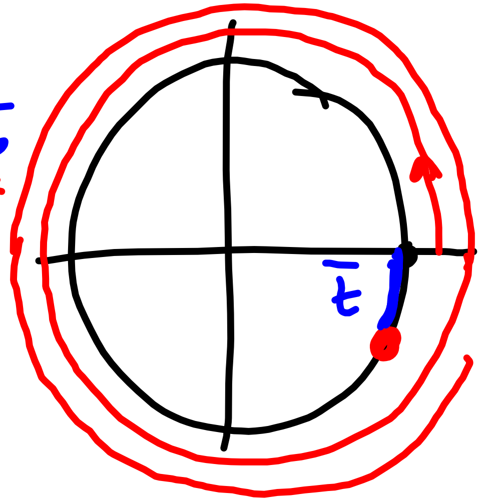
1. Find the terminal points determined by the following values of t.

(a) $t = \frac{23\pi}{6}$

$\frac{23}{6} = 3\frac{5}{6}$



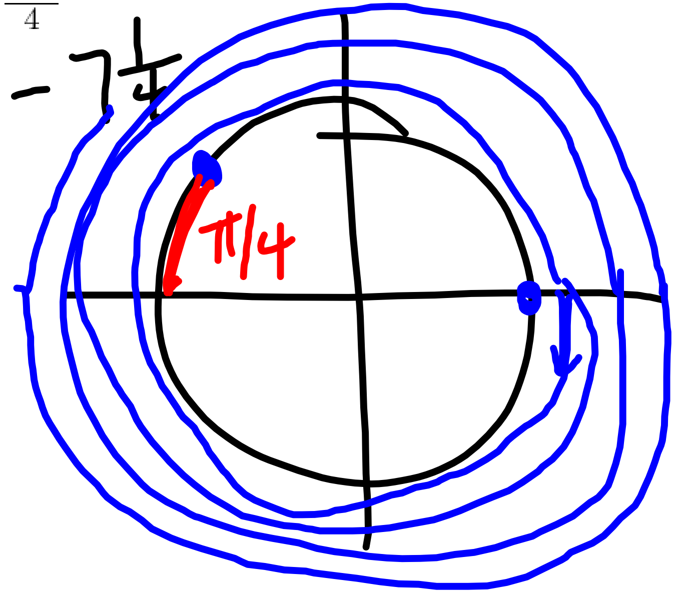
$\bar{t} = \frac{\pi}{6}$

$\boxed{\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)}$

In QIV, x+, y−

(b) $t = -\frac{29\pi}{4}$

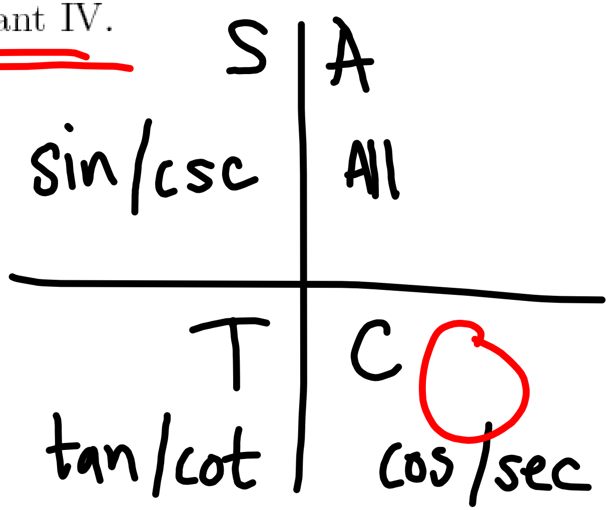
$-\frac{29}{4} = -7\frac{1}{4}$



$\boxed{\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)}$

In QII, x−, y+

2. Determine the sign of the expression $\sec t \csc t \tan^2 t$ in Quadrant IV.



$\sec t > 0$ + in QIV

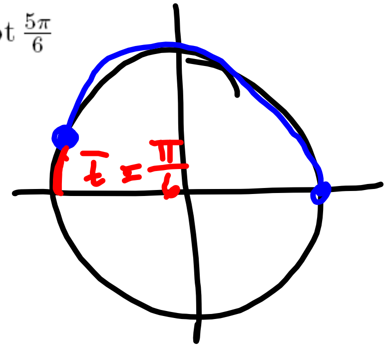
$\csc t < 0$ − in QIV

$\tan^2 t > 0$ + always

$\boxed{-}$ Negative.

