1. Find the equation of the line that

(a) passes through the y-intercept of the line $-2x + 3y = 9$ and is parallel to the line $7x - 4y = 6$.

y-int of $-2x+3y=9$

Set $x=0$: $-2(0)+3y=9$

$3y=9$

$y=3$

OR

$-2x+3y=9$

$3y=2x+9$

$y=\frac{2}{3}x+3$

y-int is $(0,3)$

y-int: $(0,3)$

parallel $\Rightarrow$ same slope.

$7x-4y=6$

$-4y=-7x+6$

$y=\frac{7}{4}x-\frac{3}{2}$

$m=\frac{7}{4}$

$y=mx+b$; $\boxed{y=\frac{7}{4}x+3}$

(b) passes through the x-intercept of the line $3x - 8y = 12$ and is perpendicular to the line $x = 3$.

x-int: Set $y=0$

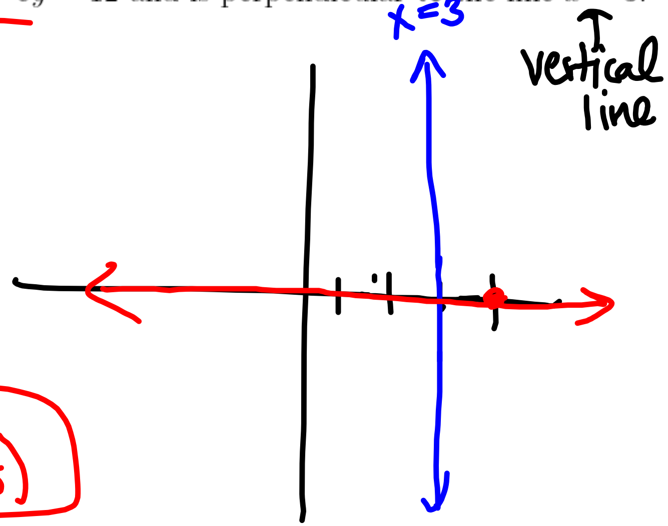
$3x-8(0)=12$

$3x=12$

$x=4$

$(4,0)$

$\boxed{y=0 \text{ (the x-axis)}}$



2. Suppose that the relationship between the cost of utilities and the average temperature in a month is linear. If the average temperature in a month is 96°, your utilities bill is \$100. If the average temperature in a month is 81°, your utilities bill is \$75.

   (a) Find an equation that expresses the cost of your utilites, C, in terms of the average temperature, T, in any given month.

$(x, y) \to (T, C)$

$(96, 100)$

$(81, 75)$

$$m = \frac{\Delta C}{\Delta T} = \frac{100-75}{96-81} = \frac{25}{15} = \frac{5}{3}$$

$$y - y_1 = m(x - x_1)$$

$$C - C_1 = m(T - T_1)$$

$$C - 100 = \frac{5}{3}(T - 96)$$

$$C - 100 = \frac{5}{3}T - 160$$

$$\boxed{C = \frac{5}{3}T - 60}$$

   (b) How much will your utilities bill increase if the average temperature in the current month is 6° higher than the average temperature last month?

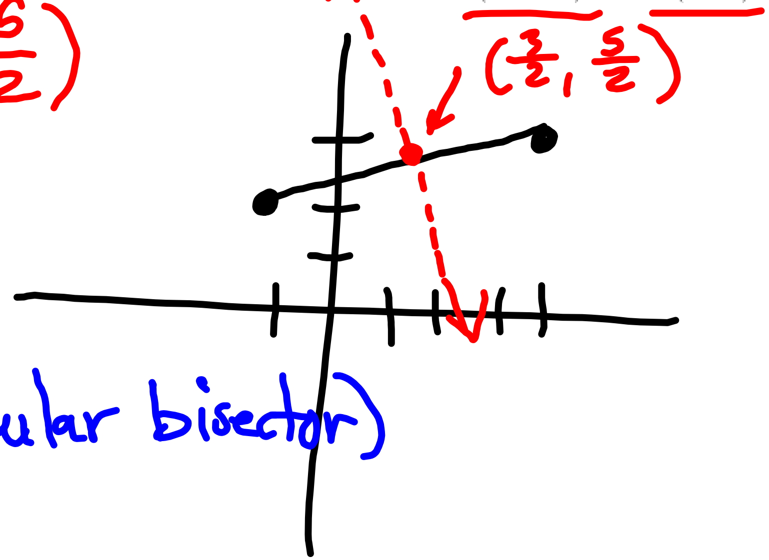
$$m = \frac{\Delta C}{\Delta T} = \frac{5}{3}$$

