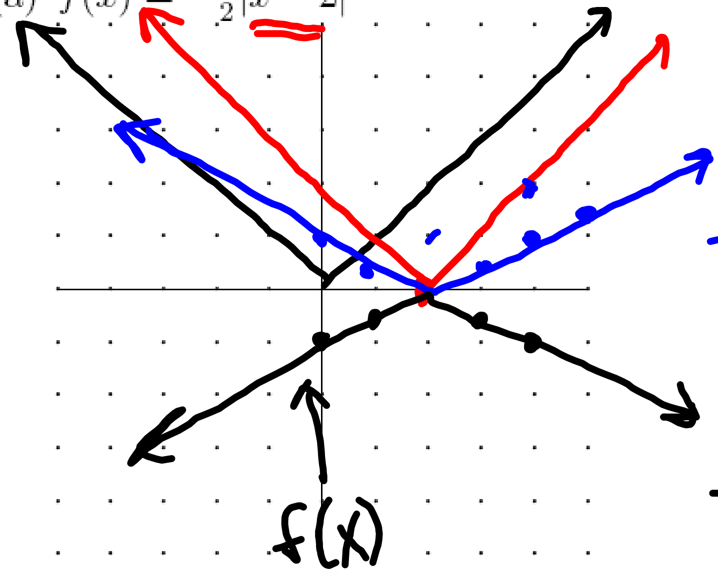
1. Graph the following functions by using transformations.

(a) $f(x) = -\frac{1}{2}|x-2|$



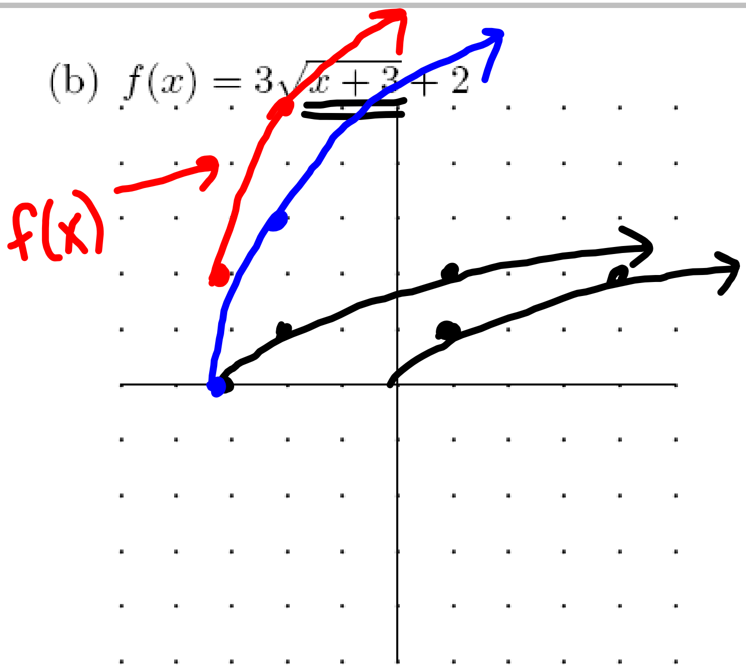
Start with $g(x) = |x|$

$g(x-2) = |x-2| \rightarrow$ Shift right 2

$\frac{1}{2}g(x-2) = \frac{1}{2}|x-2| \rightarrow$ Vertically shrink graph by 1/2.

$-\frac{1}{2}g(x-2) = -\frac{1}{2}|x-2| \rightarrow$ Reflect across x-axis

(b) $f(x) = 3\sqrt{x+3} + 2$



$g(x) = \sqrt{x}$

$g(x+3) = \sqrt{x+3} \rightarrow$ shift left 3

$3g(x+3) = 3\sqrt{x+3} \rightarrow$ Vertically stretch graph by 3.

