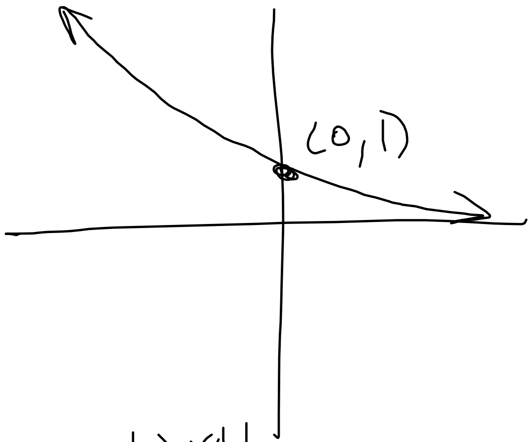
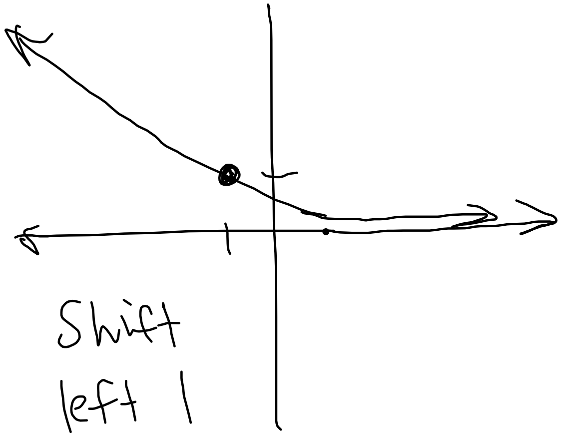
1. Sketch graphs for the following exponential and logarithmic functions by using transformations. Find the domain, range, and asymptotes for each.

   (a) $f(x) = -(\frac{1}{4})^{x+1} + 2$

$\left(\frac{1}{4}\right)^{x}$

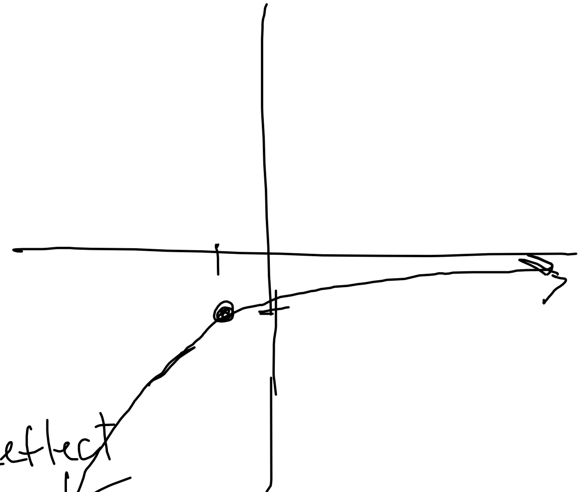


$\left(\frac{1}{4}\right)^{x+1}$



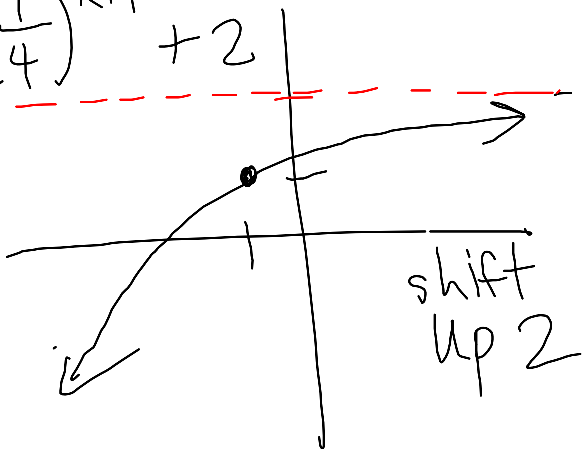
Shift left 1

$-\left(\frac{1}{4}\right)^{x+1}$



Reflect across x-axis

$-\left(\frac{1}{4}\right)^{x+1} + 2$



shift up 2

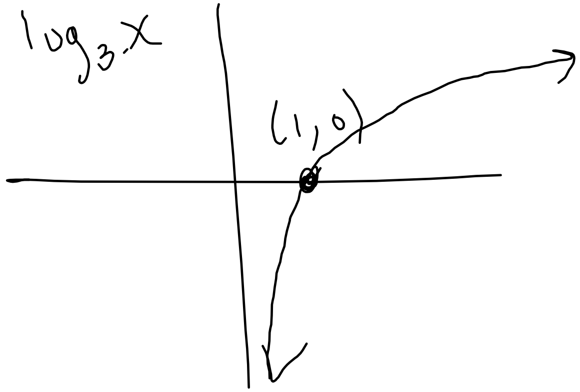
Domain: $\mathbb{R}$: $(-\infty, \infty)$