Given that $\Delta T = 6°$, $\Delta C = \frac{5}{3} \cdot \Delta T = \frac{5}{3} \cdot 6 = 10$

$\boxed{\$10 \text{ higher}}$

3. Find an equation of the perpendicular bisector of the line segment joining the points $(-1, 2)$ and $(4, 3)$.

Midpoint: $\left(\frac{-1+4}{2}, \frac{2+3}{2}\right) = \left(\frac{3}{2}, \frac{5}{2}\right)$



$m = \frac{3-2}{4-(-1)} = \frac{1}{5}$

(of line segment)

$m_\perp = -5$ (slope of perpendicular bisector)

$$y - y_1 = m(x - x_1)$$

$$y - \frac{5}{2} = -5\left(x - \frac{3}{2}\right)$$

$$y - \frac{5}{2} = -5x + \frac{15}{2}$$

$$\boxed{y = -5x + 10}$$

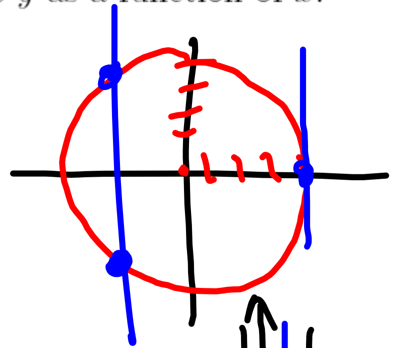
4. Determine whether the following equations define y as a function of x.

(a) $x^2 + y^2 = 16$

$y^2 = 16 - x^2$

$y = \pm\sqrt{16 - x^2}$

NOT a function

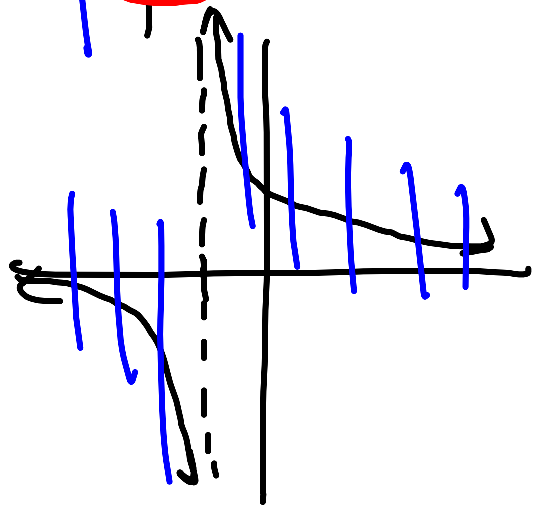


(b) $x^3y + 4y = 12$

$y(x^3 + 4) = 12$

$y = \dfrac{12}{x^3 + 4}$

YES, is a function

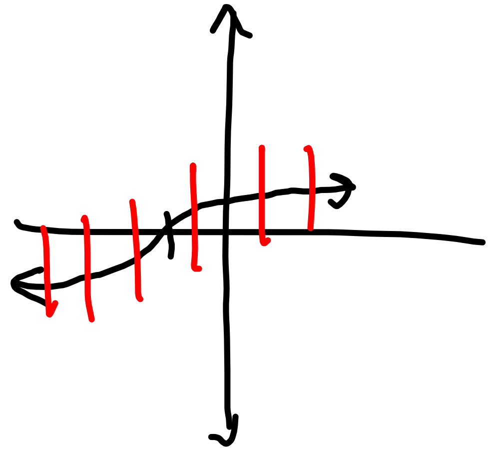


(c) $y^3 - x = 1$

$y^3 = 1 + x$

$y = \sqrt[3]{1 + x}$

YES, is a function

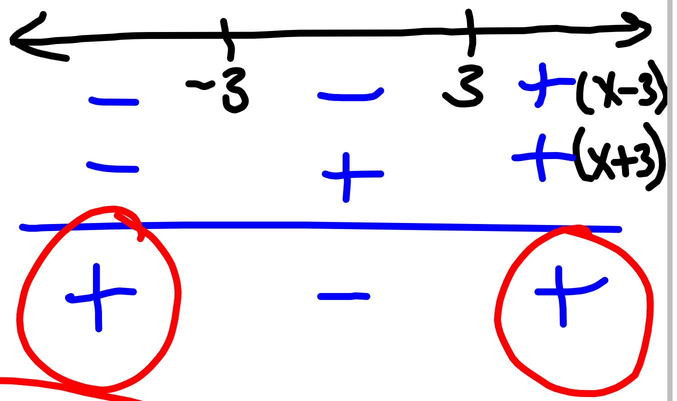


5. Find the domains of the following functions.

(a) $f(x) = \frac{x^3}{\sqrt{x^2 - 9}}$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$