$3g(x+3)+2 = 3\sqrt{x+3}+2$

$\rightarrow$ shift up 2

2. A function $f(x)$ is horizontally stretched by a factor of 5, reflected across the y-axis, vertically stretched by a factor of 5, and then shifted down 4. Write a function $g(x)$ in terms of $f(x)$ that represents the resulting graph.

$$g(x) = 5f\left(-\frac{1}{5}x\right) - 4$$

3. Determine whether the following functions are even, odd, or neither.

(a) $f(x) = x^2 - \sqrt[5]{x}$

$f(-x) = (-x)^2 - \sqrt[5]{-x}$

$= x^2 + \sqrt[5]{x}$

Neither

(b) $f(x) = |x| - 5x^{-4}$

$f(-x) = |-x| - 5(-x)^{-4}$

$= |x| - 5x^{-4}$

$= f(x)$

EVEN

Even: $f(-x) = f(x) \rightarrow$ symmetric about y-axis

Odd: $f(-x) = -f(x) \rightarrow$ symmetric about origin.

4. For the quadratic function below, write in standard form, find the vertex of the parabola, and find the maximum or minimum value.

$f(x) = -3x^2 - 18x - 31$

$f(x) = a(x-h)^2 + k$

Vertex: (h, k)

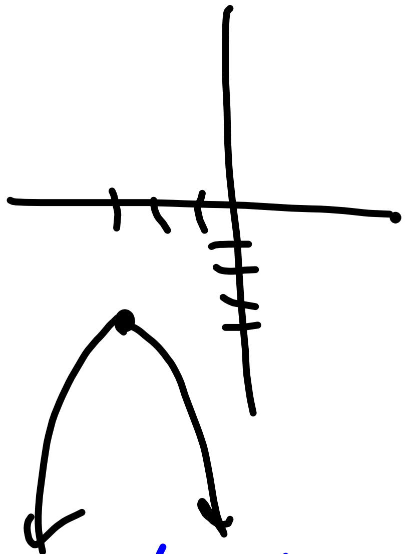
$f(x) = (-3x^2 - 18x \quad) - 31$

$= -3(x^2 + 6x + 9) - 31 + 27$

$= -3(x+3)^2 - 4$

$(x-(-3))^2$

Vertex: $(-3, -4)$



$a = -3 < 0 \Rightarrow$ parabola opens downward $\Rightarrow$ vertex is a maximum.

$\boxed{\text{Maximum value is } -4.}$

5. For the quadratic function below, find the maximum or minimum value and state the range.

$f(x) = 5x^2 + 6x + 4$

$a = 5 > 0 \Rightarrow$ parabola opens upward $\Rightarrow$ vertex is a minimum

Minimum value occurs when $x = \frac{-6}{2(5)} = \frac{-6}{10} = \frac{-3}{5}$

Minimum value is $f\left(-\frac{3}{5}\right) = 5\left(-\frac{3}{5}\right)^2 + 6\left(-\frac{3}{5}\right) + 4$

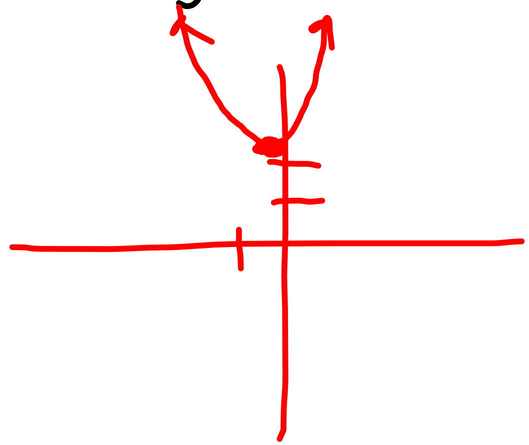
$$= 5\left(\frac{9}{25}\right) - \frac{18}{5} + 4$$