3. Evaluate the following.

(a) $\cot \frac{5\pi}{6}$

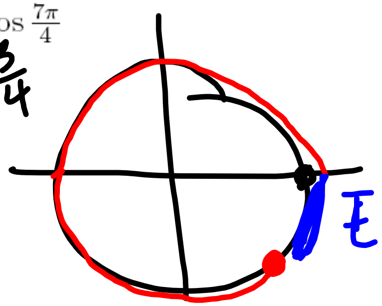


In QII, $\cot t < 0$

$$\cot\left(\frac{5\pi}{6}\right) = -\cot\left(\frac{\pi}{6}\right) = -\frac{\cos\left(\frac{\pi}{6}\right)}{\sin\left(\pi/6\right)} = -\frac{\sqrt{3}/2}{1/2} = \boxed{-\sqrt{3}}$$

(b) $\cos \frac{7\pi}{4}$

$\frac{7}{4} = 1\frac{3}{4}$



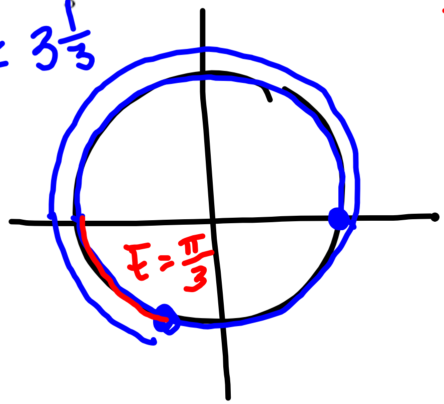
$t = \frac{\pi}{4}$

In QIV, $\cos t > 0$

$$\cos\left(\frac{7\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \boxed{\frac{1}{\sqrt{2}}}$$

(c) $\csc \frac{10\pi}{3}$

$\frac{10}{3} = 3\frac{1}{3}$



In QIII, $\csc t < 0$

$$\csc\left(\frac{10\pi}{3}\right) = -\csc\left(\frac{\pi}{3}\right) = -\frac{1}{\sin\frac{\pi}{3}} = -\frac{1}{\sqrt{3}/2} = \boxed{-\frac{2}{\sqrt{3}}}$$

4. The terminal point of a number t is $(-\frac{4\sqrt{3}}{7}, y)$ and is in Quadrant II.

(a) Find the y-coordinate.

$$x^2 + y^2 = 1$$

$$\left(\frac{-4\sqrt{3}}{7}\right)^2 + y^2 = 1$$

$$\frac{16 \cdot 3}{49} + y^2 = 1$$

$$\frac{48}{49} + y^2 = 1$$

$$y^2 = 1 - \frac{48}{49} = \frac{1}{49}$$

$$y = \pm \frac{1}{7}$$

In QII, $y > 0$

$$\boxed{y = \frac{1}{7}}$$

(b) Find all trig values of t.

$$\left(-\frac{4\sqrt{3}}{7}, \frac{1}{7}\right)$$

$$\sin t = y = \frac{1}{7} \qquad \csc t = \frac{1}{\sin t} = 7$$

$$\cos t = x = -\frac{4\sqrt{3}}{7} \qquad \sec t = \frac{1}{\cos t} = \frac{-7}{4\sqrt{3}}$$

$$\tan t = \frac{\sin t}{\cos t} = \frac{1/7}{-4\sqrt{3}/7} = -\frac{1}{4\sqrt{3}} \qquad \cot t = \frac{1}{\tan t} = -4\sqrt{3}$$

