MATH 152  
SPRING 2018  

Sample Exam (covering sections 5.5-7.2)  

1. Find the area of the region bounded by \( y = x^3, \ y = x \) from \( x = 0 \) to \( x = 2 \).  
   a) \( \frac{3}{2} \)  
   b) 2  
   c) \( \frac{1}{2} \)  
   d) \( \frac{5}{2} \)  
   e) 3  

2. If we revolve the region bounded by \( x = 2y^2 \) and \( x = 2 \) about the line \( x = 2 \), which of the following integrals gives the resulting volume?  
   a) \( \int_{-1}^{1} \pi (4 - 4y^4) \, dy \)  
   b) \( \int_{-1}^{1} \pi (4 - (2 - 2y^2)^2) \, dy \)  
   c) \( \int_{-1}^{1} 4\pi y^4 \, dy \)  
   d) \( \int_{-1}^{1} \pi (2 - 2y^2)^2 \, dy \)  
   e) \( \int_{-1}^{1} \pi (4y^4 - 4) \, dy \)  

3. A spring has a natural length of 1 m. The force required to keep it stretched to a length of 2 m is 10 N. Find the work required to stretch the spring from a length of 2 m to a length of 4 m.  
   a) \( \frac{75}{4} \) J  
   b) 45 J  
   c) \( \frac{75}{2} \) J  
   d) 30 J  
   e) 40 J
4. Evaluate \( \int_0^{\sqrt{\pi/2}} x^5 \cos(x^3) \, dx \)
   a) \( \frac{\pi}{6} - \frac{1}{3} \)
   b) \( \frac{\pi}{3} - \frac{1}{6} \)
   c) \( \frac{\pi}{2} - \frac{1}{3} \)
   d) \( \frac{\pi}{3} - \frac{1}{2} \)
   e) \( \frac{\pi}{6} - \frac{1}{2} \)

5. \( \int_1^e x \ln x \, dx = \)
   a) \( \frac{7e^8 + 1}{4} \)
   b) \( \frac{9e^8 + 1}{4} \)
   c) \( \frac{8e^8 + 1}{4} \)
   d) \( \frac{7e^8 - 1}{4} \)
   e) \( \frac{8e^8 - 1}{4} \)

6. \( \int \sin^2(x) \, dx = \)
   a) \( \frac{x}{2} + \frac{1}{4} \sin(2x) + C \)
   b) \( \frac{x}{2} - \frac{1}{4} \sin(2x) + C \)
   c) \( \frac{4}{3} \sin^3(x) + C \)
   d) \( \frac{x}{2} + 2 \sin(2x) + C \)
   e) \( \frac{1}{3} \sin^3(x) + C \)
7. A 15 pound rope, 30 feet long, hangs from the top of a cliff. How much work is done in pulling \( \frac{1}{3} \) of this rope to the top of the cliff?

   a) 125 foot-pounds
   b) 25 foot-pounds
   c) 35 foot-pounds
   d) 2255 foot-pounds
   e) 75 foot-pounds

8. \( \int_{0}^{\pi/4} \sec^4 x \, dx \)

   a) \( \frac{16}{3} \)
   b) \( \frac{4}{3} \)
   c) \( \frac{8}{3} \)
   d) \( \frac{1}{6} \)
   e) None of these

9. \( \int \frac{x}{(x - 1)^2} \, dx \)

   a) \( \ln |x - 1| + \frac{1}{x - 1} + C \)
   b) \( \ln |x - 1| - \frac{1}{x - 1} + C \)
   c) \( \ln |x - 1| + \frac{1}{3(x - 1)^2} + C \)
   d) \( \ln |x - 1| - \frac{1}{3(x - 1)^2} + C \)
   e) \( \ln |x - 1| + \frac{3}{(x - 1)^2} + C \)
Part II - Work Out Problems
10. Find the volume of the solid obtained by revolving the region bounded by $y = 4 - x^2$ and $y = 3$ about the $x$-axis.

11. The base of a solid is the region bounded by $y = x^2$ and $y = 1$. Cross-sections perpendicular to the $y$-axis are equilateral triangles. Set up but do not evaluate an integral that gives the volume of the solid.
12. A 15 m long trough with semicircular ends of radius 2 m is full of water. Set up but do not evaluate an integral that will compute the work required to pump all of the water out of a 1 m high spout. Indicate on the picture where you are placing the axis and which direction is positive. Note: The density of water is $\rho = 1000 \text{ kg/m}^3$ and the acceleration due to gravity is $9.8 \text{ m/s}^2$.

13. Using cylindrical shells, set up but do not evaluate an integral that gives the volume of the solid formed by rotating the region bounded by $y = \sqrt{x}$ and $y = x^2$ about the line $y = -1$. 
14. Consider the region $R$ bounded by $y = \sqrt{x} + 3$, $y = 3$, $x = 16$

a.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region $R$ about the $x$-axis

b.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region $R$ about the $y$-axis

c.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region $R$ about the line $x = -1$

d.) Set up but do not evaluate an integral that gives the volume of the solid obtained by rotating the region $R$ about the line $y = 10$. 
15. Find $\int \sec^5 x \tan^3 x \, dx$.

16. Find $\int \sin^5(3x) \cos^2(3x) \, dx$.

17. Evaluate $\int \arccos x \, dx$. 