$$= \frac{9}{5} - \frac{18}{5} + 4$$

$$= \frac{9 - 18 + 20}{5} = \frac{11}{5}$$

$\boxed{\text{Minimum value is } \frac{11}{5}}$

$\boxed{\text{Range: } \left[\frac{11}{5}, \infty\right)}$

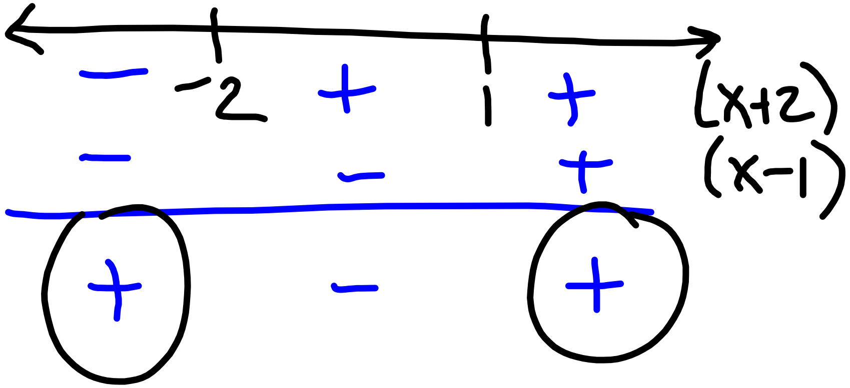


6. Suppose $f(x) = \dfrac{1}{\sqrt{x^2 + x - 2}}$ and $g(x) = \dfrac{\sqrt{x+3}}{x^2 - 9x + 20}$.

(a) Find the domain of f.

$x^2 + x - 2 > 0$

$(x+2)(x-1) > 0$



$\boxed{(-\infty, -2) \cup (1, \infty)}$

(b) Find the domain of g.

$x^2 - 9x + 20 \neq 0$

$(x-4)(x-5) \neq 0$

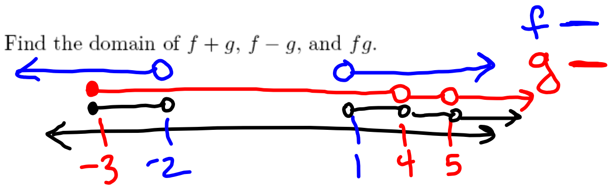
$x \neq 4, 5$

$x + 3 \geq 0$

$x \geq -3$

$\boxed{[-3, 4) \cup (4, 5) \cup (5, \infty)}$

(c) Find the domain of $f+g$, $f-g$, and fg.



$$\boxed{[-3,-2) \cup (1,4) \cup (4,5) \cup (5,\infty)}$$

(d) Calculate $(f+g)(2)$ and $(fg)(6)$.

$$(f+g)(2) = f(2) + g(2) = \frac{1}{\sqrt{2^2+2-2}} + \frac{\sqrt{2+3}}{2^2-9\cdot 2+20}$$

$$= \boxed{\frac{1}{2} + \frac{\sqrt{5}}{6} = \frac{3+\sqrt{5}}{6}}$$

$$(fg)(6) = f(6)\,g(6) = \left(\frac{1}{\sqrt{6^2+6-2}}\right)\left(\frac{\sqrt{6+3}}{6^2-9\cdot 6+20}\right)$$

$$= \left(\frac{1}{\sqrt{40}}\right)\left(\frac{3}{2}\right) = \frac{3}{2\sqrt{40}} = \frac{3}{2\sqrt{4}\sqrt{10}}$$

$$= \boxed{\frac{3}{4\sqrt{10}} = \frac{3\sqrt{10}}{40}}$$

(e) Find $\frac{f}{g}$ and its domain.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{1}{\sqrt{x^2+x-2}}}{\frac{\sqrt{x+3}}{x^2-9x+20}}$$

$$\boxed{\left(\frac{f}{g}\right)(x) = \frac{x^2-9x+20}{\sqrt{x^2+x-2}\,\sqrt{x+3}}} \qquad x \neq -3$$

Domain: $(-3,-2) \cup (1,4) \cup (4,5) \cup (5,\infty)$

7. Let $f(x) = \frac{x}{x+6}$ and $g(x) = x - 1$.

(a) Find $f \circ g$ and its domain.

$$(f \circ g)(x) = f(g(x)) = f(\underline{x-1}) = \frac{(x-1)}{(x-1)+6}$$

$$= \boxed{\frac{x-1}{x+5}}$$

Domain of g: $\mathbb{R}$

$f(g(x))$ is defined when $x \neq -5$

Domain of $f \circ g$: $\boxed{(-\infty, -5) \cup (-5, \infty)}$

(b) Find $f \circ f$ and its domain.

