

1. (a) Express 200° in radians.

$$200^\circ \times \frac{\pi \text{ rad}}{180^\circ}$$

$$= \frac{200\pi}{180} \text{ rad}$$

$$= \boxed{\frac{10\pi}{9} \text{ rad}}$$

(b) Express $\frac{5\pi}{4}$ radians in degrees.

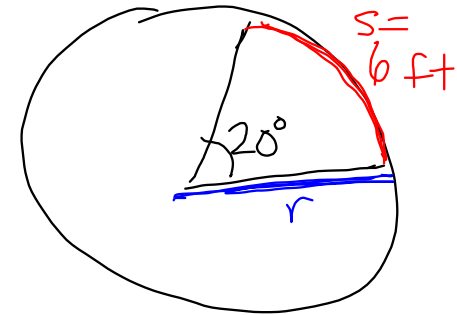
$$\frac{5\pi}{4} \text{ rad} \times \frac{180^\circ}{\pi \text{ rad}}$$

$$= \frac{5 \cdot 180^\circ}{4} = \boxed{225^\circ}$$

2. An arc of length 6 ft subtends a central angle of 20° . What is the radius of this circle?

$$S = r\theta \quad \theta \text{ must be in radians}$$

$$\theta = 20^\circ \times \frac{\pi \text{ rad}}{180^\circ} = \frac{\pi}{9} \text{ rad.}$$



$$6 = r \cdot \frac{\pi}{9}$$

$$\frac{9}{\pi} \cdot 6 = r$$

$$\frac{54}{\pi} = r$$

$$r = \frac{54}{\pi} \text{ ft} \approx 17.1887 \text{ ft}$$

3. In a pizza with diameter 16 in, the area of a certain piece is 24 in^2 . What is the length of the crust on this piece?

diameter : 16 in
radius : 8 in

$$s = r\theta$$

Use area of sector to find θ :

$$A = \frac{1}{2} r^2 \theta$$

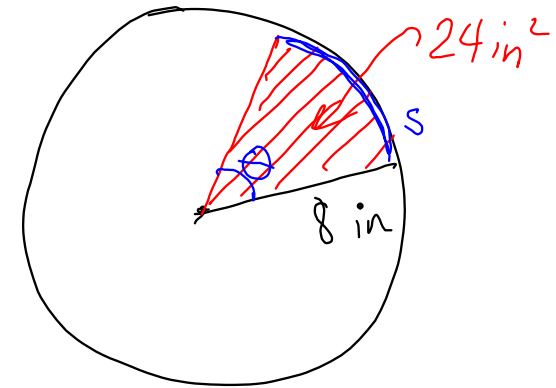
$$24 = \frac{1}{2} (8)^2 \cdot \theta$$

$$48 = 64\theta$$

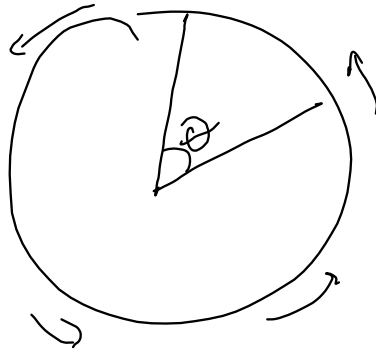
$$\frac{48}{64} = \theta$$

$$\theta = \frac{3}{4} \text{ rad.}$$

$$s = r\theta = (8)\left(\frac{3}{4}\right) = \boxed{6 \text{ in}}$$



4. If a record player makes 8 revolutions every 5 seconds, what is the angular speed of the player?



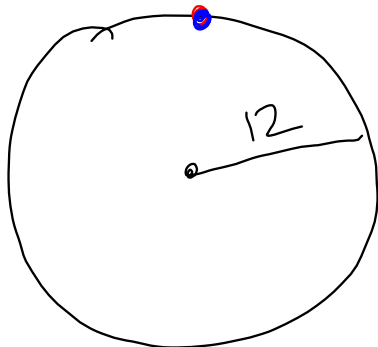
$$\omega = \frac{\theta}{t}$$

In 5 seconds, the player rotates $8 \times 2\pi = 16\pi$ rad.

