is the car heading south going?

Let x = speed of car going south.



$d = rt$

$$[1.5(x+2)]^2 + (1.5x)^2 = 15^2$$

$$\left[\frac{3}{2}(x+2)\right]^2 + \left(\frac{3}{2}x\right)^2 = 15^2$$

$$\frac{4}{9}\left[\frac{9}{4}(x^2+4x+4) + \frac{9}{4}x^2\right] = 225\left(\frac{4}{9}\right)$$

$$x^2+4x+4+x^2 = 100$$

$$2x^2+4x-96 = 0$$

$$2(x^2+2x-48) = 0$$

$$2(x+8)(x-6) = 0$$

$$x = \cancel{-8}, 6$$

$x = 6$ mph

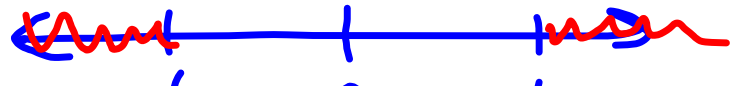
8. Solve the following inequalities:

(a) $2|-11-7x|-2>10$

$2|-11-7x| > 12$

$|-11-7x| \geq 6$

$|a| > 6$

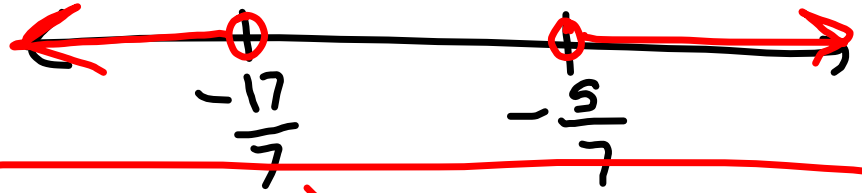


$a < -6$ OR $a > 6$

$-11-7x < -6$ OR $-11-7x > 6$