5. Find all other trig values of t if $\csc t = -5$ and the terminal point of t is in Quadrant III.

$$\boxed{\csc t = -5}$$

$$\frac{1}{\sin t} = -5 \implies \boxed{\sin t = -\frac{1}{5}}$$

$$\sin^2 t + \cos^2 t = 1$$

$$\left(-\frac{1}{5}\right)^2 + \cos^2 t = 1$$

$$\frac{1}{25} + \cos^2 t = 1$$

$$\cos^2 t = \frac{24}{25}$$

$$\cos t = \pm\sqrt{\frac{24}{25}} = \pm\frac{2\sqrt{6}}{5}$$

In QIII, $\cos t < 0$, so $\boxed{\cos t = \frac{-2\sqrt{6}}{5}}$

$$\tan t = \frac{\sin t}{\cos t} = \frac{-1/5}{-2\sqrt{6}/5} = \boxed{\frac{1}{2\sqrt{6}} = \frac{\sqrt{6}}{12}}$$

$$\sec t = \frac{1}{\cos t} = \frac{1}{(-2\sqrt{6}/5)} = \boxed{-\frac{5}{2\sqrt{6}} = \frac{-5\sqrt{6}}{12}}$$

$$\cot t = \frac{1}{\tan t} = \boxed{2\sqrt{6}}$$

6. Express $\sin t$ in terms of $\cot t$ if the terminal point of t is in Quadrant IV.

$$1 + \cot^2 t = \csc^2 t \quad \text{(a Pythagorean Identity)}$$

$$1 + \cot^2 t = \frac{1}{\sin^2 t}$$

$$\sin^2 t = \frac{1}{1+\cot^2 t}$$

$$\sin t = \pm\sqrt{\frac{1}{1+\cot^2 t}}$$

In QIV, $\sin t < 0$

$$\boxed{\sin t = -\sqrt{\frac{1}{1+\cot^2 t}}}$$

7. Find the amplitude, period, and phase shift, and sketch a graph of the following function.
$f(x) = -3\sin(2x - \frac{2\pi}{3})$

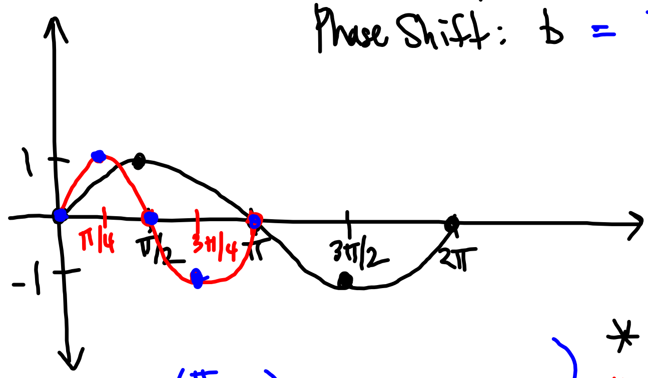
$y = a\sin k(x-b)$

$= -3\sin 2(x - \frac{\pi}{3})$

Amplitude: $|a| = |-3| = 3$

Period: $\frac{2\pi}{k} = \frac{2\pi}{2} = \pi$

Phase Shift: $b = \frac{\pi}{3}$



$(0,0) \to (\frac{\pi}{3}, 0)$

$(\frac{\pi}{4}, 1) \to (\frac{7\pi}{12}, 1)$ $\quad \frac{\pi}{4} + \frac{\pi}{3} = \frac{7\pi}{12}$

$(\frac{\pi}{2}, 0) \to (\frac{5\pi}{6}, 0)$ $\quad \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$

$(\frac{3\pi}{4}, -1) \to (\frac{13\pi}{12}, -1)$ $\quad \frac{3\pi}{4} + \frac{\pi}{3} = \frac{13\pi}{12}$

