Semiclassical theory of mesoscopic transport

with

spin-orbit interactions

Quantum chaos: Routes to RMT and beyond

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Jens Bolte
Dept. of Mathematics
Royal Holloway,
Univ. of London

jens.bolte@rhul.ac.uk

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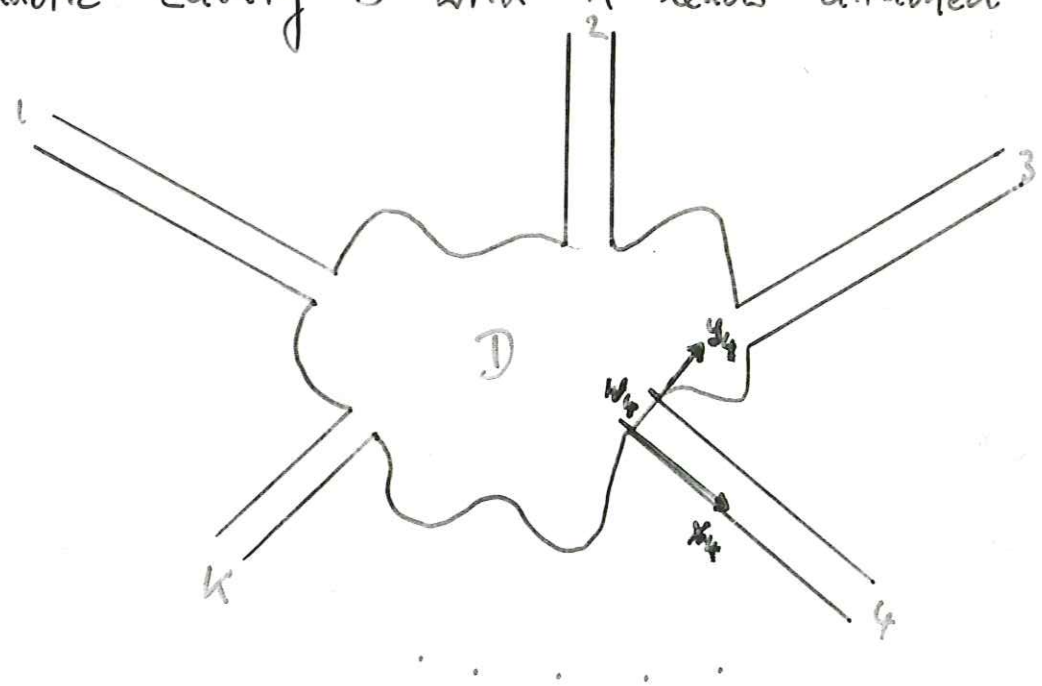
Mesoscopic transport

chaotic cavity D with K leads attached (no disorder)



Hamiltonian:

$$\hat{H} = \frac{1}{2m}\left(\hat{\underline{p}} - \frac{e}{c}\underline{A}(\underline{x})\right)^2 + \hat{\underline{S}}\cdot\underline{C}(\hat{\underline{p}},\hat{\underline{x}})$$

where

- $\underline{B} = \mathrm{rot}\,\underline{A} \equiv 0$ in leads
- spin-orbit interactions $\hat{\underline{S}}\cdot\hat{\underline{C}}$ (spin s) including

$$\left.\begin{aligned} &\text{Zeeman:} && \underline{C}_Z = -\frac{e}{mc}\underline{B} \\ &\text{Rashba:} && \underline{C}_R = \varkappa_R(i\sigma_y)\left(p - \frac{e}{c}\underline{A}\right) \\ &\text{Dresselhaus:} && \underline{C}_D = \varkappa_D\,\sigma_z\left(p - \frac{e}{c}\underline{A}\right) \end{aligned}\right\} \begin{array}{l}\equiv 0 \\ \text{in leads}\end{array}$$

Schrödinger eq. $\rightarrow$ linear response:

Kubo - Greenwood expression for conductivity, including spin-orbit contribution in current density

(cf. Fisher, Lee '81, Baranger, Stone '89, Nöckel, Stone, Baranger '93)