$-7x < 5$ $\quad -7x > 17$

$x > -\frac{5}{7}$ $\quad x < -\frac{17}{7}$



$\left(-\infty, -\frac{17}{7}\right) \cup \left(-\frac{5}{7}, \infty\right)$

(b) $|5x - 2| \leq 8$

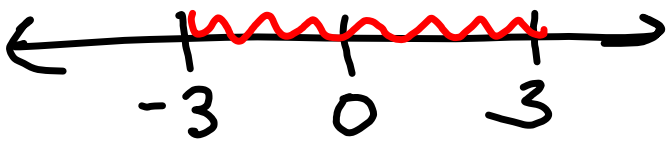
$-8 \leq 5x - 2 \leq 8$

$-6 \leq 5x \leq 10$

$-\frac{6}{5} \leq x \leq 2$

$\left[-\frac{6}{5}, 2\right]$

$|a| \leq 3$



$-3 \leq a \leq 3$

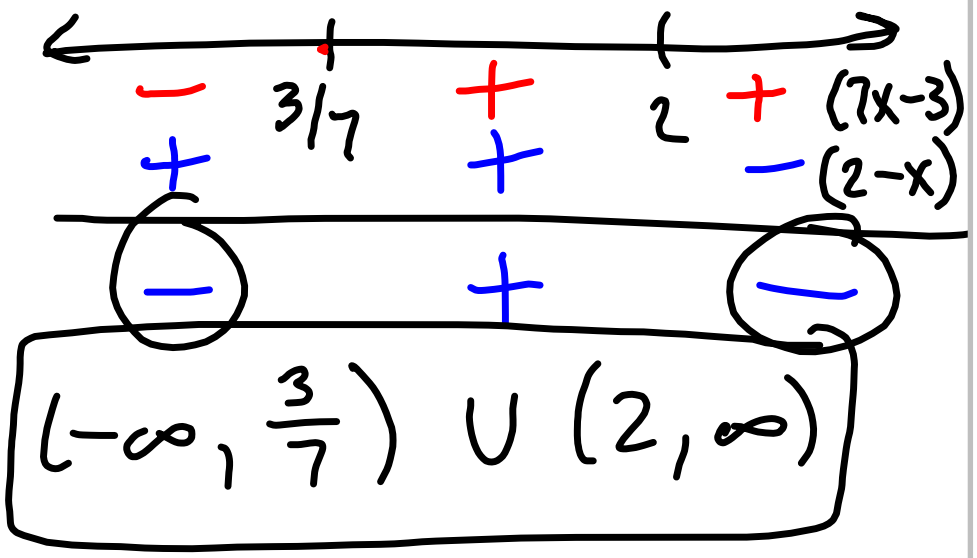
(c) $\dfrac{3x+5}{2-x} < 4$

$$\frac{3x+5}{2-x} - 4 < 0$$

$$\frac{3x+5-4(2-x)}{2-x} < 0$$

$$\frac{3x+5-8+4x}{2-x} < 0$$

$$\frac{7x-3}{2-x} < 0$$



(d) $\dfrac{x}{2x-1} \geq \dfrac{3}{x+2}$

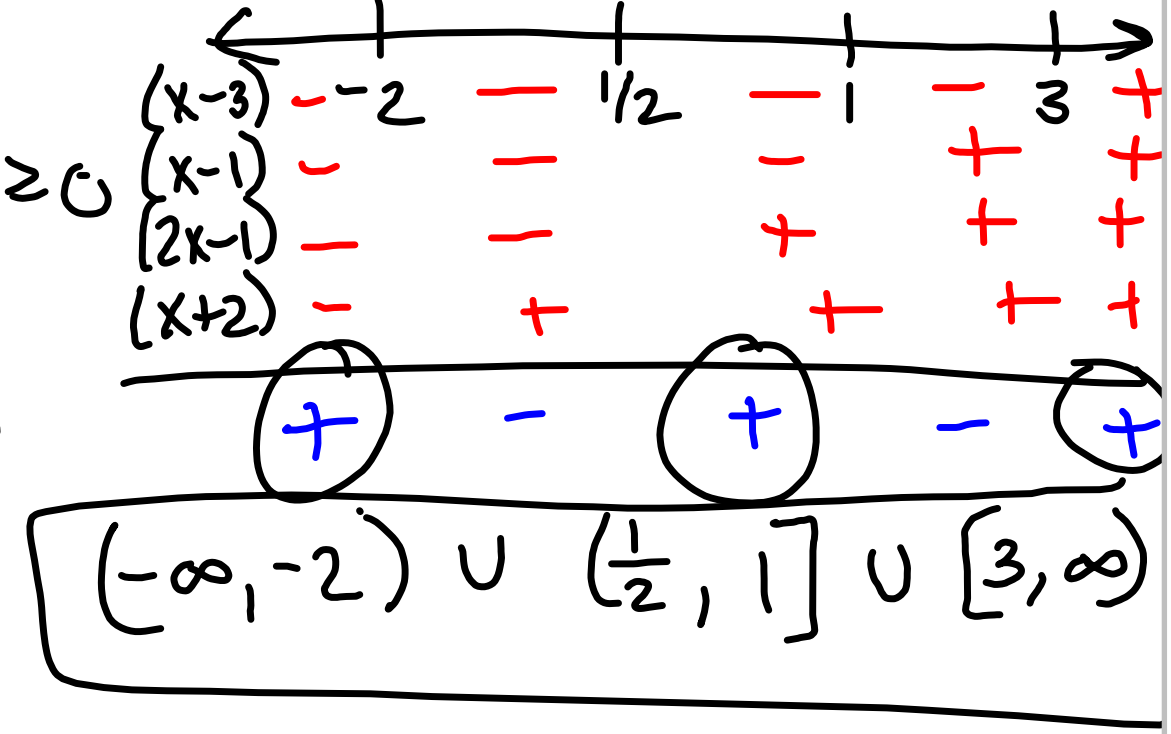
$$\frac{x}{2x-1} - \frac{3}{x+2} \geq 0$$