$(\pi, 0) \to (\frac{4\pi}{3}, 0)$

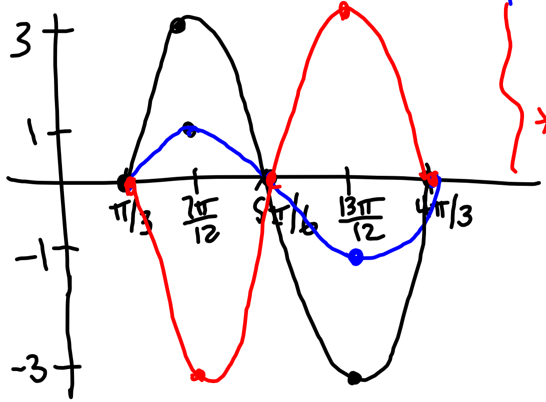
* : $\sin x$

* $\sin 2x$
- Horizontally shrink by $1/2$.

* $\sin 2(x - \pi/3)$
Shift right $\pi/3$.

* $3\sin 2(x - \pi/3)$
Vertically stretch by 3

* $-3\sin 2(x - \pi/3)$
Reflect across x-axis



8. Find the period and describe how the following functions would be graphed.

(a) $f(x) = \tan(\frac{1}{4}x + 3\pi)$

$= \tan \frac{1}{4}(x + 12\pi)$

$y = \tan k(x - b)$

Period: $\frac{\pi}{k}$

Period: $\frac{\pi}{(1/4)} = 4\pi$

Shift: -12π

Start with $\tan x$, horizontally stretch by a factor of 4, shift left 12π.

(b) $f(x) = 2\sec(5x - \pi) + 1$

$= 2\sec 5\left(x - \frac{\pi}{5}\right) + 1$

Period: $\frac{2\pi}{5}$

Shift: $\frac{\pi}{5}$

Start with sec x, horizontally shrinks by 1/5, shift right $\frac{\pi}{5}$, vertically stretch by 2, shift up 1.

9. A population that grows exponentially quadruples in size in 7 days. Find the initial population if it is known that the population is 2000 after 10 days.

$$n(t) = n_0 e^{rt}$$

When $t=7$, $n(t) = 4n_0$

$$4\cancel{n_0} = \cancel{n_0} e^{r(7)}$$

$$4 = e^{7r}$$

$$\ln 4 = \ln e^{7r} = 7r$$

$$\frac{\ln 4}{7} = r$$

$$n(t) = n_0 e^{\frac{\ln 4}{7} \cdot t}$$

When $t=10$, $n(t) = 2000$

$$2000 = n_0 e^{\frac{\ln 4}{7} \cdot 10}$$

$$\boxed{n_0 = \frac{2000}{e^{\frac{\ln 4}{7} \cdot 10}} \approx 276}$$

10. Suppose the amount of a radioactive substance in grams after t years is modeled by the equation $m(t) = 25e^{-0.05t}$.

(a) What is the half-life of this substance?

When $t = h$, $m(t) = \frac{25}{2}$.

$$h = \frac{\ln 2}{r} = \boxed{\frac{\ln 2}{0.05}}$$

$$\frac{25}{2} = 25e^{-0.05h}$$

$$\frac{1}{2} = e^{-0.05h}$$

$$\ln\left(\frac{1}{2}\right) = \ln e^{-0.05h} = -0.05h$$

$$h = \frac{\ln\left(\frac{1}{2}\right)}{-0.05} = \frac{\cancel{\ln 1} - \ln 2}{-0.05} = \boxed{\frac{\ln 2}{0.05}}$$

(b) When will there be 7 grams remaining?