Landauer: $I_n = \sum_m g_{nm} V_m$

V_n: voltage at lead n

I_n: current through —"—

conductance coefficients

$$g_{nm} = -\frac{e^2}{2\pi\hbar} \int_0^\infty f_\beta'(E) \left[\sum_{\alpha_n, \alpha_m'} \left| S_{\alpha_n \alpha_m'} \right|^2 - \delta_{nm} (2s+1) N_n \right] dE$$

S - Matrix

$$S^{nm}_{\alpha_n \alpha_m'} = \frac{2\hbar^2}{im} \sqrt{\frac{k_n k_m}{W_n W_m}} \int_0^{W_n} \int_0^{W_m} \sin\left(\frac{a_n \pi y_n}{W_n}\right) \sin\left(\frac{a_m' \pi y_m'}{W_m}\right) *$$

$$* \, G_{\sigma\sigma'}(\underline{x}_n, \underline{x}_m', E) \, dy_m' \, dy_n + \delta_{nm} \delta_{\alpha_n \alpha_m'}$$

N_n: # of transversal modes in lead n

$f_\beta(E) = \frac{1}{1 + e^{\beta(E-\mu)}}$ Fermi fct.

Spin-Orbit Coupling, Antilocalization, and Parallel Magnetic Fields in Quantum Dots

D. M. Zumbühl,[1] J. B. Miller,[1,2] C. M. Marcus,[1] K. Campman,[3] and A. C. Gossard[3]

[1]*Department of Physics, Harvard University, Cambridge, Massachusetts 02138*

[2]*Division of Engineering and Applied Sciences, Harvard University, Cambridge, Massachusetts 02138*

[3]*Department of Electrical and Computer Engineering, University of California, Santa Barbara, California 93106*

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We investigate antilocalization due to spin-orbit coupling in ballistic GaAs quantum dots. Antilocalization that is prominent in large dots is suppressed in small dots, as anticipated theoretically. Parallel magnetic fields suppress both antilocalization and also, at larger fields, weak localization, consistent with random matrix theory results once orbital coupling of the parallel field is included. *In situ* control of spin-orbit coupling in dots is demonstrated as a gate-controlled crossover from weak localization to antilocalization.

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The combination of quantum coherence and electron spin rotation in mesoscopic systems produces a number of interesting transport properties. Numerous proposals for potentially revolutionary electronic devices that use spin-orbit (SO) coupling have appeared in recent years, including gate-controlled spin rotators [1] as well as sources and detectors of spin-polarized currents [2]. It has also been predicted that the effects of some types of SO coupling will be strongly suppressed in small 0D systems, i.e., quantum dots [3–5].

In this Letter, we investigate SO effects in ballistic-chaotic GaAs/AlGaAs quantum dots. We identify the signature of SO coupling in ballistic quantum dots to be *antilocalization* (AL), leading to characteristic magnetoconductance curves, analogous to known cases of disordered 1D and 2D systems [6–11]. AL is found to be prominent in large dots and suppressed in smaller dots, as anticipated theoretically [3–5]. Results are generally in excellent agreement with a new random matrix theory (RMT) that includes SO and Zeeman coupling [5]. Moderate magnetic fields applied in the plane of the 2D electron gas (2DEG) in which the dots are formed cause a crossover from AL to weak localization (WL). This can be understood as a result of Zeeman splitting, consistent with RMT [5]. At larger parallel fields WL is also suppressed, which is not expected within RMT. The suppression of WL is explained by orbital coupling of the parallel field, which breaks time-reversal symmetry [12]. Finally, we demonstrate *in situ* electrostatic control of the SO coupling by tuning from AL to WL in a dot with a center gate.

In mesoscopic conductors, coherent backscattering of time-reversed electron trajectories leads to a conductance *minimum* (WL) at $B = 0$ in the spin-invariant case, and a conductance *maximum* (AL) in the case of strong SO coupling [6]. In semiconductor heterostructures, SO coupling results mainly from electric fields [13] (appearing as magnetic fields in the electron frame), leading to momentum dependent spin precessions due to crystal inversion asymmetry (Dresselhaus term [14]) and hetero-interface asymmetry (Rashba term [15]).