Range: $(-\infty, 2)$

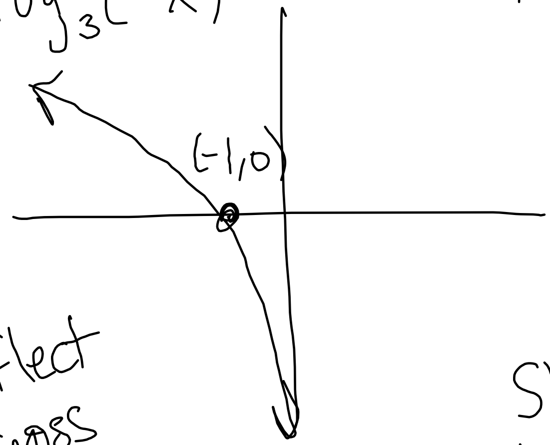
Horizontal Asymptote: $y=2$

(b) $f(x) = \log_3(-x) - 1$

$\log_3 x$

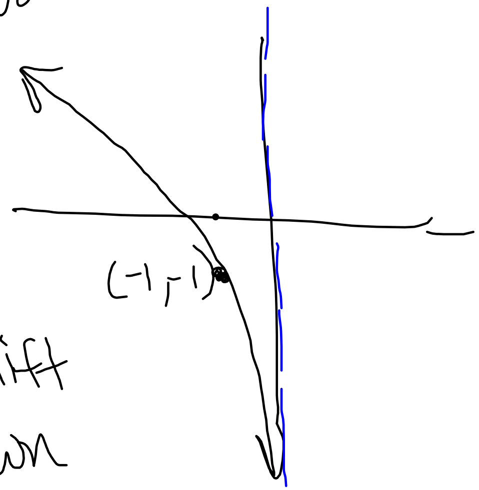


$\log_3(-x)$



Reflect across y-axis

$\log_3(-x) - 1$



Shift down 1.

Domain: $(-\infty, 0)$

Range: $\mathbb{R}$; $(-\infty, \infty)$

Vertical Asymptote: $x = 0$

2. Find the domain of the logarithmic function $f(x) = \log_2(\underbrace{x^2 - 9x + 18})$.

$$x^2 - 9x + 18 > 0$$

$$(x-6)(x-3) > 0$$

		3		6		
	$-$		$-$		$+$	$x-6$
	$-$		$+$		$+$	$x-3$
	$+$		$-$		$+$	

$$\boxed{\text{Domain: } (-\infty, 3) \cup (6, \infty)}$$

$$\log_a AB = \log_a A + \log_a B$$

$$\log_a \left(\frac{A}{B}\right) = \log_a A - \log_a B$$

$$\log_a A^c = c \log_a A$$

3. Use the Laws of Logarithms to fully expand the following expressions.

(a) $\ln\left(\dfrac{3x^5(x+2)^2}{\sqrt[4]{y}}\right)$

$$= \ln 3x^5(x+2)^2 - \ln \sqrt[4]{y}$$

$$= \ln 3 + \ln x^5 + \ln (x+2)^2 - \ln y^{1/4}$$

$$= \boxed{\ln 3 + 5\ln x + 2\ln(x+2) - \tfrac{1}{4}\ln y}$$

$$\ln(x+2) \neq \ln x + \ln 2$$

(b) $(\log 4xy)^2 \log \sqrt[5]{7x+14}$

$$= (\log 4 + \log x + \log y)^2 \log (7x+14)^{1/5}$$

$$= (\log 4 + \log x + \log y)^2 \cdot \tfrac{1}{5}\log(7x+14)$$

$$= (\log 4 + \log x + \log y)^2 \tfrac{1}{5}\log 7(x+2)$$

$$= \boxed{(\log 4 + \log x + \log y)^2 \cdot \tfrac{1}{5}\left[\log 7 + \log(x+2)\right]}$$

4. Use the Laws of Logarithms to combine the following expressions. Write as a single logarithm when possible.

(a) $\frac{1}{3}\log(x-7) - 2\log y - 5\log(z^2+16)$

$$\underbrace{\log(x-7)^{1/3} - \log y^2} - \log(z^2+16)^5$$

$$= \log\left(\frac{\sqrt[3]{x-7}}{y^2}\right) - \log(z^2+16)^5$$

$$= \boxed{\log\left(\frac{\sqrt[3]{x-7}}{y^2(z^2+16)^5}\right)}$$

$$\frac{\frac{\sqrt[3]{x-7}}{y^2}}{(z^2+16)^5} = \frac{\sqrt[3]{x-7}}{y^2(z^2+16)^5}$$

