Math 151 Exam 2 Review

1. Use a linear approximation to estimate the value of $\sqrt[5]{31.8}$.

2. Find the linear approximation to $f(x) = (x - 3)^3$ at $x = 5$ and use it to approximate the value of $2.05^3$.

3. The diameter of a sphere was measured to be 10 inches with a possible error in measurement of 0.2 inches. Use differentials to estimate the maximum error in the volume of the sphere.

4. Calculate the following limits.
   (a) $\lim_{x \to 6^+} \left( \frac{1}{2} \right)^{\frac{x-8}{x-6}}$
   (b) $\lim_{x \to -\infty} \frac{4e^{-5x} + 3}{6 + 3e^{-5x}}$

5. Find an equation of the tangent line to the graph of $f(x) = xe^{x^3+x}$ at $x = 1$.

6. Find $y''$ if $y = e^{2x} \cos x$.

7. A 5-meter drawbridge is raised so that the angle of elevation changes at a rate of 0.1 rad/s. At what time is the height of the drawbridge changing when it is 2 m off the ground?

8. Ship A is 100 km west of ship B. Ship A begins sailing south at 35 km/h while ship B begins sailing north at 25 km/h. How fast is the distance between the ships changing 4 hours later?

9. A 25-ft long trough has ends which are isosceles triangles with height 5 ft and a length of 4 ft across the top. Water is poured in at a rate of 15 ft$^3$/min, but water is also leaking out of the trough at a rate of 3 ft$^3$/min. At what rate is the height of the water changing when the width of the water across the trough is 2 ft?

10. Calculate the following limits.
    (a) $\lim_{x \to 0} \frac{\cos x + \tan 9x - 1}{\sin x}$
    (b) $\lim_{x \to 0} \frac{x \cos 7x \sin 4x}{3 \sin^2 10x}$

11. Find the values of $x$, $0 \leq x \leq 2\pi$, where the tangent line to $f(x) = \sin^2 x + \cos x$ is horizontal.

12. Find $y'$ for the equation $\sin 2y + xy^2 = e^{x^2}y$.

13. For what value(s) of $a$ are the curves $\frac{x^2}{a^2} + \frac{y^2}{9} = \frac{5}{4}$ and $y^2 - 4x^2 = 5$ orthogonal at the point $(1, 3)$?

14. Find a tangent vector to the curve $\mathbf{r}(t) = \langle \cos 3t, \ 2 \sin 5t + \cos 4t \rangle$ at the point where $t = \frac{\pi}{4}$.

15. The position of an object is given by the function $\mathbf{r}(t) = \langle \sqrt{t^2 + 8t}, \frac{t}{3t - 2} \rangle$.
    (a) What is the speed of the particle at time $t = 1$?
    (b) What is the acceleration at time $t = 1$?

16. Find $f^{(29)}(x)$ where $f(x) = e^{-4x} + \cos 3x$. 


17. Consider the curve \( x = t^2 + 6t, \ y = 2t^3 - 9t^2 \).
   
   (a) Find the slope of the tangent line at the point \((-5, -11)\).
   
   (b) Find the points on the curve where the tangent line is horizontal or vertical.

18. Find an equation of the tangent line to the curve \( x = \tan 2t + \frac{4}{(t+1)^2}, \ y = (t^2 + 3t + 2)^4 - (t + 1)^{5/2} \) at the point where \( t = 0 \).

19. Suppose \( F(x) = \sqrt[3]{g(2x + 1)} + f(5x + g(\cos x)) + g(f(3x)) \) where \( f(x) \) and \( g(x) \) are differentiable functions. Find \( F'(x) \).

20. For functions \( f \) and \( g \), we are given that \( f(1) = 2, f'(1) = 5, g(1) = 3, \) and \( g'(1) = 4 \). Suppose that \( U(x) = f(x^2)e^{g(x)} \). Find an equation of the tangent line to \( U(x) \) at the point where \( x = 1 \).

21. Given that \( y = 3x + 7 \) is the tangent line to the graph of \( f(x) = ax^2 + bx \) at the point where \( x = 2 \), find the values of \( a \) and \( b \).

22. Differentiate the following.
   
   (a) \( f(x) = \tan^2 5x + \sec(\sqrt{3x^2 - x^3}) \)
   
   (b) \( g(t) = \left( \frac{3t^2 + 5t}{4t^3 - t^2} \right)^8 \)
   
   (c) \( h(x) = \left( \frac{6}{x^2} - \sqrt[4]{8x^6 - x} \right)(\cot 9x + e^x) \)

23. A particle has position function \( s(t) = t^3 - 3t^2, \ t \geq 0 \).
   
   (a) At what rate is the particle’s velocity changing when \( t = 3 \)?
   
   (b) What is the total distance traveled by the particle from \( t = 1 \) to \( t = 4 \) seconds?

24. Find the values of \( a \) and \( b \) so that the following function is differentiable everywhere.

\[
f(x) = \begin{cases} 
x^2 + ax + b & \text{if } x < 0 \\
e^x + \cos x & \text{if } x \geq 0
\end{cases}
\]