Random, Pure State $|\Psi\rangle$:

$x_{i,\alpha}$: independent and identically distributed Gaussian variables, real or complex

$W = XX^{+}$: is then (almost) a random Wishart matrix $(\sum \lambda_i = 1)$

Joint distribution of the N (non-negative) eigenvalues of W

$$P^{W}(\lambda_1, \dots, \lambda_N) = B_{M,N}\, \delta\left(\sum_{i=1}^{N} \lambda_i - 1\right) \cdot \prod_{i=1}^{N} \lambda_i^{\frac{\beta}{2}(M-N+1)-1} \cdot \prod_{j<k} |\lambda_j - \lambda_k|^{\beta}$$

$$\mathrm{Prob}\{\lambda_{min} \geq x\} = \mathrm{Prob}\{\lambda_1 \geq x, \lambda_2 \geq x, \dots, \lambda_N \geq x\} =$$

$$\left.\begin{matrix} M=N \\ \beta=2 \end{matrix}\right\} = Q_N(x) = B_{N,N} \int_{x}^{\infty} \cdots \int_{x}^{\infty} \delta\left(\sum \lambda_i - 1\right) \prod_{j<k} (\lambda_j - \lambda_k)^2 \cdot \prod_{i=1}^{N} d\lambda_i$$

distribution function of λ_{min}

its derivative : probability density $P_N(x)$

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$1 << N \leq M$ $\qquad \langle S \rangle \approx \ln N - \frac{N}{2M}$

Complex case $\beta = 2$

$$p_N(x) = N(N^2-1)(1-Nx)^{N^2-2} \, \theta(1-Nx)$$

exact, finite N

Real case $\beta = 1$

Complicated expression in terms of hypergeometric function

$$\langle \lambda_{min} \rangle = \frac{1}{N^3}$$

(exact, finite N)

For large N

$$\langle \lambda_{min} \rangle \approx \frac{c}{N^3}$$

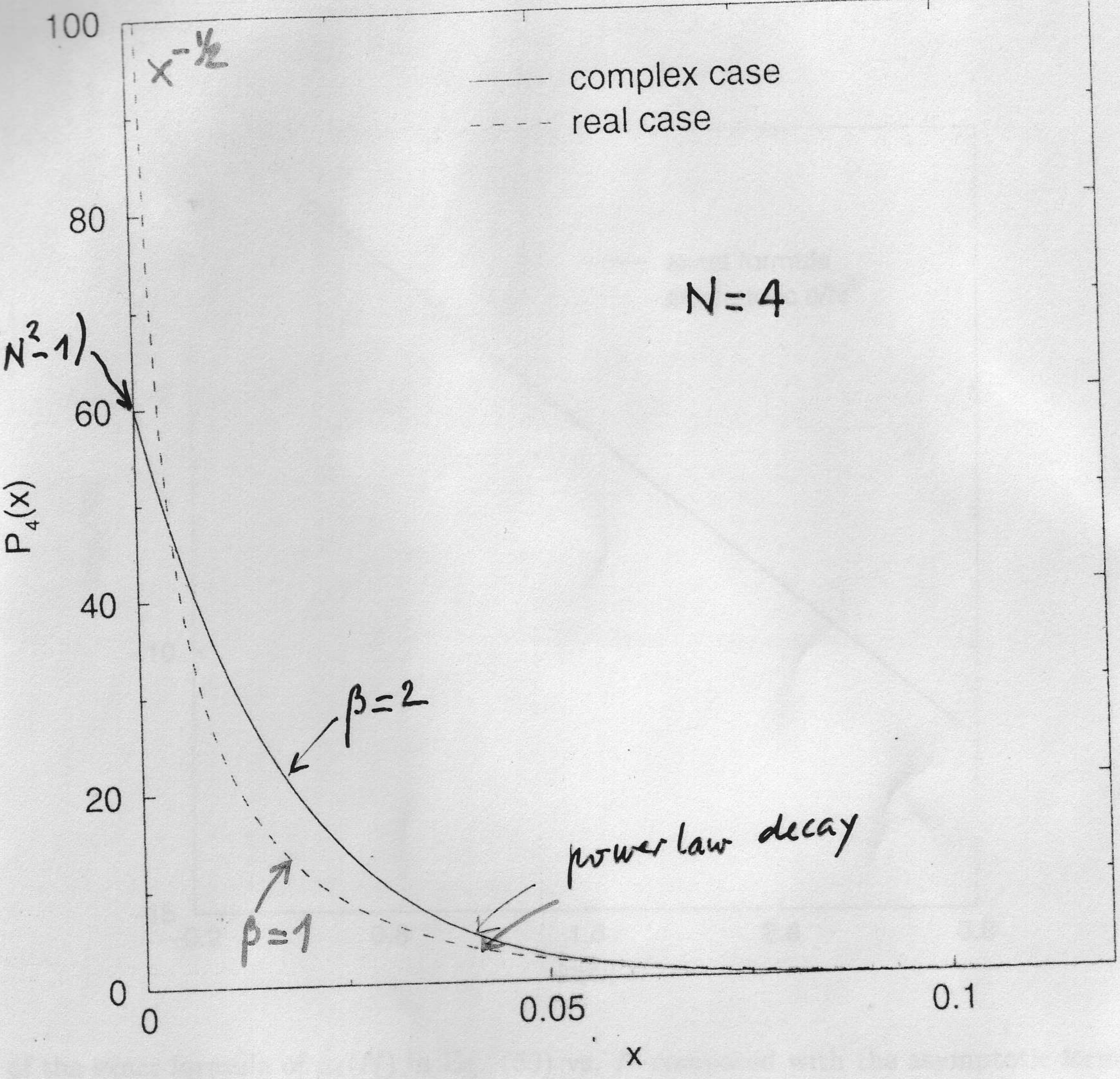
$$c = 2\left[1 - \sqrt{\frac{\pi e}{2}} \, \mathrm{erfc}\left(\frac{1}{\sqrt{2}}\right)\right] = 0.688641\ldots$$