(b) $\dfrac{2\ln x - 4\ln\frac{1}{y} - 3\ln xy}{4\ln x + \ln y}$

$$= \frac{\ln x^2 - \ln\left(\frac{1}{y}\right)^4 - \ln(xy)^3}{\ln x^4 + \ln y} = \frac{\ln\left(\frac{x^2}{\left(\frac{1}{y}\right)^4 (xy)^3}\right)}{\ln(x^4 y)}$$

$$= \frac{\ln\left(\frac{x^2}{\frac{1}{y^4}\cdot x^3 \cdot y^3}\right)}{\ln(x^4 y)} = \boxed{\frac{\ln\left(\frac{y}{x}\right)}{\ln(x^4 y)}}$$

$$\frac{x^2}{\frac{1}{y^4}\cdot x^3 y^3} = \frac{x^2}{y^{-4}x^3y^3} = \frac{y^{\cancel{4}}\cancel{x^2}}{x^{\cancel{3}}\cancel{y^3}} = \frac{y}{x}$$

5. Evaluate the following logarithmic expressions.

(a) $\log_3 \sqrt{27} = x$

$$3^x = \sqrt{27}$$
$$3^x = \sqrt{3^3}$$
$$3^x = (3^3)^{1/2}$$
$$3^x = 3^{3/2}$$
$$\boxed{x = 3/2}$$

(b) $\log_{10} 10000 + \ln \frac{1}{e^5} + 4^{\log_4 7}$

$$4 + \ln e^{-5} + 4^{\log_4 7}$$
$$4 + (-5) + 7 = \boxed{6}$$

$$\ln e^{-5} = \log_e e^{-5}$$

(c) $\log_4 5 - \log_4 80$

$$= \log_4\left(\frac{5}{80}\right) = \log_4\left(\frac{1}{16}\right) = \log_4 4^{-2} = \boxed{-2}$$

(d) $\log_2 144 + \log_2 9 - \log_2 81$

$$\log_2\left(\frac{144 \cdot 9}{81}\right) = \log_2(16) = \boxed{4}$$
$$= \log_2 2^4 =$$

(e) $\log_3 7$

$$\log_3 7 = \frac{\log 7}{\log 3} =$$
$$\log_3 7 = \frac{\ln 7}{\ln 3} =$$
$$\approx \boxed{1.7712}$$

6. Solve the following equations for x.

(a) $125^{4x+1} = 25$

$$(5^3)^{4x+1} = 5^2$$

$$5^{12x+3} = 5^2 \implies \log 5^{12x+3} = \log 5^2$$

$$(12x+3)\log 5 = 2 \log 5$$

$$12x+3 = 2$$

$$12x+3 = 2$$

$$12x = -1$$

$$\boxed{x = -1/12}$$

(b) $\log_x 8 = \frac{3}{4}$

$$x^{3/4} = 8 \implies (\sqrt[4]{x})^3 = 8$$

$$(\sqrt[4]{x})^4 = (2)^4$$

$$\boxed{x = 16}$$

(c) $6^{3x-7} = 2$

$$\log 6^{3x-7} = \log 2$$

$$(3x-7)\log 6 = \log 2$$

$$3x-7 = \frac{\log 2}{\log 6}$$

$$3x = \frac{\log 2}{\log 6} + 7$$

$$\boxed{x = \frac{1}{3}\left[\frac{\log 2}{\log 6} + 7\right] = \frac{1}{3}\left[\log_6 2 + 7\right]}$$

(d) $\ln(x+1) - \ln 4 = 3$

$$\ln\left(\frac{x+1}{4}\right) = 3$$

$$\log_e\left(\frac{x+1}{4}\right) = 3$$

$$e^3 = \frac{x+1}{4}$$

$$4e^3 = x+1$$

$$\boxed{4e^3 - 1 = x}$$

(e) $2e^{-20x} = 3$

$$e^{-20x} = \frac{3}{2}$$

$$\ln e^{-20x} = \ln\left(\frac{3}{2}\right)$$

$$-20x = \ln\left(\frac{3}{2}\right)$$

$$\boxed{x = \frac{\ln\left(\frac{3}{2}\right)}{-20} = \frac{\ln 3 - \ln 2}{-20}}$$

$\log_a x = y$

$a^y = x$

(f) $\log_3(x-6) + \log_3(x+2) = 2$

$$\log_3 (x-6)(x+2) = 2$$

$$3^2 = (x-6)(x+2)$$

$$9 = x^2 - 4x - 12$$

$$0 = x^2 - 4x - 21$$

$$0 = (x-7)(x+3)$$

$$x = 7, \cancel{-3}$$

$$\boxed{x = 7}$$

$x = -3$ is extraneous

(g) $2^{2x+3} = 3^{x-2}$

$$\log 2^{2x+3} = \log 3^{x-2}$$

$$(2x+3)\log 2 = (x-2)\log 3$$

$$2x\log 2 + 3\log 2 = x\log 3 - 2\log 3$$

$$2x\log 2 - x\log 3 = -2\log 3 - 3\log 2$$

$$x(2\log 2 - \log 3) = -2\log 3 - 3\log 2$$

$$\boxed{x = \frac{-2\log 3 - 3\log 2}{2\log 2 - \log 3}} = \frac{-\log 3^2 - \log 2^3}{\log 2^2 - \log 3} = \frac{\log\left(\frac{1}{9\cdot 8}\right)}{\log\left(\frac{4}{3}\right)}$$