$$\frac{x(x+2) - 3(2x-1)}{(2x-1)(x+2)} \geq 0$$

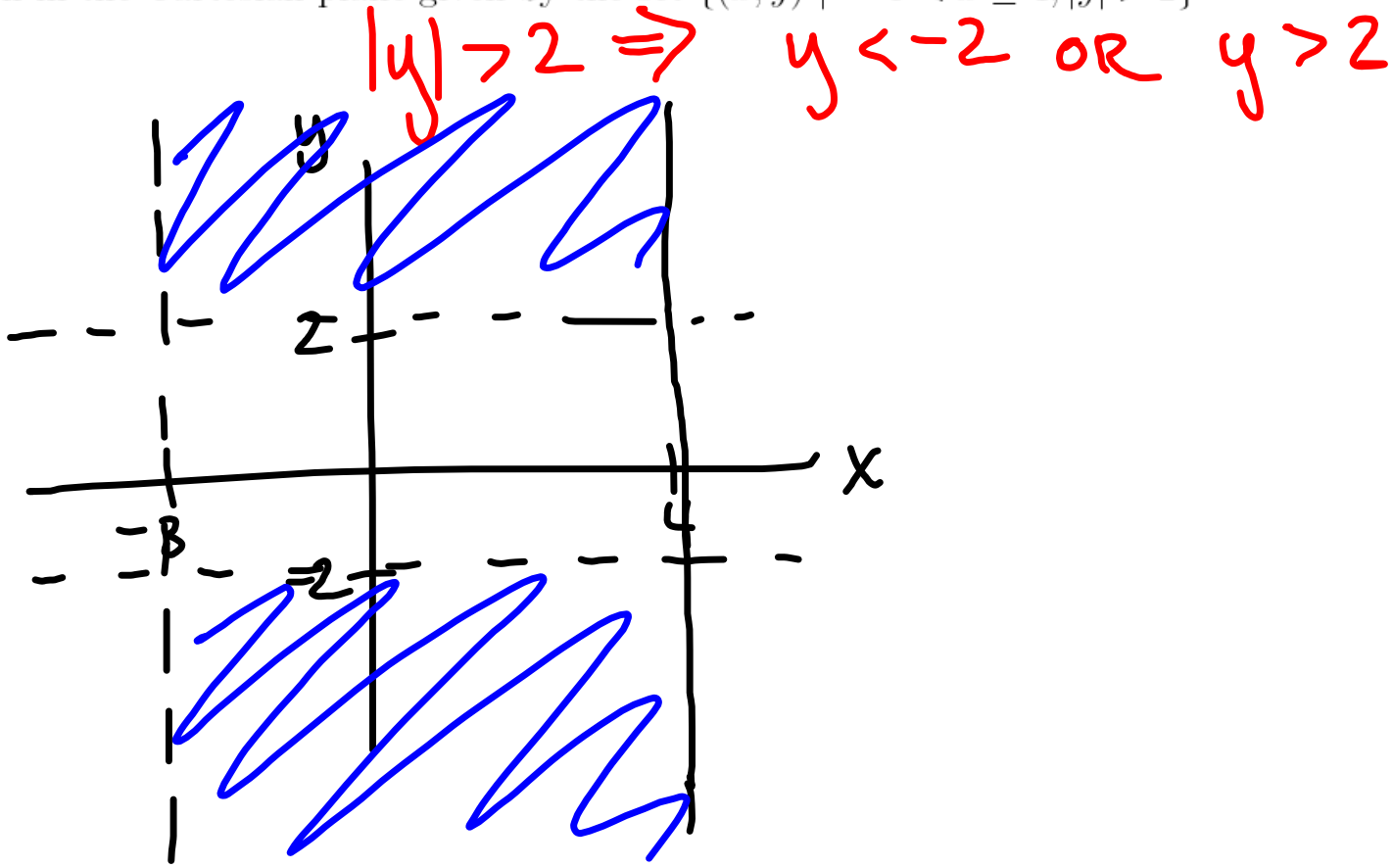
$$\frac{x^2+2x-6x+3}{(2x-1)(x+2)} \geq 0$$