$$7 = 25e^{-0.05t}$$

$$\frac{7}{25} = e^{-0.05t}$$

$$\ln\left(\frac{7}{25}\right) = \ln e^{-0.05t}$$

$$\ln\left(\frac{7}{25}\right) = -0.05t$$

$$\boxed{\frac{\ln\left(\frac{7}{25}\right)}{-0.05} = t \approx 25.459 \text{ years}}$$

11. Evaluate $\frac{\log_4 8}{\log_4 32} \cdot (\log_4 \frac{3}{8} + \log_4 \frac{1}{6})$.

$$\frac{\log_4 8}{\log_4 32}\left(\log_4\left(\frac{3}{8}\cdot\frac{1}{6}\right)\right)$$

$$= \frac{\log_4 8}{\log_4 32} \cdot \log_4 \frac{1}{16} = \frac{\frac{3}{2}}{\frac{5}{2}} \cdot -2 = \frac{3}{\cancel{2}} \cdot \frac{\cancel{2}}{5}(-2)$$

$$= \boxed{\frac{-6}{5}}$$

$$\log_4 8 = x \Rightarrow 4^x = 8$$
$$(2^2)^x = 2^3$$
$$2^{2x} = 2^3$$
$$2x = 3$$
$$x = 3/2$$

$$\log_4 \frac{1}{16} = \log_4 4^{-2}$$
$$= -2$$

$$\log_4 32 = y \Rightarrow 4^y = 32$$
$$(2^2)^y = 2^5$$
$$2^{2y} = 2^5$$
$$2y = 5$$
$$y = 5/2$$

