Math 151 Week in Review 9
Sections 4.2-4.4

1. Consider the function \( f(x) = \frac{2x - 4}{x - 9} \).
   
   (a) Show that \( f(x) \) is one-to-one.
   (b) Find the inverse of \( f(x) \).

2. Consider the function \( f(x) = \sqrt{x - 4} + 3 \). Find \( f^{-1}(x) \) and state its domain and range.

3. Given \( f(x) = e^{3x+3} - 4x^3 - 2 \), find \( g'(3) \) where \( g \) is the inverse of \( f \).

4. Evaluate the following.
   
   (a) \( \ln e^{3} + \log_{5} 25^{10} + \log \frac{1}{1000} \)
   (b) \( \log_{6} 16 + 2\log_{6} 3 - \log_{6} 4 + (\log_{5} 3)^2 \)

5. Given that \( \log a = 9, \log b = 6, \log c = 8, \) and \( \log d = 5 \), what is \( \log \left( \frac{b^3 \sqrt{a}}{cd} \right) \)?

6. Write the following as a single logarithm. \( 4 \log_{3} x - \frac{1}{4} \log_{3} y + e \log_{3}(x + z) \)

7. Solve the following equations for \( x \).
   
   (a) \( \log_{3}(\log_{2}(\log_{5}(x + 7))) = 0 \)
   (b) \( 4^{3x+7} = \log 1000 \)
   (c) \( e^{5x} = 4 \)
   (d) \( \ln(6 - x) + \ln(2 - x) = \ln 5 \)
   (e) \( \log_{2}(x^2 - 38) - \log_{2}(5 - x) = 1 \)

8. Calculate the following limits.
   
   (a) \( \lim_{x \to 4^-} \frac{x - 6}{x - 4} \)
   (b) \( \lim_{x \to \infty} \ln \left( \frac{3}{x + 1} \right) \)
   (c) \( \lim_{x \to \infty} \ln(2e^{-3x} + 4) - \ln(5e^{-3x} + 2) \)

9. Find the inverse function of \( f(x) = \log(e^x + 2) \).

10. Find the domain of \( f(x) = \ln(x^2 - 2x - 8) \).

11. Express \( \log_{8} 23 \) in terms of natural logarithms.

12. Differentiate the following.
   
   (a) \( f(x) = \ln(x^4 - 3x^2) + \ln |e^{3x} - 2x| \)
   (b) \( g(t) = \log_{5}(\sin 3t) \)
   (c) \( f(x) = \ln \left( \ln \left( \frac{3x - 9}{x^4 + x} \right) \right) \)
   (d) \( g(x) = \log \left( \sqrt[3]{x^2 + \sec x} \right) \)
(e) \( h(x) = 6x^2 \ln x \)
(f) \( g(u) = 4e^{2u} \ln(\tan u) \)
(g) \( f(x) = x^{\sqrt{x}} \)

13. Use logarithmic differentiation to find the derivative of \( y = \frac{(\cos^2 x)(5 - 2x)^6}{e^{5x} \sqrt{3x^6 - x}} \).

14. Find the equation of the tangent line to \( y = (3^x + 1)^{\cos x} \) at the point where \( x = 0 \).