$\boxed{(-\infty, -3) \cup (3, \infty)}$

(b) $f(x) = \dfrac{\sqrt[4]{x^2 - 6x - 16}}{x^2 + 4x - 21}$.

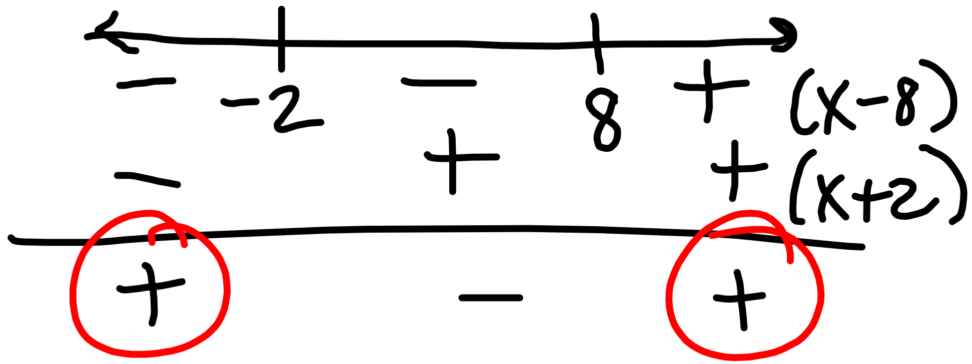
$x^2 + 4x - 21 \neq 0$

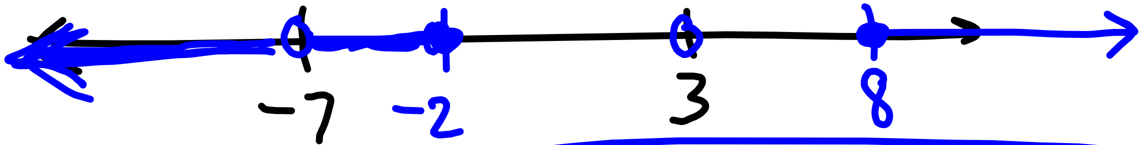
$(x+7)(x-3) = 0$

$x \neq -7, 3$

$x^2 - 6x - 16 \geq 0$

$(x-8)(x+2) \geq 0$





$\boxed{(-\infty, -7) \cup (-7, -2] \cup [8, \infty)}$

6. Let $f(x) = \dfrac{x^2+1}{2-x}$. Evaluate the following.

(a) $f(\frac{1}{x})$ $= \dfrac{\left(\frac{1}{x}\right)^2 + 1}{2 - \left(\frac{1}{x}\right)} = \dfrac{\frac{1}{x^2} + 1}{2 - \frac{1}{x}} = \dfrac{\frac{1+x^2}{x^2}}{\frac{2x-1}{x}}$

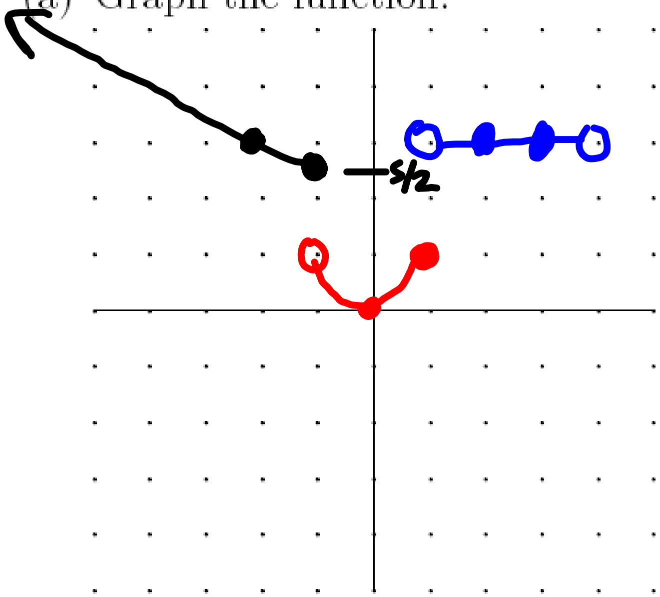
$= \dfrac{1+x^2}{x^{\cancel{2}}} \cdot \dfrac{\cancel{x}}{2x-1} = \boxed{\dfrac{1+x^2}{x(2x-1)}}$