SO coupling effects have been previously measured using AL in GaAs 2DEGs [8–10] and other 2D heterostructures [11]. Other means of measuring SO coupling in heterostructures, such as from Shubnikov–de Haas oscillations [16] and Raman scattering [17] are also quite developed. SO effects have also been reported in

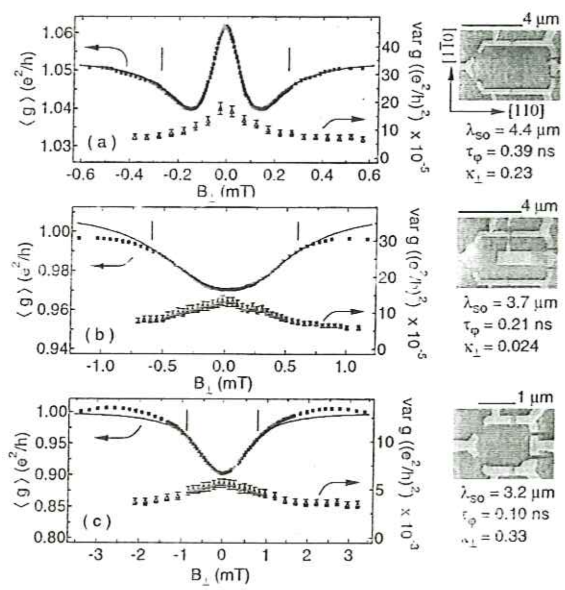


FIG. 1. Average conductance $\langle g \rangle$ (squares) and variance of conductance var(g) (triangles) calculated from ~200 statistically independent samples (see text) as a function of perpendicular magnetic field $B_\perp$ for (a) 8.0 μm^2 dot, (b) 5.8 μm^2 center-gated dot, and (c) 1.2 μm^2 dot at $T = 0.3$ K, along with fits to RMT (solid curves). In (b), the center gate is fully depleted. Vertical lines indicate the fitting range; error bars of $\langle g \rangle$ are about the size of the squares.

Weak (anti-) localisation, depending on relevance of spin-orbit interaction:

WL: no relevant s-o interaction

$$L_{escape} \ll L_{so} \quad \text{(small quantum dot)}$$

AL: s-o interactions relevant

$$L_{escape} \gg L_{so} \quad \text{(large quantum dot)}$$

For semiclassics: $L_{so} \gg \lambda_{Fermi}$

$$\updownarrow$$

$$E_{orbital} \gg E_{so} \quad \text{(energy scales)}$$

no relevant regime $L_{escape} \gg L_{so} \gg \lambda_{Fermi}$

Semiclassics

Approximation for Green's fct. when $E_{orb} \gg E_{so}$, or

$$\langle \frac{1}{2m}(\hat{\underline{p}} - \frac{e}{c}\underline{A})^2 \rangle \quad \gg \quad \langle \hat{\underline{S}} \cdot \underline{C} \rangle \quad , \quad \text{or}$$

$$\hat{H} = \frac{1}{2m}\left(\hat{\underline{p}} - \frac{e}{c}\underline{A}\right)^2 + \hbar \frac{1}{2} \underline{\sigma} \cdot \underline{C}$$

scl. parameter: $\hbar \to 0$

$\hat{=}$ $E_{orb} \gg E_{so}$

Thus: Use $\hbar$ as single scl. parameter, i.e., investigate Schrödinger eq. as $\hbar \to 0$.