$$\frac{x^2-4x+3}{(2x-1)(x+2)} \geq 0$$

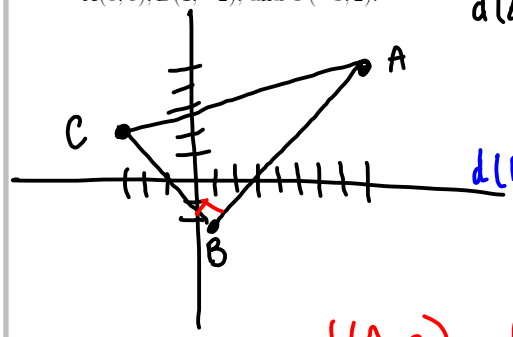
$$\frac{(x-3)(x-1)}{(2x-1)(x+2)} \geq 0$$



9. Shade the region in the Cartesian plane given by the set $\{(x, y) \mid -3 < x \leq 4, |y| > 2\}$



10. Verify that a triangle with the following vertices is a right triangle and find its area: $A(8,5), B(1,-2)$, and $C(-3,2)$.



$d(A,B) = \sqrt{(1-8)^2 + (-2-5)^2}$

$= \sqrt{49+49} = \sqrt{2 \cdot 49}$

$= 7\sqrt{2}$

$d(B,C) = \sqrt{(-3-1)^2 + (2-(-2))^2}$

$= \sqrt{16+16} = \sqrt{2 \cdot 16}$

$= 4\sqrt{2}$

$d(A,C) = \sqrt{(-3-8)^2 + (2-5)^2}$

$= \sqrt{121+9}$

$= \sqrt{130}$

Show $d(A,B)^2 + d(B,C)^2 = d(A,C)^2$

$(7\sqrt{2})^2 + (4\sqrt{2})^2$

$= 49 \cdot 2 + 16 \cdot 2$

$= 98 + 32$

$= 130$ ✓

$d(A,C)^2 = (\sqrt{130})^2 = 130$ ✓

Area $= \frac{1}{2}bh = \frac{1}{2} d(A,B)\, d(B,C)$

$= \left(\frac{1}{2}\right) 7\sqrt{2} \cdot 4\sqrt{2}$

$= \frac{1}{2} \cdot 28 \cdot 2$

Area = 28

11. Find the x and y-intercepts of the following graphs and test for symmetry.

(a) $x^3 + y = 8$

x-int: Set $y=0$

$x^3 + 0 = 8$

$x^3 = 8$

$x = 2$: $(2,0)$

y-int: Set $x=0$

$y = 8$: $(0,8)$

x-axis: $x^3 - y = 8$ NO

y-axis: $(-x)^3 + y = 8$

$-x^3 + y = 8$ NO

origin: $(-x)^3 - y = 8$

$-x^3 - y = 8$ NO

(b) $y = |3x| + 4$

x-int: $0 = |3x| + 4$

$-4 \neq |3x|$ None

y-int: $y = |3(0)| + 4$

$y = 4$: $(0,4)$

x-axis: $-y = |3x| + 4$ NO

y-axis: $y = |3(-x)| + 4$

$y = |-3x| + 4$

$y = |3x| + 4$ YES

origin: $-y = |-3x| + 4$

$-y = |3x| + 4$ NO

(c) $x = y^5 - 4y^3$

x-int: $x = 0^5 - 4(0)^3 = 0$

$(0,0)$

y-int: $0 = y^5 - 4y^3$

$0 = y^3(y^2 - 4)$

$0 = y^3(y-2)(y+2)$

$y = 0, 2, -2$

$(0,0), (0,2), (0,-2)$

x-axis: $x = (-y)^5 - 4(-y)^3$

$x = -y^5 + 4y^3$ NO

y-axis: $-x = y^5 - 4y^3$ NO

origin: $-x = (-y)^5 - 4(-y)^3$

$-x = -y^5 + 4y^3$

$x = y^5 - 4y^3$ YES

12. Find the equation of the circle that:

(a) has center $(5, -6)$ and radius 12.