$$= \boxed{\frac{\log\left(\frac{1}{72}\right)}{\log\left(\frac{4}{3}\right)}}$$

(h) $\log(x-4) - \log(3x-10) = \log(\frac{1}{x})$

$$\log\left(\frac{x-4}{3x-10}\right) = \log\left(\frac{1}{x}\right)$$

$$\frac{x-4}{3x-10} = \frac{1}{x}$$

$$x(x-4) = 3x-10$$

$$x^2 - 4x = 3x - 10$$

$$x^2 - 7x + 10 = 0$$

$$(x-5)(x-2) = 0$$

$$x = 5, \cancel{2}$$

$$\boxed{x = 5}$$

$x = 2$ is extraneous

If $\log A = \log B$
then $A = B$

$$\log A = \log B$$

$$\log A - \log B = 0$$

$$\log\left(\frac{A}{B}\right) = 0$$

$$10^0 = \frac{A}{B}$$

$$1 = \frac{A}{B}$$

$$B = A$$

(i) $e^{4x} - e^{2x} - 20 = 0$ Let $w = e^{2x}$ $e^{4x} = (e^{2x})^2 = w^2$

$w^2 - w - 20 = 0$

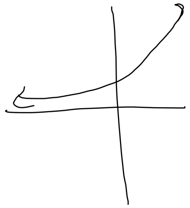
$(w-5)(w+4) = 0$

$(e^{2x}-5)(e^{2x}+4) = 0$

$e^{2x} - 5 = 0$ $e^{2x} + 4 = 0$

$e^{2x} = 5$ $e^{2x} = -4$

$\ln e^{2x} = \ln 5$ $\cancel{\ln e^{2x} = \ln(-4)}$



$2x = \ln 5$

$\boxed{x = \frac{\ln 5}{2}}$

(j) $\log_3 x - \underbrace{\log_9(x+42)} = 0$

$$\log_3 x - \frac{\log_3(x+42)}{\log_3 9} = 0$$

$$2\left(\log_3 x - \frac{\log_3(x+42)}{2}\right) = (0)2$$

$$2\log_3 x - \log_3(x+42) = 0$$

$$\log_3 x^2 - \log_3(x+42) = 0$$

$$\log_3\left(\frac{x^2}{x+42}\right) = 0$$

$$3^0 = \frac{x^2}{x+42}$$

$$1 = \frac{x^2}{x+42}$$

$$x + 42 = x^2$$

$$0 = x^2 - x - 42$$

$$0 = (x-7)(x+6)$$

$$x = 7, \cancel{-6}$$

$$\boxed{x = 7}$$

-6 is extraneous

7. (a) If $2000 is invested in an account at an interest rate of 6%/yr compounded quarterly, how much is ~~in the~~ account after 21 months?

$P=$ $\quad r = .06 \quad n=4$

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

21 months $= \frac{21}{12}$ yrs

$t = \frac{21}{12} = \frac{7}{4}$

$$A(t) = 2000\left(1+\frac{.06}{4}\right)^{4t}$$

$$A\left(\tfrac{7}{4}\right) = 2000\left(1+\frac{.06}{4}\right)^{4\left(\frac{7}{4}\right)}$$

$$\approx \boxed{\$2219.69}$$

(b) When will the account have $5000 in it?

$$5000 = 2000\left(1+\frac{.06}{4}\right)^{4t}$$

$$\frac{5}{2} = \left(1+\frac{.06}{4}\right)^{4t}$$

$$\log\frac{5}{2} = \log\left(1+\frac{.06}{4}\right)^{4t}$$

$$\log\frac{5}{2} = 4t\cdot\log\left(1+\frac{.06}{4}\right)$$

$$\frac{\log\frac{5}{2}}{4\log\left(1+\frac{.06}{4}\right)} = t$$

$$\boxed{t \approx 15.39 \text{ yrs}}$$

(c) If interest was compounded continuously, how long would it take to have $5000 in the account?

$$A(t) = Pe^{rt}$$

$$A(t) = 2000e^{.06t}$$

$$5000 = 2000e^{.06t}$$

$$\frac{5}{2} = e^{.06t}$$

$$\ln\frac{5}{2} = \ln e^{.06t}$$

$$\ln\frac{5}{2} = .06t$$

$$\frac{\ln\frac{5}{2}}{.06} = t$$

$$\boxed{t \approx 15.27 \text{ yrs}}$$