$$\omega = \frac{\theta}{t} = \boxed{\frac{16\pi}{5} \text{ rad/sec}}$$

$$\approx 10.0531 \text{ rad/sec}$$

5. A car tire has a radius of 12 inches. The tire is rotating at a rate of 700 rpm. What is the speed of the car? (What is the linear speed of the tire?)



$$v = \frac{s}{t} = \frac{\text{distance}}{\text{time}} \quad \begin{array}{l} \text{rev per minute} \\ \downarrow \end{array}$$

In 1 revolution, we travel $2\pi r = 2\pi \cdot 12 = 24\pi$ in

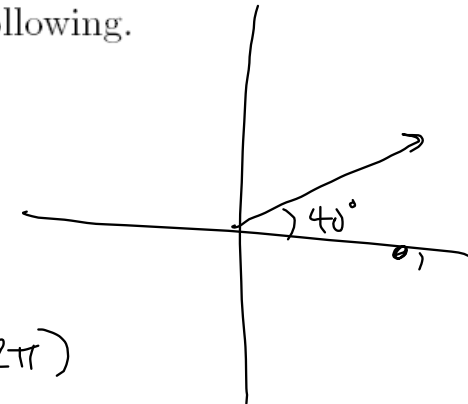
$$\begin{aligned} \text{In 1 minute, we travel } & (24\pi)(700) \text{ inches} \\ & = 16800\pi \text{ inches} \end{aligned}$$

$$v = 16800\pi \text{ inches/min}$$

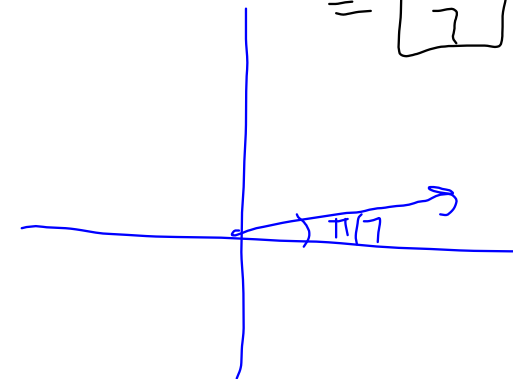
16800π inches	1 ft	1 mi	60 min	≈ 49.9799 mi/hr
1 min	12 in	5280 ft	1 hr	

6. Find coterminal angles between 0° and 360° or 0 and 2π for the following.

(a) $-680^\circ + 360^\circ = -320^\circ + 360^\circ = \boxed{40^\circ}$



(b) $\frac{43\pi}{7} - 2\pi = \frac{43\pi}{7} - \frac{14\pi}{7} = \frac{29\pi}{7} - \frac{14\pi}{7} = \frac{15\pi}{7} - \frac{14\pi}{7} = \boxed{\frac{\pi}{7}}$



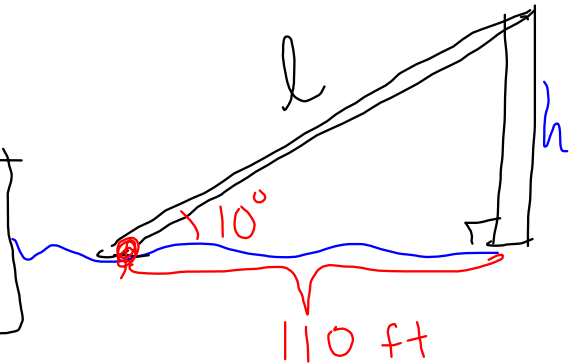
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7. A water slide at a water park forms an angle of elevation of 10° from the water to the top of the slide. The endpoint of the slide is 110 ft horizontally from the start of the slide.