(h,k)

Standard form of circle with center (h,k) and radius r

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-5)^2 + (y-(-6))^2 = 12^2$$

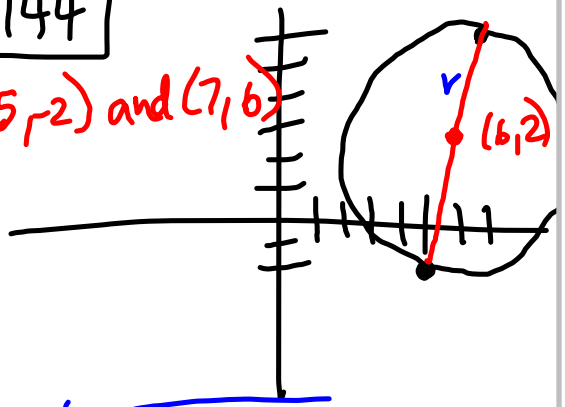
$$\boxed{(x-5)^2 + (y+6)^2 = 144}$$

(b) has a diameter with endpoints $(5, -2)$ and $(7, 6)$.

Center is midpoint between $(5,-2)$ and $(7,6)$

Center: $\left(\frac{5+7}{2}, \frac{-2+6}{2}\right)$

$= (6, 2)$



$$r = \frac{1}{2}\sqrt{(7-5)^2 + (6-(-2))^2} = \frac{1}{2}\sqrt{4+64} = \frac{1}{2}\sqrt{68}$$

$$= \frac{1}{2} \cdot 2\sqrt{17} = \sqrt{17}$$

$$(x-6)^2 + (y-2)^2 = (\sqrt{17})^2$$

$$\boxed{(x-6)^2 + (y-2)^2 = 17}$$

13. Find the center and radius of the circle $2x^2 + 2y^2 - 6x + 5y = 1$.

$(2x^2 - 6x \quad) + (2y^2 + 5y \quad) = 1$

$2(x^2 - 3x + \frac{9}{4}) + 2(y^2 + \frac{5}{2}y + \frac{25}{16}) = 1 + \frac{9}{2} + \frac{25}{8}$

$2(x - \frac{3}{2})^2 + 2(y + \frac{5}{4})^2 = \frac{8 + 36 + 25}{8}$

$2(x - \frac{3}{2})^2 + 2(y + \frac{5}{4})^2 = \frac{69}{8}$

$(x - \frac{3}{2})^2 + (y + \frac{5}{4})^2 = \frac{69}{16}$

$(\frac{3}{2})^2 = \frac{9}{4}$

$(\frac{5}{4})^2 = \frac{25}{16}$

Center: $(\frac{3}{2}, -\frac{5}{4})$

$r = \sqrt{\frac{69}{16}} = \frac{\sqrt{69}}{4}$

14. Solve the following graphically.

(a) $x^2 + \sqrt{x^4 + 5} = x^3 - 7x$

$Y_1 = x^2 + \sqrt{x^4+5}$

$Y_2 = x^3 - 7x$

> Graph and find intersection points.

$x = -1.7542, -0.3424, 3.8362$

(b) $-0.58x \leq 0.87x - 5x^{1/3}$

Graph and find on what intervals $Y_1 \leq Y_2$

$Y_1 = -0.58x$

$Y_2 = 0.87x - 5x^{1/3}$

$\Downarrow$

$[-6.4033, 0] \cup (6.4033, \infty)$