SOME EXTREME VALUE STATISTICS PROBLEMS IN RMT

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'Quantum Chaos: Routes to RMT
and beyond', Banff 24-29/02/2008

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- Large deviations and random matrices.

Gaussian
Wishart-Laguerre } ensembles of RM

Maximum eigenvalue (edge)

Tracy-Widom distribution

Far from the edge (large deviations)

for { Gaussian
Wishart

Coulomb gas

D.S. Dean, S.N. Majumdar, Phys. Rev. Lett. 97 (2006) 160201
P. Vivo, S.N. Majumdar, O. Bohigas, J. Phys. A40 (2007) 4317
Acta Physica Polonica 38 (2007) 4139

- Entanglement of a random pure state.

Bipartite system. Reduced density matrix

Entanglement and minimum eigenvalue of reduced density matrix

Probability distribution of the minimum eigenvalue

for complex
real } random state

S.N. Majumdar, O. Bohigas, A. Lakshminarayan,
J. Stat. Phys. February 2008

Wigner-Dyson (Gaussian)

H : $N \times N$ hermitean matrix

H : real eigenvalues

$$P(H) \propto e^{-\frac{\beta}{2} \operatorname{Tr} H^2}$$

Wishart

X : $M \times N$ matrix $M \geq N$ $\left(\frac{N}{M} = c \leq 1\right)$

$W = X^{+} X$: hermitean $N \times N$ positive definite (Wishart) matrix

X^{+} : conjugate transpose

$$P(X) \propto e^{-\frac{\beta}{2} \operatorname{Tr} X^{+} X}$$

X : gaussian entries (i.i.d.)

Joint distribution of eigenvalues

Gaussian

$$\propto e^{-\frac{\beta}{2}\sum_1^N \lambda_i^2} \prod_{j<k} |\lambda_j - \lambda_k|^\beta \prod_1^N d\lambda_i$$

Wishart

$$\propto \prod_1^N \lambda_i^{\frac{\beta}{2}(1+M-N)-1} \, e^{-\frac{\beta}{2}\lambda_i} \cdot \prod_{j<k} |\lambda_j - \lambda_k|^\beta \cdot \prod_1^N d\lambda_i$$

for $c=1$, $\beta=2$

$$\propto \prod_1^N e^{-\lambda_i} \cdot \prod_{j<k} |\lambda_j - \lambda_k|^2 \prod_1^N d\lambda_i$$

Gaussian

2d Coulomb gas submitted to external quadratic potential

Wishart

2d Coulomb gas confined to positive half line subject to an external linear + logarithmic potential

Eigenvalue density

Gaussian

$$\rho_N(\lambda) = \frac{1}{\sqrt{N}} f\left(\frac{\lambda}{\sqrt{N}}\right)$$

$$f(x) = \sqrt{\frac{1}{\pi}(2-x^2)} \qquad [-\sqrt{2}, \sqrt{2}]$$

Wigner Semi-circle

Wishart

$$\rho_N(\lambda) = \frac{1}{N} f\left(\frac{\lambda}{N}\right)$$

Marčenko-Pastur

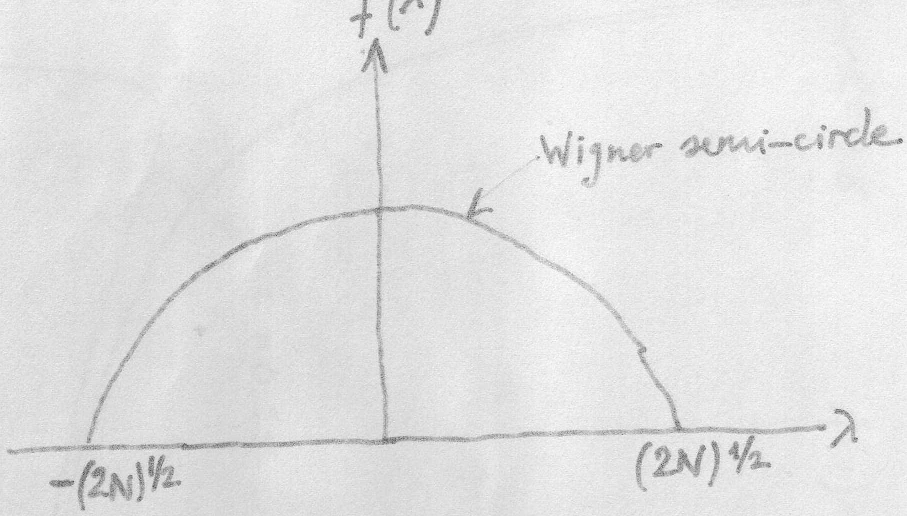
$$f(x) = \frac{1}{2\pi x} \sqrt{(x_+ - x)(x - x_-)}$$