(a) How long is the slide?

$$\cos 10^\circ = \frac{110}{l}$$

$$l = \frac{110}{\cos 10^\circ} \text{ ft} \approx 111.6969 \text{ ft}$$

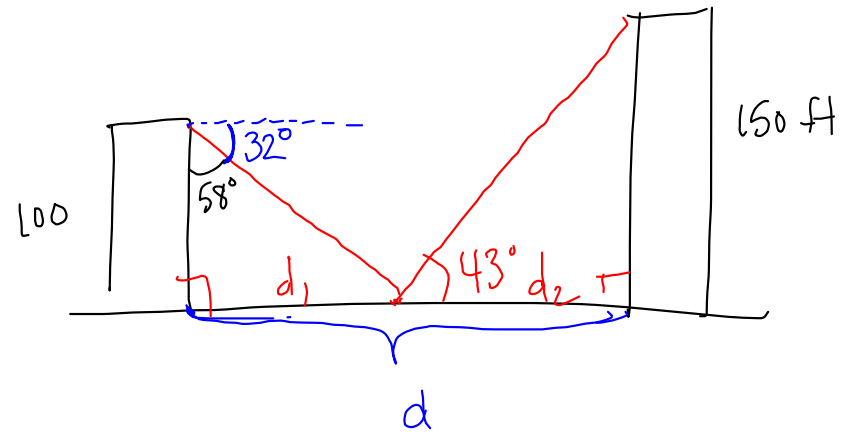


- (b) How high does the slide start?

$$\tan 10^\circ = \frac{h}{110}$$

$$h = 110 \cdot \tan 10^\circ \text{ ft} \approx 19.3960 \text{ ft}$$

8. An ant is sitting between 2 towers. One tower is 100 ft tall and the other is 150 ft tall. The angle of depression from the 100 ft tower to the ant is 32° . The angle of elevation from the ant to the top of the 150 ft tower is 43° . How far apart are the towers?



$$\tan 58^\circ = \frac{d_1}{100}$$

$$100 \cdot \tan 58^\circ = d_1$$

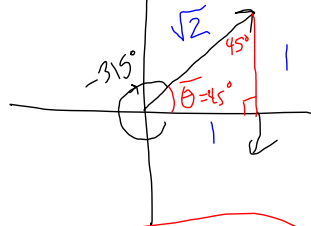
$$\tan 43^\circ = \frac{150}{d_2}$$

$$d_2 = \frac{150}{\tan 43^\circ}$$

$$d = d_1 + d_2 = 100 \tan 58^\circ + \frac{150}{\tan 43^\circ} \text{ ft}$$
$$\approx 320.8888 \text{ ft}$$

9. Find the reference angle for the given angle and then evaluate the trig function exactly.

(a) $\csc(-315^\circ)$



$$\bar{\theta} = 360^\circ - 315^\circ = 45^\circ$$

Since $\csc \theta > 0$ in QI.

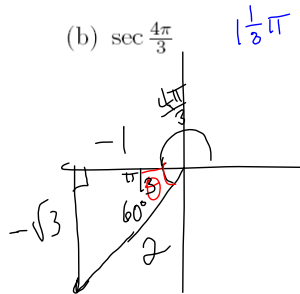
$$\csc(-315^\circ) = +\csc(45^\circ)$$

$$= \frac{1}{\sin 45^\circ} = \boxed{\sqrt{2}}$$

OR

$$\csc(-315^\circ) = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

(b) $\sec \frac{4\pi}{3}$



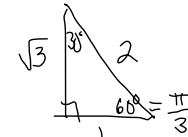
$$\bar{\theta} = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$$

$$\sec\left(\frac{4\pi}{3}\right) = -\sec\left(\frac{\pi}{3}\right) = -\frac{1}{\cos \pi/3} = -\frac{1}{1/2} = \boxed{-2}$$

$\sec \theta < 0$ in QIII

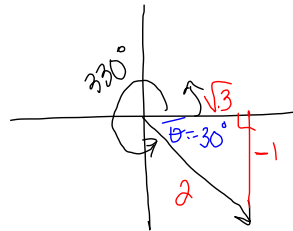
OR

$$\sec\left(\frac{4\pi}{3}\right) = \frac{\text{hyp}}{\text{adj}} = \frac{2}{-1} = -2$$