(b) $f(-x^2)$ $= \dfrac{(-x^2)^2 + 1}{2 - (-x^2)} = \boxed{\dfrac{x^4+1}{2+x^2}}$

7. Consider the function:

$$f(x) = \begin{cases} -\frac{1}{2}x + 2 & \text{if } x \leq -1 \\ x^2 & \text{if } -1 < x \leq 1 \\ 3 & \text{if } 1 < x < 4 \end{cases}$$

(a) Graph the function.



x	$f(x)$
-2	$-\frac{1}{2}(-2)+2 = 3$
-1	$-\frac{1}{2}(-1)+2 = \frac{5}{2}$
0	$0^2 = 0$
1	$1^2 = 1$
2	3
3	3

(b) What are the domain and range of f?

Domain: $(-\infty, 4)$

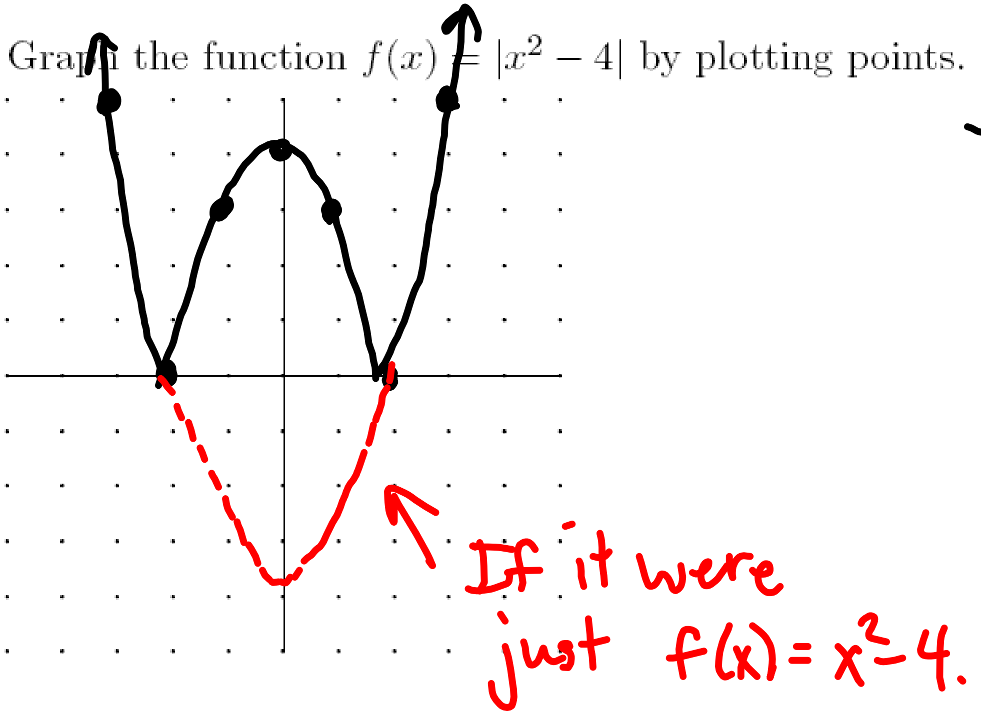
Range: $[0, 1] \cup [5/2, \infty)$

(c) On what intervals is f increasing? decreasing?

Increasing on $(0, 1)$

Decreasing on $(-\infty, -1) \cup (-1, 0)$

8. Graph the function $f(x) = |x^2 - 4|$ by plotting points.



x	$f(x)$
-3	$\lvert(-3)^2-4\rvert = 5$
-2	$\lvert(-2)^2-4\rvert = 0$
-1	$\lvert(-1)^2-4\rvert = 3$
0	$\lvert 0^2-4\rvert = 4$
1	$\lvert 1^2-4\rvert = 3$
2	$\lvert 2^2-4\rvert = 0$
3	$\lvert 3^2-4\rvert = 5$