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{x}{x+6}\right) = \frac{\left(\frac{x}{x+6}\right)}{\left(\frac{x}{x+6}\right)+6}$$

$$= \frac{\left(\frac{x}{x+6}\right)}{\left(\frac{x}{x+6}+6\right)} \cdot \frac{x+6}{x+6} = \frac{x}{x+6(x+6)}$$

$$= \boxed{\frac{x}{7x+36}}$$

Domain of f: $\{x \mid x \neq -6\}$

$f(f(x))$ is defined for: $\{x \mid x \neq -\frac{36}{7}\}$

Domain of $f \circ f$: $\{x \mid x \neq -6, -\frac{36}{7}\}$

$\underline{(-\infty, -6) \cup (-6, -\frac{36}{7}) \cup (-\frac{36}{7}, \infty)}$

Title: Sep 22-4:27 PM (9 of 28)

8. Let $f(x) = 2x^2 - 3x$ and $g(x) = 2x^3 + x$. Calculate the following. Expand fully to polynomial form.

(a) $f \circ g$

$(f \circ g)(x) = f(g(x)) = f(2x^3 + x)$

$= 2(2x^3 + x)^2 - 3(2x^3 + x)$

$= 2[4x^6 + 4x^4 + x^2] - 6x^3 - 3x$

$= 8x^6 + 8x^4 + 2x^2 - 6x^3 - 3x$

$\boxed{8x^6 + 8x^4 - 6x^3 + 2x^2 - 3x}$

(b) $g \circ f$

$(g \circ f)(x) = g(f(x)) = g(2x^2 - 3x)$

$= 2(2x^2 - 3x)^3 + (2x^2 - 3x)$

$= 2[8x^6 - 3(2x^2)^2(3x) + 3(2x^2)(3x)^2 - (3x)^3]$ $+ 2x^2 - 3x$

$= 2[8x^6 - 36x^5 + 54x^4 - 27x^3] + 2x^2 - 3x$

$= \boxed{16x^6 - 72x^5 + 108x^4 - 54x^3 + 2x^2 - 3x}$

$(A - B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$

9. Determine whether the following define y as a function of x. If y IS a function of x, state whether it is a one-to-one function.