(c) $\tan 690^\circ$

$$690^\circ - 360^\circ = 330^\circ$$



$$\bar{\theta} = 360^\circ - 330^\circ = 30^\circ$$

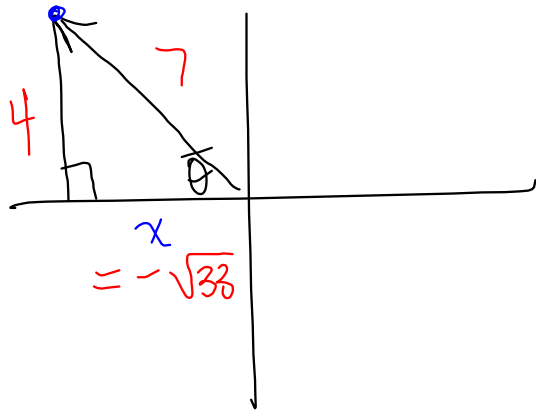
$$\tan 690^\circ = -\tan 30^\circ = \boxed{-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}}$$

$\tan \theta < 0$ in QIV.

OR

$$\tan 690^\circ = \frac{\text{opp}}{\text{adj}} = \frac{-1}{\sqrt{3}}$$

10. Given that $\csc \theta = \frac{7}{4}$ and that $\cot \theta < 0$, find the other trig functions for θ .



$\csc \theta > 0$ and $\cot \theta < 0$ in QII

$$\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{7}{4}$$

$$x^2 + 4^2 = 7^2$$

$$x^2 = 49 - 16 = 33$$

$$x = \pm \sqrt{33}$$

$x < 0$ in QII, so $x = -\sqrt{33}$

$$\sin \theta = \frac{4}{7}$$

$$\cos \theta = \frac{-\sqrt{33}}{7}$$

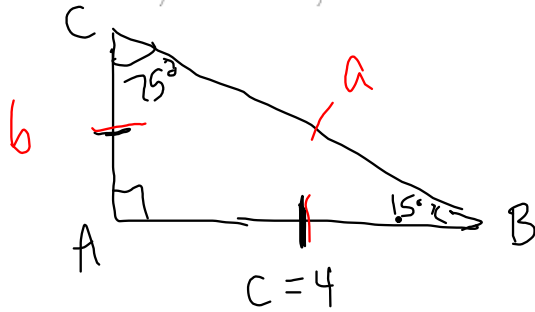
$$\tan \theta = \frac{4}{-\sqrt{33}} = -\frac{4\sqrt{33}}{33}$$

$$\sec \theta = -\frac{7}{\sqrt{33}} = -\frac{7\sqrt{33}}{33}$$

$$\cot \theta = -\frac{\sqrt{33}}{4}$$

11. Solve the following triangles.

(a) $A = 90^\circ, B = 15^\circ, c = 4$



$$C = 180 - 90 - 15 = \underline{\underline{75^\circ}}$$

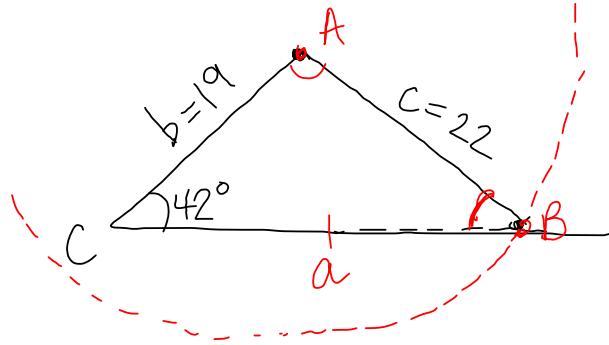
$$\cos 15^\circ = \frac{4}{a}$$

$$a = \frac{4}{\cos 15^\circ} \approx \underline{\underline{4.1411}}$$

$$\tan 15^\circ = \frac{b}{4}$$

$$b = 4 \tan 15^\circ \approx \underline{\underline{1.0718}}$$

(b) $C = 42^\circ, b = 19, c = 22$



Since $c = 22 > b = 19$, there is exactly one triangle.