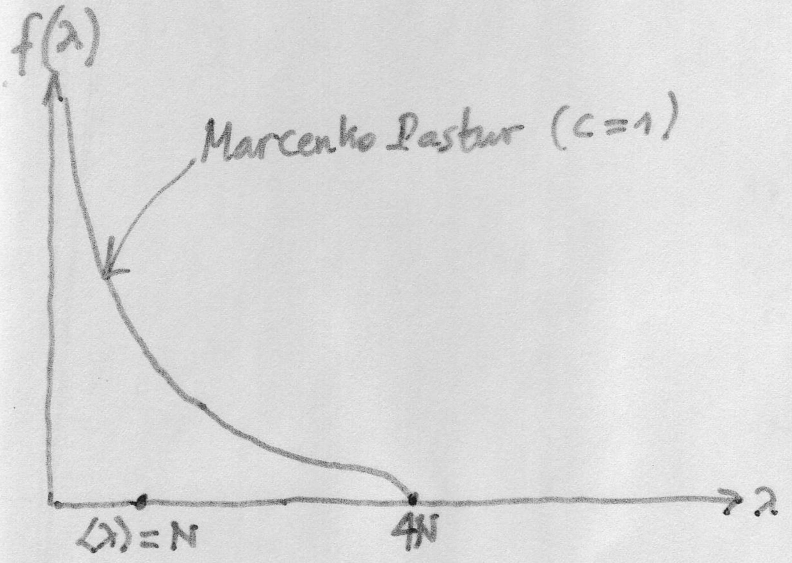
$$x_\pm = \left(\frac{1}{\sqrt{c}} \pm 1\right)^2 \qquad [x_-, x_+]$$

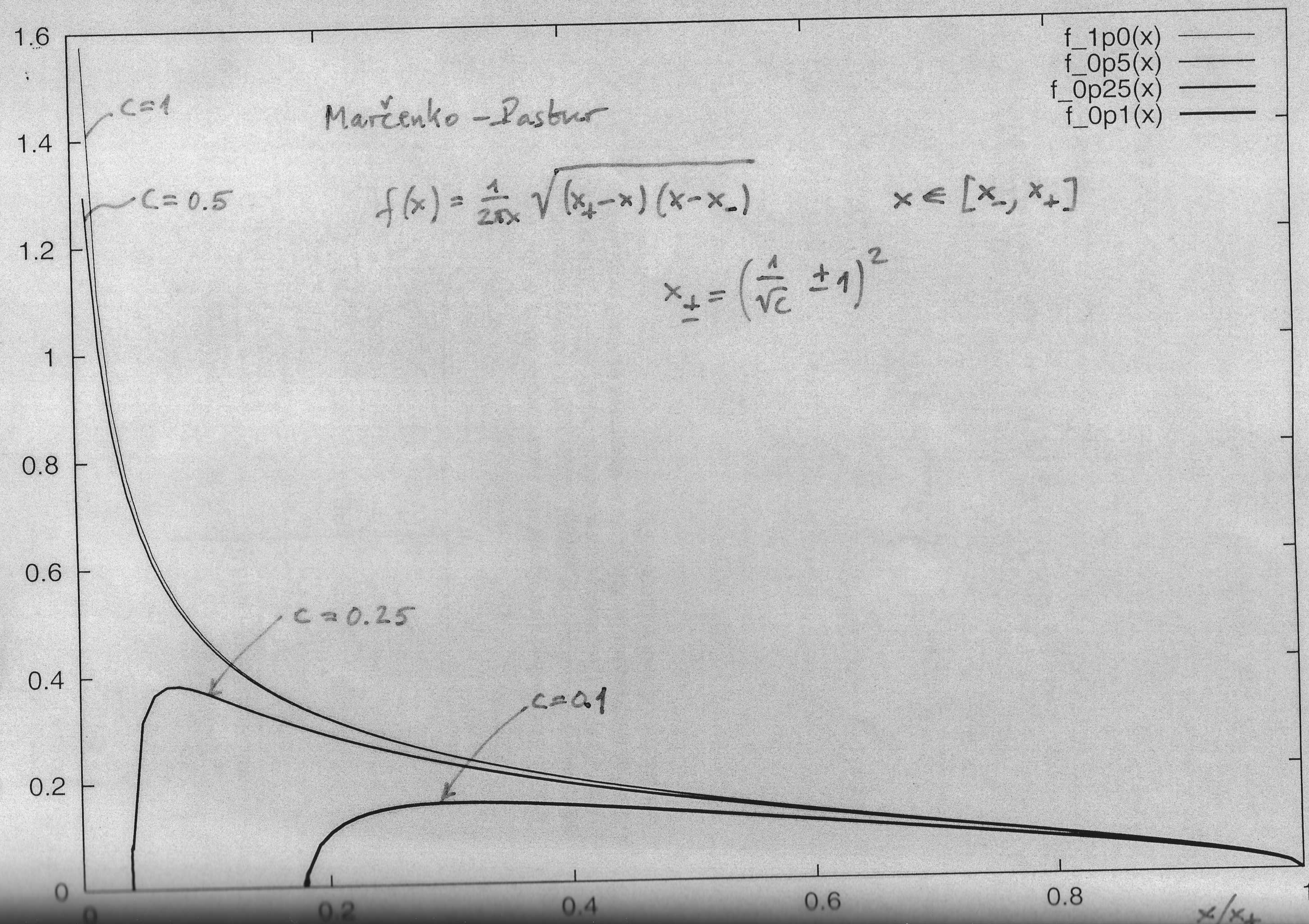
$$c = \frac{N}{M} \leq 1$$

For $c = 1$ $(M = N)$

$$f(x) = \frac{1}{2\pi} \sqrt{\frac{4-x}{x}} \qquad [0, 4]$$


f(λ)
Wigner semi-circle
λ
-(2N)^1/2
(2N)^1/2


f(λ)
Marcenko Pastur (c=1)
λ
⟨λ⟩ = N
4N



Marčenko – Pastur
$f(x) = \frac{1}{2\pi x}\sqrt{(x_+ - x)(x - x_-)}$
$x \in [x_-, x_+]$
$x_\pm = \left(\frac{1}{\sqrt{c}} \pm 1\right)^2$
c=1
c=0.5
c=0.25
c=0.1
f_1p0(x)
f_0p5(x)
f_0p25(x)
f_0p1(x)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
x/x+