(a) $y - 5 = 3|x - 2|$

$y = 5 + 3|x-2|$

IS a function

NOT one-to-one

(b) $xy^2 + 3y^2 = x$

$y^2(x+3) = x$

$y^2 = \frac{x}{x+3}$

$y = \pm\sqrt{\frac{x}{x+3}}$

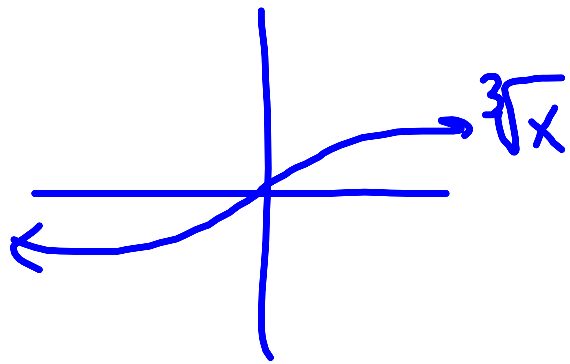
NOT a function

(c) $y^3 = -8x$

$y = \sqrt[3]{-8x}$

IS a function

IS one-to-one



10. Find inverse functions for the following.

(a) $f(x) = \sqrt[3]{x^5 + 9}$

$$y = \sqrt[3]{x^5+9}$$

$$x = \sqrt[3]{y^5+9}$$

$$x^3 = y^5 + 9$$

$$x^3 - 9 = y^5$$

$$\sqrt[5]{x^3-9} = y \Rightarrow \boxed{f^{-1}(x) = \sqrt[5]{x^3-9}}$$

(b) $f(x) = \dfrac{x}{x+4}$

$$y = \frac{x}{x+4}$$

$$x = \frac{y}{y+4}$$

$$x(y+4) = y$$

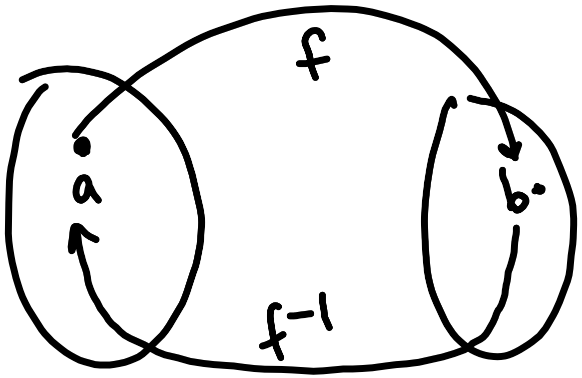
$$xy + 4x = y$$

$$4x = y - xy$$

$$4x = y(1-x)$$

$$\frac{4x}{1-x} = y$$

$$\boxed{f^{-1}(x) = \frac{4x}{1-x}}$$



$f: (x, y)$

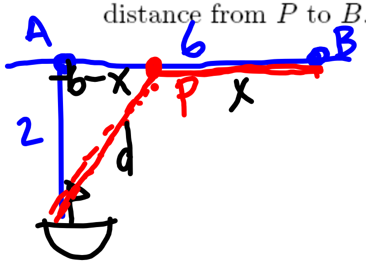
$f^{-1}: (y, x)$

11. (Taken from *Precalculus: Functions and Graphs* by Swokowski/Cole)

A man is in a rowboat that is 2 miles from the nearest point A on a straight shoreline. He wishes to reach his house, which is located at a point B that is 6 miles farther down the shoreline from A. He plans to row to a point P that is between A and B and then walk the remainder of the distance. Suppose he can row at a rate of 3 mi/hr and can walk at a rate of 5 mi/hr.

$d = rt \Rightarrow t = \frac{d}{r}$

(a) If T is the total time required to reach the house, express T as a function of x, where x is the distance from P to B.



$$T = \frac{d}{3} + \frac{x}{5}$$

$$2^2 + (6-x)^2 = d^2$$

$$4 + 36 - 12x + x^2 = d^2$$

$$x^2 - 12x + 40 = d^2$$

$$\sqrt{x^2 - 12x + 40} = d$$

$$\boxed{T(x) = \frac{\sqrt{x^2-12x+40}}{3} + \frac{x}{5}}$$

(b) What is the shortest possible travel time? What distance x will result in the shortest travel time?

Find minimum of $T(x)$ on calculator.

Minimum value of $T(x)$ = minimum travel time = $\boxed{1.73 \text{ hrs}}$

Minimum occurs when $\boxed{x = 4.50 \text{ miles}}$

12. A very large bottle contains 2000 mL of 10% acid solution. An 80% acid solution is being poured into the bottle at a rate of 10 mL/sec.

(a) Express the concentration C of the bottle as a function of time t.

$$\underset{2000\text{ mL}}{\boxed{10\%}} + \underset{10t\text{ mL}}{\boxed{80\%}} = \underset{2000+10t\text{ mL}}{\boxed{C}}$$

After t seconds, we've added $10t$ mL

Acid: $.1(2000) + .8(10t) = C(2000+10t)$

$$200 + 8t = C(2000+10t)$$

$$\boxed{\frac{200+8t}{2000+10t} = C(t)}$$

(C is concentration as a decimal)

(b) When will the concentration be 60%?

For what t does $C = 0.60$?

$$\frac{200+8t}{2000+10t} = 0.60$$

$$200 + 8t = 0.60(2000+10t)$$

$$200 + 8t = 1200 + 6t$$

$$2t = 1000$$

$$\boxed{t = 500 \text{ seconds}}$$

13. Find the average rate of change of the function $f(x) = \dfrac{x^2}{x+1}$ from $x = 5$ to $x = 5 + h$.

$$\frac{f(5+h) - f(5)}{5+h-5} = \frac{\dfrac{(5+h)^2}{5+h+1} - \dfrac{5^2}{5+1}}{h}$$

$$= \frac{\left(\dfrac{(5+h)^2}{6+h} - \dfrac{25}{6}\right)}{h} \cdot \frac{6(6+h)}{6(6+h)}$$

$$= \frac{6(5+h)^2 - 25(6+h)}{6h(6+h)} = \frac{6(25+10h+h^2) - 150 - 25h}{6h(6+h)}$$