12. Solve the following equations.

(a) $2^{3x-4} = 6e^x$

$$\ln 2^{3x-4} = \ln 6e^x$$

$$(3x-4)\ln 2 = \ln 6 + \ln e^x$$

$$(3x-4)\ln 2 = \ln 6 + x$$

$$3x\ln 2 - 4\ln 2 = \ln 6 + x$$

$$3x\ln 2 - x = \ln 6 + 4\ln 2$$

$$x(3\ln 2 - 1) = \ln 6 + 4\ln 2$$

$$\boxed{x = \frac{\ln 6 + 4\ln 2}{3\ln 2 - 1}}$$

(b) $\log_{16} x + 2\log_{16}(x-2) - \log_{16}(3x-4) = \frac{1}{4}$

$$\log_{16} x + \log_{16}(x-2)^2 - \log_{16}(3x-4) = \frac{1}{4}$$

$$\log_{16}\left(\frac{x\cdot(x-2)^2}{3x-4}\right) = \frac{1}{4}$$

$$16^{1/4} = \frac{x(x-2)^2}{3x-4}$$

$$2 = \frac{x(x-2)^2}{3x-4}$$

$$6x-8 = x(x-2)^2$$

$$6x-8 = x(x^2-4x+4)$$

$$6x-8 = x^3-4x^2+4x$$

$$0 = \underbrace{x^3-4x^2}-\underbrace{2x+8}$$

$$0 = x^2(x-4)-2(x-4)$$

$$0 = (x-4)(x^2-2)$$

$$x-4=0 \qquad x^2-2=0$$

$$x=4 \qquad x^2=2$$

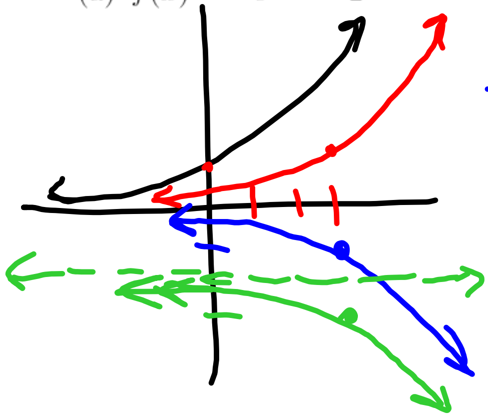
$$x=\underbrace{\pm\sqrt{2}}$$

$$\boxed{x=4}$$

extraneous solutions

13. Find the domain, range, and asymptotes of the following functions.

(a) $f(x) = -e^{x-3} - 2$



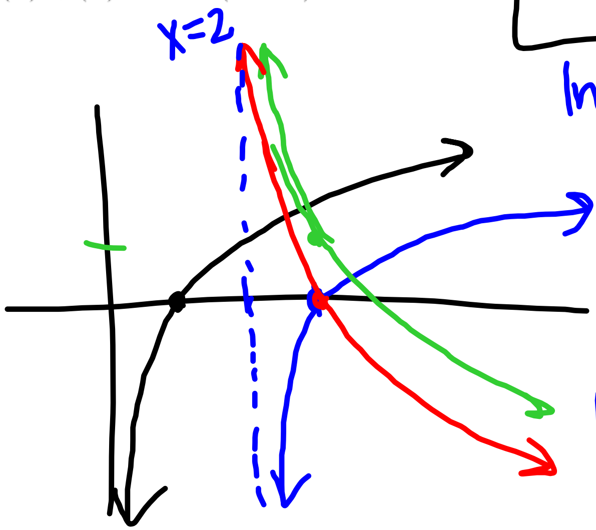
e^{x-3}: Shift right 3

$-e^{x-3}$: Reflect across x-axis

$-e^{x-3} - 2$: Shifts down 2

$\boxed{\text{HA: } y = -2 \quad \text{Domain: } \mathbb{R}\ (-\infty, \infty) \quad \text{Range: } (-\infty, -2)}$

(b) $f(x) = -\ln(x-2) + 1$



$\ln(x-2)$: Shift right 2

$-\ln(x-2)$: Reflects across x-axis

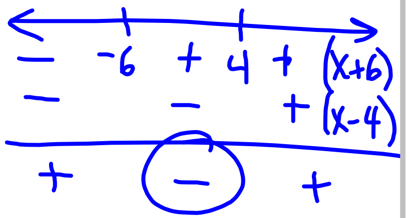
$-\ln(x-2)+1$: Shift up 1

$\boxed{\text{VA: } x = 2 \quad \text{Domain: } (2, \infty) \quad \text{Range: } \mathbb{R} : (-\infty, \infty)}$

14. Find the domains of the following functions.

(a) $f(x) = \log_9(\underbrace{-x^2 - 2x + 24}).$

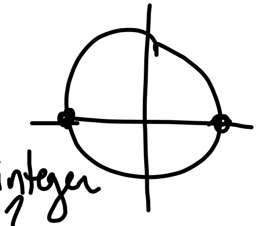
$$-x^2 - 2x + 24 > 0$$
$$x^2 + 2x - 24 < 0$$
$$(x+6)(x-4) < 0$$
$$\boxed{(-6, 4)}$$



(b) $f(x) = \csc x$

$$f(x) = \csc x = \frac{1}{\sin x}$$

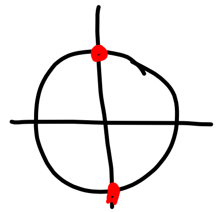
undefined when $\sin x = 0$



Domain : $\{x \mid x \neq n\pi$, for any integer $n\}$

(c) $f(x) = \dfrac{\tan x}{x^2 - 9}$ ✓ $x^2 - 9 \neq 0$

$$x^2 \neq 9$$
$$x \neq 3, -3$$



$\tan x = \dfrac{\sin x}{\cos x}$: undefined when $\cos x = 0$.