Result (for Green's fct., J.B., S. Keppeler '99)

$$G(\underline{x}, \underline{x}', E) \underset{\hbar \to 0}{\sim} \sum_{\gamma(\underline{x},\underline{x}')} \operatorname{tr} d_\gamma(\underline{p}', \underline{x}', t) \; A_\gamma(\underline{x}, \underline{x}', E) \, e^{\frac{i}{\hbar} S_\gamma(\underline{x}, \underline{x}', E)}$$

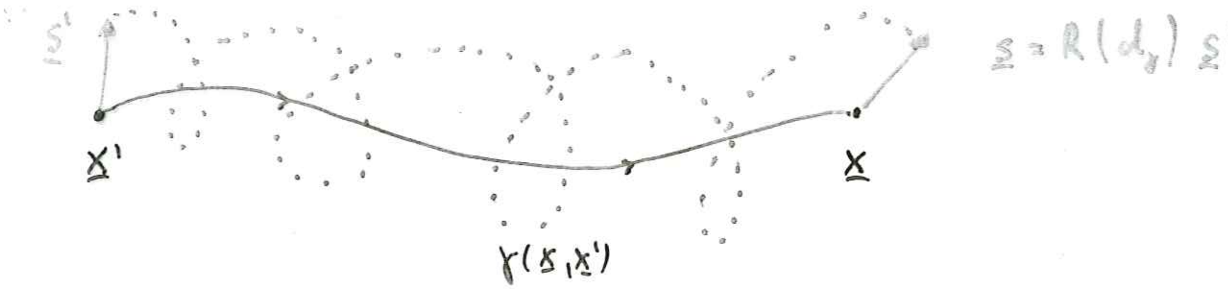
where

$\gamma(\underline{x}, \underline{x}')$: classical trajectory from $\underline{x}'$ to $\underline{x}$ at energy E, generated by

$$H_0(\underline{p}, \underline{x}) = \frac{1}{2m}\left(\underline{p} - \frac{e}{c}\underline{A}(\underline{x})\right)^2$$

$S_\gamma = \int_\gamma \underline{p} \cdot d\underline{x}$ action

A_γ stability amplitude, Maslov phases



$d_\gamma \in SU(2)$ spin transport along γ, solution of

$$\dot{d}_\gamma + i\,\tfrac{1}{2}\,\underline{\sigma}\cdot\underline{C}\; d_\gamma = 0$$

Yielding: classical spin-orbit dynamics

$$(\underline{p}', \underline{x}', \underline{s}') \longmapsto (\underline{p}, \underline{x}, \underline{s})$$

on spin-orbit phase space $\mathbb{R}^2 \times \mathcal{D} \times S^2$

invariant measure

$$\text{const.} * \delta\left(H_0(\underline{p},\underline{x}) - E\right)\, dp\, dx\, ds$$

$\rightsquigarrow$ ergodicity, mixing, ... (closed system)

The spin transport is multiplicative:

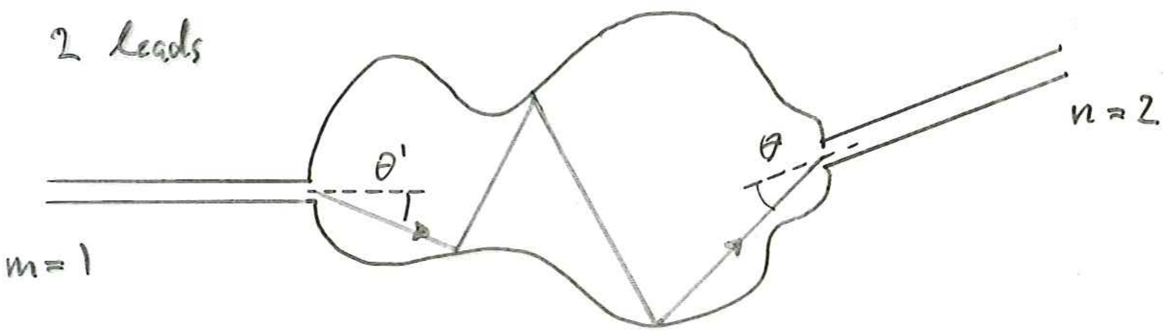
$\gamma = \gamma_2\gamma_1$

$\Downarrow$

$d_\gamma = d_{\gamma_2}\, d_{\gamma_1}$

Open system, transmission

2 leads



$$S^{nm}_{\alpha_n \alpha'_m} \underset{\hbar\to 0}{\sim} \sum_{\gamma\theta\theta'} B_\gamma \, d_{\gamma,\sigma\sigma'} \, e^{\frac{i}{\hbar} S_\gamma}$$