$$= \frac{\cancel{150} + 60h + 6h^2 - \cancel{150} - 25h}{6h(6+h)} = \frac{6h^2 + 35h}{6h(6+h)}$$

$$= \frac{\cancel{h}(6h+35)}{6\cancel{h}(6+h)} = \boxed{\frac{6h+35}{6(6+h)}}$$

14. Solve the following equations.

(a) $3x^{\boxed{1/3}} + 2x^{-2/3} - 2x^{-5/3} = 0$

$\frac{1}{3} + {+}\frac{5}{3} = \frac{6}{3} = 2$

$-\frac{2}{3} + {+}\frac{5}{3} = \frac{3}{3} = 1$

$x^{-5/3}(3x^2 + 2x - 2) = 0$

$x^{-5/3} = 0$

$\frac{1}{x^{5/3}} = 0$

Never $= 0$

$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-2)}}{2(3)}$

$= \frac{-2 \pm \sqrt{4+24}}{6} = \frac{-2 \pm \sqrt{28}}{6}$

$= \frac{-2 \pm 2\sqrt{7}}{6} = \boxed{\frac{-1 \pm \sqrt{7}}{3}}$

$(x-2)(x+2)(3x+1)$

(b) $\left(\dfrac{1}{x-2} + \dfrac{11}{3x+1}\right) = \left(\dfrac{28}{\cancel{(x^2-4)}\cancel{(3x+1)}}\right)$ $\cancel{(x-2)(x+2)(3x+1)}$

$(x-2)(x+2)$

$(x+2)(3x+1) + 11(x-2)(x+2) = 28$

$3x^2 + x + 6x + 2 + 11(x^2-4) = 28$

$3x^2 + 7x + 2 + 11x^2 - 44 = 28$

$14x^2 + 7x - 70 = 0$

$7(2x^2 + x - 10) = 0$

$7(2x+5)(x-2) = 0$

$x = -5/2,\ 2$

$x=2$ makes denominator 0 in original equation.

$x=2$ is extraneous

$\boxed{x = -5/2}$

(c) $9x^3 - 18x^2 - 4x + 8 = 0$

$$(9x^3 - 18x^2) + (-4x + 8) = 0$$

$$\underbrace{9x^2(x-2)} - \underbrace{4(x-2)} = 0$$

$$(x-2)(9x^2 - 4) = 0$$

$$(x-2)(3x-2)(3x+2) = 0$$

$$\boxed{x = 2,\ \frac{2}{3},\ -\frac{2}{3}}$$

(d) $\sqrt{4x-19}+4=x$

$$\left(\sqrt{4x-19}\right)^2=(x-4)^2$$

$$4x-19=x^2-8x+16$$

$$0=x^2-12x+35$$

$$0=(x-5)(x-7)$$

$$x=5,7$$

Check: $x=5$: $\sqrt{20-19}+4=5$ ✓

$x=7$: $\sqrt{28-19}+4=7$ ✓

$$\boxed{x=5,7}$$

(e) $16x - 24\sqrt{x} + 9 = 0$

$16(\sqrt{x})^2 - 24(\sqrt{x}) + 9 = 0$

Let $w = \sqrt{x}$

$16w^2 - 24w + 9 = 0$

$(4w - 3)^2 = 0$

$w = \frac{3}{4}$

$\Downarrow$

$\sqrt{x} = \frac{3}{4}$

$\boxed{x = \frac{9}{16}}$

Check:

$16\left(\frac{9}{16}\right) - 24\sqrt{\frac{9}{16}} + 9$

$= 9 - 24\left(\frac{3}{4}\right) + 9$

$= 9 - 18 + 9 = 0$ ✓

15. Solve the following inequalities.

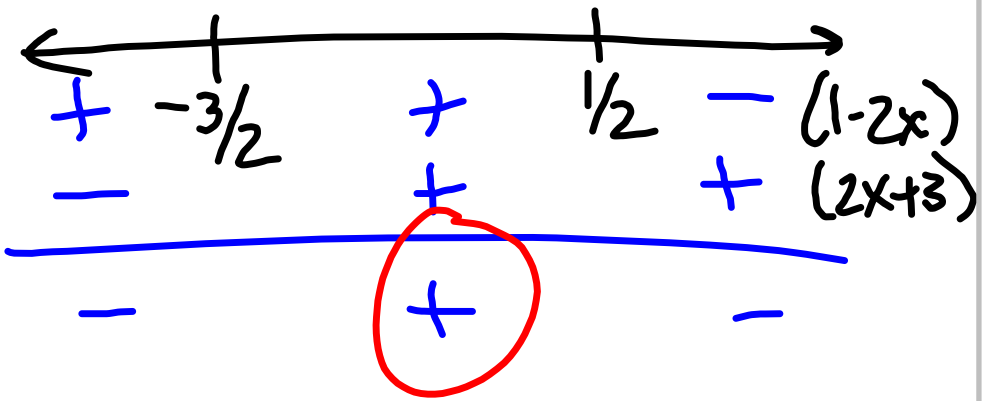
(a) $\dfrac{4}{2x+3} \geq 1$

$$\frac{4}{2x+3} - 1 \geq 0$$

$$\frac{4-(2x+3)}{2x+3} \geq 0$$

$$\frac{4-2x-3}{2x+3} \geq 0$$

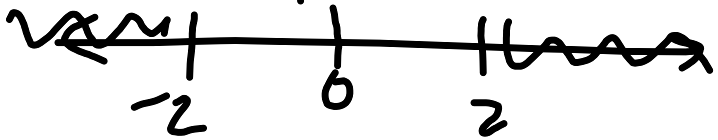
$$\frac{1-2x}{2x+3} \geq 0$$



$$\boxed{\left(-\frac{3}{2}, \frac{1}{2}\right]}$$

(b) $-|6x-11|+5 \le 3$

$|x| \ge 2$



$-|6x-11| \le -2$

$|6x-11| \ge 2$

$6x-11 \le -2$ OR $6x-11 \ge 2$

$6x \le 9$ $\qquad$ $6x \ge 13$

$x \le \frac{9}{6}$ $\qquad$ $x \ge \frac{13}{6}$

$x \le \frac{3}{2}$

$$\boxed{\left(-\infty, \tfrac{3}{2}\right] \cup \left[\tfrac{13}{6}, \infty\right)}$$

16. Simplify the following expression and write without negative exponents: $\left(\frac{25x^4y^{-2}}{z^6}\right)^{3/2}\left(\frac{y^{-3}z}{x^5}\right)^{-3}$

$$\left(\frac{25x^4y^{-2}}{z^6}\right)^{3/2}\left(\frac{y^{-3}z}{x^5}\right)^{-3} = \left(\frac{25^{3/2}x^6y^{-3}}{z^9}\right)\left(\frac{y^9z^{-3}}{x^{-15}}\right)$$

$$= \left(\frac{25^{3/2}x^6}{y^3z^9}\right)\left(\frac{y^9x^{15}}{z^3}\right)$$

$$= \frac{25^{3/2}x^{6}y^{\cancel{9}\,6}x^{15}}{\cancel{y^3}z^9z^3}$$

$$= \frac{25^{3/2}x^{21}y^6}{z^{12}} = \boxed{\frac{125x^{21}y^6}{z^{12}}}$$

$$25^{3/2} = (\sqrt{25})^3 = 5^3 = 125$$

17. Simplify the following expression: $\sqrt[6]{3^{15}x^{22}y^{14}}$

$$\sqrt[6]{3^{15}x^{22}y^{14}} = \sqrt[6]{3^{12}\cdot 3^3\cdot x^{18}\cdot x^4\cdot y^{12}\cdot y^2}$$

$$= \sqrt[6]{(3^2)^6\cdot 3^3\cdot (x^3)^6\cdot x^4 (y^2)^6\cdot y^2}$$

(Need absolute value because we're taking an <u>even</u> root.)

$$= |3^2x^3y^2|\cdot \sqrt[6]{3^3x^4y^2}$$

$$= |3^2|\cdot|x^3|\cdot|y^2|\cdot \sqrt[6]{27x^4y^2}$$

$$\boxed{= 9|x^3|\cdot y^2\cdot \sqrt[6]{27x^4y^2} \quad \text{OR} \quad 9|x|^3y^2\sqrt[6]{27x^4y^2}}$$

Since y^2 is always positive, $|y^2| = y^2$.