Domain: $\{x \mid x \neq -3, 3, \frac{\pi}{2} + n\pi$ for any integer $n\}$

(d) $f(x) = \dfrac{\ln(-3x+5)}{e^{4x}}$

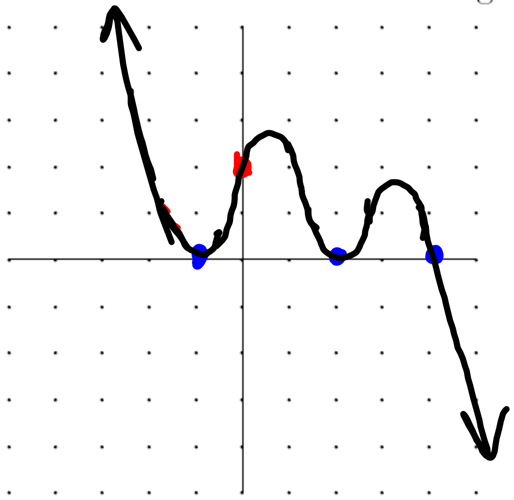
$$e^{4x} \neq 0$$

$\ln(-3x+5)$ defined when $-3x + 5 > 0$

$$-3x > -5$$
$$x < 5/3$$

$$\boxed{(-\infty, 5/3)}$$

15. Describe the end behavior and graph the polynomial $P(x) = -\frac{1}{8}(x-2)^2(x+1)^2(x-4)$.



Zeros: (x-int.)

$x = 2, -1, 4$

$(2,0), (-1,0), (4,0)$

y-int: $P(0) = -\frac{1}{8}(0-2)^2(0+1)^2 \cdot (0-4)$

$= -\frac{1}{8} \cdot 4 \cdot 1 \cdot (-4)$

$= 2$

$(0,2)$

$x = -1$ has multiplicity 2 (touches)

$x = 2$ " " " (touches)

$x = 4$ " " 1 (crosses)

Leading Term: $-\frac{1}{8}x^5 \ldots\ldots$

Degree: 5 : odd

Leading Coeff.: $-\frac{1}{8}$: Negative

As $x \to \infty$, $y \to -\infty$

As $x \to -\infty$, $y \to \infty$

16. Graph the rational function $r(x) = \dfrac{(3x^2-27)(x^2+2x-35)}{(x^2-6x+5)(x+4)^2}$.

$$= \frac{3(x^2-9)(x+7)(x-5)}{(x-5)(x-1)(x+4)^2}$$

$$= \frac{3(x-3)(x+3)(x+7)\cancel{(x-5)}}{\cancel{(x-5)}(x-1)(x+4)^2}$$

$$= \frac{3(x-3)(x+3)(x+7)}{(x-1)(x+4)^2}, x \neq 5$$

Hole at $x=5$

VA: $x=1$, $x=-4$

HA:

$$r(x) = \frac{3x^3 + \cdots}{x^3 + \cdots}$$

Deg Num = Deg Den

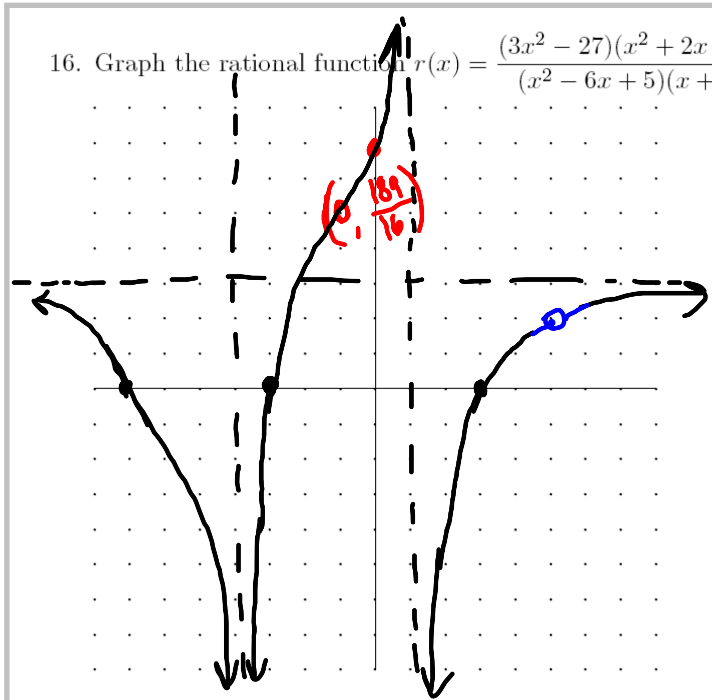
$y = 3$

x-int: $x = 3, -3, -7$

$(3,0), (-3,0), (-7,0)$

y-int: $r(0) = \dfrac{3(-3)(3)(7)}{(-1)(4)^2} = \dfrac{189}{16}$

$\left(0, \dfrac{189}{16}\right)$



	-7		-4		-3		1		3		
$-$		$-$		$-$		$-$		$-$		$+$	$3(x-3)$
$-$		$-$		$-$		$+$		$+$		$+$	$(x+3)$
$-$		$+$		$+$		$+$		$+$		$+$	$(x+7)$
$-$		$-$		$-$		$-$		$+$		$+$	$(x-1)$
$+$		$+$		$+$		$+$		$+$		$+$	$(x+4)^2$
$+$		$-$		$-$		$+$		$-$		$+$	

17. Find all real zeros for the polynomial $P(x) = \cancel{x^3 + 2x^2 - 12x + 15}$ and factor completely if you are told that -5 is a zero.

$x^3 + x^2 - 19x + 5$

-5 is a zero $\Rightarrow$ $x+5$ is a factor.

$$\begin{array}{r|rrrr} -5 & 1 & 1 & -19 & 5 \\ & & -5 & 20 & -5 \\ \hline & 1 & -4 & 1 & \boxed{0} \end{array}$$

$$P(x) = (x+5)(x^2-4x+1)$$

$$x^2 - 4x + 1 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$$

$$= 2 \pm \sqrt{3}$$

Zeros: -5, $2+\sqrt{3}$, $2-\sqrt{3}$

$$P(x) = (x+5)(x-(2+\sqrt{3}))(x-(2-\sqrt{3}))$$

18. Simplify the expression: $\dfrac{i^{51}(2+\sqrt{-4})(-1-3i)}{-2+5i}$. $= \dfrac{-i(2+2i)(-1-3i)}{-2+5i}$

$i^{51} = \underbrace{i^{48}}_{=1} \cdot i^{3} = i^{3} = -i$

$= \dfrac{-i(-2-6i-2i-\overset{+6}{6i^2})}{-2+5i}$

$= \dfrac{-i(4-8i)}{-2+5i} = \dfrac{-4i+8i^2}{-2+5i} = \dfrac{-8-4i}{-2+5i} \cdot \dfrac{(-2-5i)}{(-2-5i)}$

$= \dfrac{16+40i+8i+\overset{-20}{20i^2}}{(-2)^2+5^2} = \dfrac{-4+48i}{29}$

$= \boxed{\dfrac{-4}{29} + \dfrac{48}{29}i}$

19. Find the quotient and remainder for $\dfrac{x^4 - 4x^3 - 5x^2 - 4}{2x^2 + 4x - 6}$.

$$
\begin{array}{r}
\frac{1}{2}x^2 - 3x + 5 \\
2x^2+4x-6 \,\overline{\big)\, x^4 - 4x^3 - 5x^2 + 0x - 4} \\
-(x^4 + 2x^3 - 3x^2) \\
\hline
-6x^3 - 2x^2 + 0x - 4 \\
-(-6x^3 - 12x^2 + 18x) \\
\hline
10x^2 - 18x - 4 \\
-(10x^2 + 20x - 30) \\
\hline
-38x + 26
\end{array}
$$

$$\frac{x^4}{2x^2} = \frac{1}{2}x^2$$

$$\frac{-6x^3}{2x^2} = -3x$$

$$\frac{10x^2}{2x^2} = 5$$

$$\boxed{\begin{aligned} Q(x) &= \tfrac{1}{2}x^2 - 3x + 5 \\ R(x) &= -38x + 26 \end{aligned}}$$