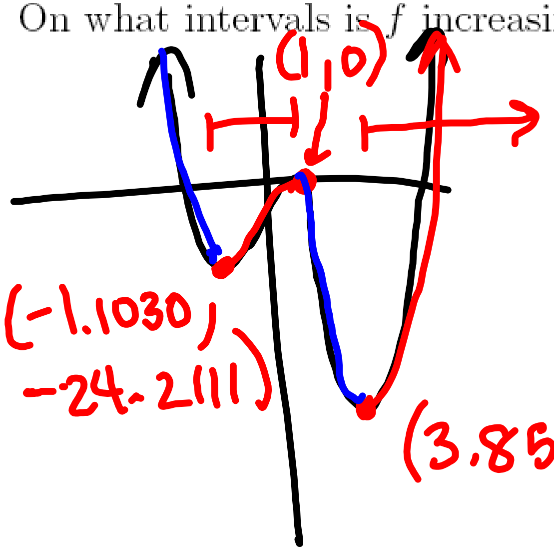
9. Graph the function $f(x) = x^4 - 5x^3 - 3x^2 + 17x - 10$ using a graphing calculator.

(a) What is the range of this function? (Round decimals to 4 places.)

$\boxed{[-54.6444, \infty)}$

2^{nd} Calc, 3: Minimum

(b) On what intervals is f increasing? decreasing? (Round decimals to 4 places.)



Increasing on:

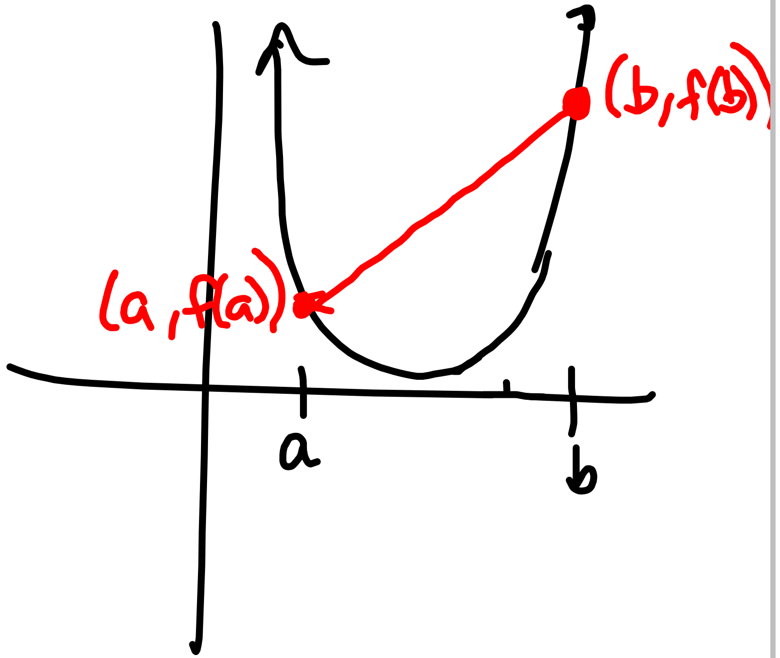
$(-1.1030, 1) \cup (3.8530, \infty)$

Decreasing on:

$(-\infty, -1.1030) \cup (1, 3.8530)$

Average rate of change of $f(x)$ from $x=a$ to $x=b$:

$$\frac{f(b)-f(a)}{b-a}$$

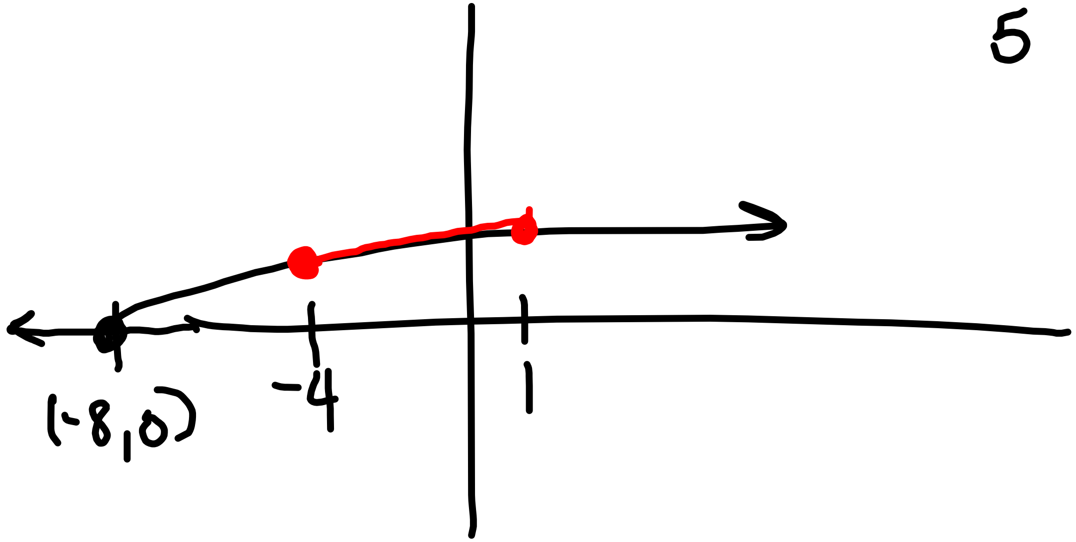


10. Find the average rate of change for the following functions on the given interval.

(a) $f(x) = \sqrt{x+8}$ from $x = -4$ to $x = 1$

$$\frac{f(1) - f(-4)}{1-(-4)} = \frac{\sqrt{1+8} - \sqrt{-4+8}}{5}$$

$$= \frac{3-2}{5} = \boxed{\frac{1}{5}}$$



(b) $f(x) = x^2 + 2x - 4$ from $x = 2$ to $x = 2 + h$

$$\frac{f(2+h) - f(2)}{2+h-2} = \frac{\left[(2+h)^2 + 2(2+h) - 4\right] - \left[2^2 + \overset{4}{\overset{\shortparallel}{4}} - 4\right]}{h}$$

$$= \frac{\cancel{4} + 4h + h^2 + \cancel{4} + 2h - \cancel{4} - \cancel{4}}{h}$$

$$= \frac{6h + h^2}{h} = \frac{\cancel{h}(6+h)}{\cancel{h}}$$

$$= \boxed{6 + h}$$

(c) $f(x) = \frac{5}{x-4}$ from $x = a$ to $x = a + h$

$$\frac{f(a+h) - f(a)}{a+h - a} = \frac{\frac{5}{a+h-4} - \frac{5}{a-4}}{h}$$

$$= \frac{\frac{5(a-4) - 5(a+h-4)}{(a+h-4)(a-4)}}{h}$$

$$= \frac{\frac{\cancel{5a} - \cancel{20} - \cancel{5a} - 5h + \cancel{20}}{(a+h-4)(a-4)}}{h} = \frac{-5\cancel{h}}{\cancel{h}(a+h-4)(a-4)}$$

$$= \boxed{\frac{-5}{(a+h-4)(a-4)}}$$

11. Suppose an object is launched into motion. After 10 seconds, the object has traveled 220 feet. After 15 seconds, the object has traveled a total of 450 feet.

(a) What was the objects average speed during the first 10 seconds?

$$\frac{\text{distance traveled}}{\text{time}} = \frac{220}{10} = \underline{22 \text{ ft/sec}}$$

$$\frac{d(10) - d(0)}{10 - 0} = \frac{220 - 0}{10 - 0} = \frac{220}{10} =$$

(b) What was the car's average speed during the last 5 seconds? from $t=10$ to $t=15$

$$\frac{\text{distance traveled}}{\text{time}} = \frac{450 - 220}{5} = \frac{230}{5} = \underline{46 \text{ ft/sec}}$$

$$\frac{d(15) - d(10)}{15 - 10} = \frac{450 - 220}{5}$$

12. If the distance in feet an object has traveled after t seconds is modeled by the function $f(t) = t^3 + 6t$, then what is the object's average speed from $t = a$ to $t = a + h$?

Average speed = average rate of change of f from t=a to t=a+h.

$$\frac{f(a+h)-f(a)}{a+h-a} = \frac{\left[(a+h)^3+6(a+h)\right]-\left[a^3+6a\right]}{h}$$

$$= \frac{\cancel{a^3}+3a^2h+3ah^2+h^3+\cancel{6a}+6h\cancel{-a^3}-\cancel{6a}}{h}$$

$$= \frac{3a^2h+3ah^2+h^3+6h}{h}$$

$$= \frac{\cancel{h}(3a^2+3ah+h^2+6)}{\cancel{h}}$$

$$= \boxed{3a^2+3ah+h^2+6 \text{ ft/sec}}$$