18. Find the center and radius of the circle $x^2 + y^2 + 8x - 10y + 37 = 0$.

Complete the square for both x and y terms:

$$\underbrace{(x^2 + 8x + 16)}_{16 = \left(\frac{8}{2}\right)^2} + \underbrace{(y^2 - 10y + 25)}_{9 = \left(\frac{-10}{2}\right)^2} = -37 + 16 + 25$$

$$(x+4)^2 + (y-5)^2 = 4$$

Standard form: $(x-h)^2 + (y-k)^2 = r^2$

Center: (h, k) ; Radius r.

$$\boxed{\text{Center: } (-4, 5) \quad \text{Radius} = 2}$$

19. Consider the points $(4,6)$ and $(-6,2)$.

(a) Find the distance between these points.

$$\sqrt{(2-6)^2+(-6-4)^2} = \sqrt{16+100}$$
$$= \sqrt{116}$$
$$= \sqrt{4\cdot 29}$$
$$= \boxed{2\sqrt{29}}$$

(b) Find the equation of the line that is parallel to the line $5x + 8y = 12$ and passes through the midpoint of the line segment between these points.

$$5x+8y=12$$
$$8y=-5x+12$$
$$y=-\frac{5}{8}x+\frac{3}{2}$$

Slope is $-\frac{5}{8}$. (Nonvertical) Parallel lines have equal slope. So the slope of the line we want is $m=-\frac{5}{8}$

Midpoint: $\left(\frac{4+(-6)}{2}, \frac{6+2}{2}\right)$
$$=(-1,4)$$

Point-Slope Form: $y-y_1=m(x-x_1)$
$$y-4=-\frac{5}{8}(x+1)$$
$$y-4=-\frac{5}{8}x-\frac{5}{8}$$
$$y=-\frac{5}{8}x-\frac{5}{8}+4$$
$$y=-\frac{5}{8}x-\frac{5}{8}+\frac{32}{8}$$
$$\boxed{y=-\frac{5}{8}x+\frac{27}{8}}$$

20. Consider the function $f(x) = \sqrt{|x-2|} + x^4 - 5x^2 + 2x + 3$.

(a) Find the x-intercepts of f.

Graph on Calculator.

Use "zero" command to find x-intercepts:

$\boxed{(-2.2066, 0) \text{ and } (-0.8368, 0)}$

(b) Where is f decreasing? → function goes down from left to right.

Use "maximum" and "minimum" commands to find points.

f is decreasing on the intervals:

$(-\infty, -1.6618) \cup (0.1649, 1.5125)$

(c) What is the range of f?

$\boxed{[-4.5916, \infty)}$

y-values that f(x) achieves.

(d) Solve the equation $f(x) = x^{1/3} - 2$.

Leave Y_1 as $f(x)$

Set $Y_2 = x^{1/3} - 2$

Use "intersect" command to find where the functions are equal ⇒ where they intersect.

$\boxed{x = -1.9739, -1.2515}$

21. True or False

(a) TRUE FALSE To rationalize the numerator of $\dfrac{\sqrt{x}+10}{x^2}$, multiply numerator and denominator by $\sqrt{x}+10$.

Multiply num. and den. by $\sqrt{x}-10$

(b) TRUE FALSE If L_1 has slope -4 and L_2 is perpendicular to L_1, then L_2 has slope 4.

negative reciprocal. L_2 should have slope $\frac{1}{4}$.

(c) TRUE FALSE The equation $y = -\frac{1}{20}x^2 + x - 5$ has exactly one real solution.

$D = b^2 - 4ac = 1 - 4\left(-\frac{1}{20}\right)(-5) = 1 - 1 = 0$

(Discriminant) If $D = 0 \Rightarrow$ exactly 1 real solution.

(d) TRUE FALSE A graph that is symmetric about the x-axis and the y-axis must also be symmetric about the origin.