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin 42^\circ}{22} = \frac{\sin B}{19}$$

$$\frac{19 \sin 42^\circ}{22} = \sin B$$

$$0.5779 = \sin B$$

$$B = \sin^{-1}(0.5779) \approx \boxed{35.3020^\circ}$$

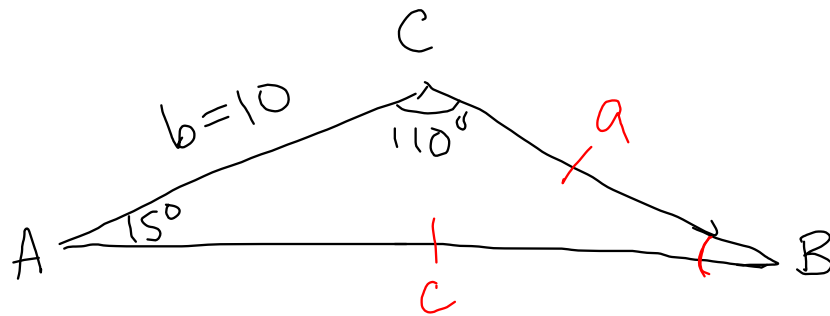
$$A = 180^\circ - 42^\circ - B = \boxed{102.6980^\circ}$$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 42^\circ}{22} = \frac{\sin 102.6980^\circ}{a}$$

$$a = \frac{22 \cdot \sin 102.6980^\circ}{\sin 42^\circ} \approx \boxed{32.0743}$$

(c) $A = 15^\circ, C = 110^\circ, b = 10$



$$B = 180 - A - C = 180 - 15 - 110 \\ = \underline{\underline{55^\circ}}$$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

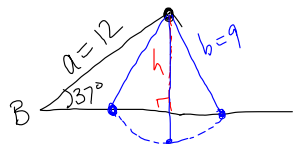
$$\frac{\sin 55^\circ}{10} = \frac{\sin 15^\circ}{a} \Rightarrow a = \frac{10 \cdot \sin 15^\circ}{\sin 55^\circ} \approx \underline{\underline{3.1596}}$$

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 55^\circ}{10} = \frac{\sin 110^\circ}{c} \Rightarrow c = \frac{10 \cdot \sin 110^\circ}{\sin 55^\circ} \approx \underline{\underline{11.0807}}$$

...

(d) $B = 37^\circ, a = 12, b = 9$



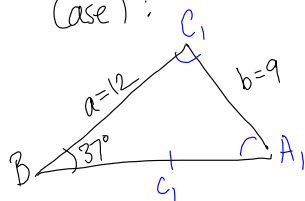
$$\sin 37^\circ = \frac{h}{12}$$

$$12 \sin 37^\circ = h$$

$$7.2218 \approx h$$

Since $b > h$ but $b < a$,
there are 2 triangles.

Case 1:



$$\frac{\sin 37^\circ}{9} = \frac{\sin A_1}{12}$$

$$\frac{12 \cdot \sin 37^\circ}{9} = \sin A_1$$

$$0.8024 = \sin A_1$$

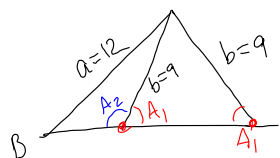
$$A_1 = \sin^{-1}(0.8024)$$

$$A_1 \approx 53.3618^\circ$$

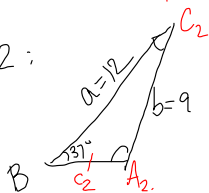
$$C_1 = 180 - B - A_1 \approx \underline{\underline{89.6382^\circ}}$$

$$\frac{\sin 37^\circ}{9} = \frac{\sin 89.6382^\circ}{c_1}$$

$$c_1 = \frac{9 \cdot \sin 89.6382^\circ}{\sin 37^\circ} \approx \underline{\underline{14.9545}}$$



Case 2:



$$A_2 = 180 - A_1 = 180 - 53.3618^\circ$$

$$\approx \underline{\underline{126.6382^\circ}}$$

$$C_2 = 180 - B - A_2$$

$$= 180 - 37 - 126.6382$$

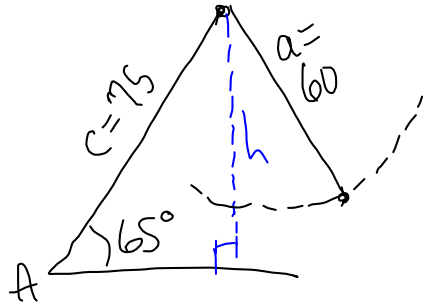
$$= \underline{\underline{16.3618^\circ}}$$

$$\frac{\sin 37^\circ}{9} = \frac{\sin 16.3618^\circ}{c_2}$$

$$c_2 = \frac{9 \cdot \sin 16.3618^\circ}{\sin 37^\circ} \approx \underline{\underline{4.2128}}$$