$x \to 0$ $\quad N(N^2-1)\exp\left[-N(N^2-1)x\right]$ $\qquad C_N \cdot x^{-1/2}$

$x \to \frac{1}{N}$ $\quad N(N^2-1)(1-Nx)^{N^2-2}$ $\qquad A_N N^{-N/2} (1-Nx)^{\frac{N^2+N}{2}}$

$\uparrow$ $\qquad\qquad \uparrow$

$\beta = 2$ $\qquad\qquad \beta = 1$



The p.d.f $P_N(x)$ of the minimum eigenvalue $\lambda_{\min}$ vs. x for $N = 4$, for the complex and the real cases (Eqs. pecively). In the complex case, the density approaches a constant as $x \to 0$, whereas for the real case, it d diverges a s $x \to 0$.

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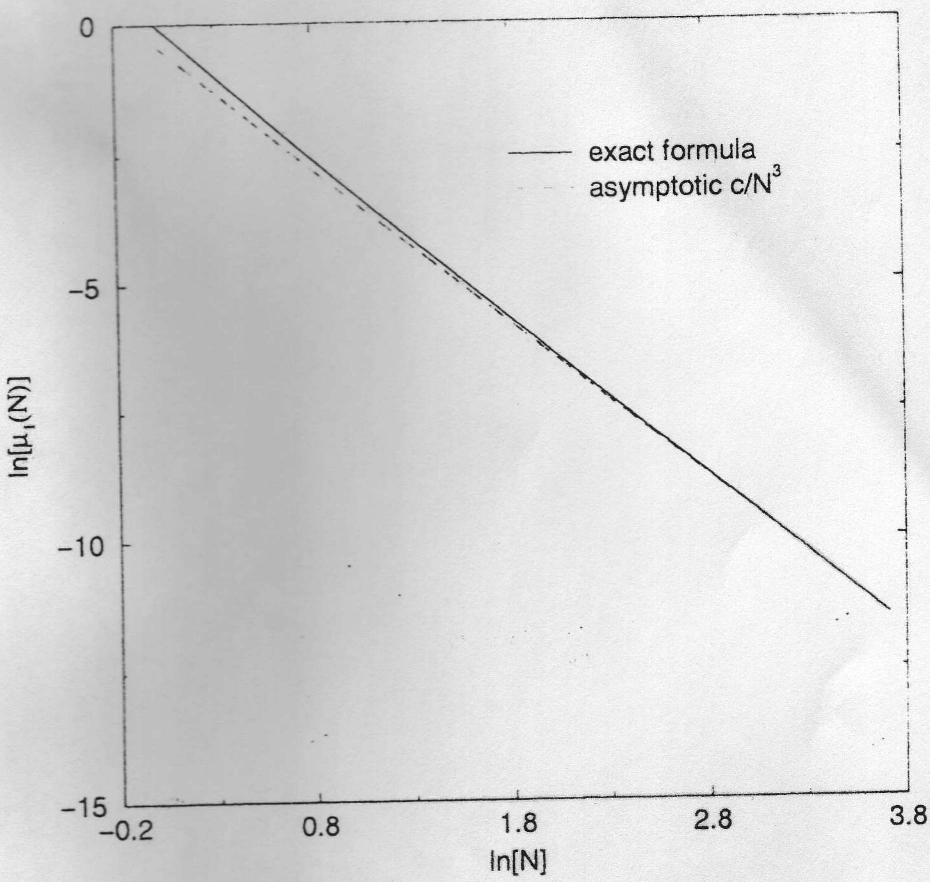
arXiv 0711.0677
J. Stat. Phys.



. A log-log plot of the exact formula of $\mu_1(N)$ in Eq. (59) vs. N compared with the asymptotic formula $\mu_1(N) \approx$ with $c = 0.688641$ for the real case.

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