12. How many triangles exist that satisfy the following?

(a) $A = 65^\circ, a = 60, c = 75$



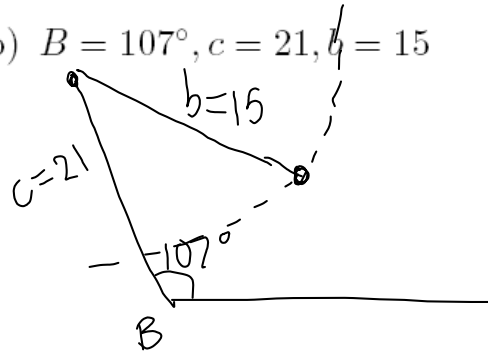
$$\sin 65^\circ = \frac{h}{75}$$

$$75 \cdot \sin 65^\circ = h$$

$$67.9731 \approx h$$

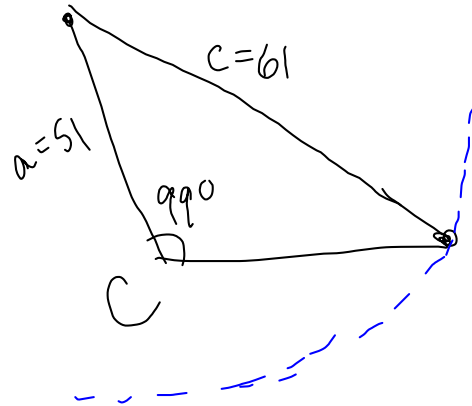
Since $a < h$, there are NO triangles.

(b) $B = 107^\circ, c = 21, b = 15$



Since $b < c$ and B is obtuse,
there are NO triangles.

(c) $C = 99^\circ, a = 51, c = 61$



Since $c > a$, there is
exactly 1 triangle.

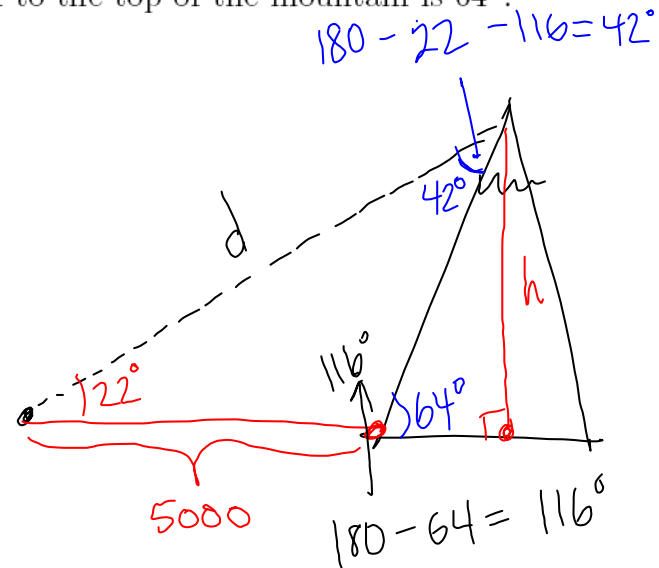
13. A ski lift takes people to the top of a mountain. The pickup point for the ski lift is 5000 ft from the base of the mountain. The angle of elevation from the pickup point to the top of the mountain is 22° and the angle of elevation from the base of the mountain to the top of the mountain is 64° .

(a) What is the distance the ski lift travels?

$$\frac{\sin 42^\circ}{5000} = \frac{\sin 116^\circ}{d}$$

$$d = \frac{5000 \cdot \sin 116^\circ}{\sin 42^\circ} \text{ ft}$$

$$\approx 6716.1331 \text{ ft}$$



(b) What is the vertical height of the mountain?

$$\sin 22^\circ = \frac{h}{d}$$

$$d \cdot \sin 22^\circ = h$$

$$6716.1331 \cdot \sin 22^\circ = h$$

$$\boxed{2515.9078 \text{ ft} \approx h}$$