Hence

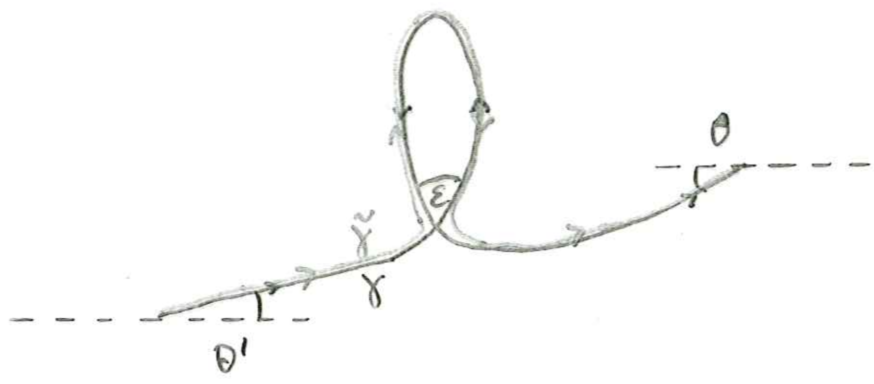
$$\sum_{\sigma,\sigma'=-s}^{s} \left| S^{21}_{\alpha_2 \alpha'_1} \right|^2 \sim \sum_{\gamma\theta\theta',\, \tilde{\gamma}\theta\theta'} B_\gamma \overline{B_{\tilde{\gamma}}} \, \mathrm{tr}\left(d_\gamma d^\dagger_{\tilde{\gamma}} \right) e^{\frac{i}{\hbar}(S_\gamma - S_{\tilde{\gamma}})}$$

Diagonal contribution $(\gamma = \tilde{\gamma})$:

$$\sum_{\sigma,\sigma'} \left| S^{21}_{\alpha_2\alpha'_1} \right|^2_{diag} \sim (2s+1) \sum_\gamma |B_\gamma|^2 \sim \frac{2s+1}{N_1 + N_2}$$

sum rule for orbital dynamics

First correction: Sieber – Richter pairs



distribution of crossing angles ε for loops of duration T

$$p_{so}(\varepsilon, T) = (-1)^{2s} \, p(\varepsilon, T)$$

sum rule for spin-orbit dynamics:

sum rule for orbital dynamics

$$\int_{SU(2)} \mathrm{tr}\left[\pi_s(g)^2\right] dg = (-1)^{2s}$$

Then

$$\sum_{\sigma,\sigma'=-s}^{s} \left| S^{21}_{\alpha_2\alpha_1'} \right|^2_{S\text{-}R} \sim \frac{-(-1)^{2s}}{(N_1+N_2)^2}$$

(see also Zaitsev, Frustaglia, Richter '05)

Higher corrections: phase space encounters (*)

trajectory pairs $(\gamma, \tilde{\gamma})$ with v_ℓ ℓ-encounters

$V = \sum_\ell v_\ell$ # encounters

$L = \sum_\ell \ell v_\ell$ # stretches in encounters

4 encounter

Poincaré section $\perp$ encounters $\rightsquigarrow$ γ cut into $L+1$ pieces:

spin transport: $d_\gamma = d_{L+1}\, d_L \cdots d_2\, d_1$

$$d_{\tilde{\gamma}} \approx d_{L+1}\, d_{k_L}^{\eta_L} \cdots d_{k_2}^{\eta_1}\, d_1 \qquad \eta_j \in \{\pm 1\}$$

Thus: $\mathrm{tr}\left(d_\gamma d_{\tilde{\gamma}}^\dagger\right) \approx \mathrm{tr}\left(d_L \cdots d_2\, d_{k_2}^{\dagger\eta_2} \cdots d_{k_L}^{\dagger\eta_L}\right)$

(*) S. Müller et. al '04, '05 ; S. Heusler et al. '06, '07
Jacquod, Whitney '06, Brouwer, Rahav '06 ...

Coordinates $(\underline{s}, \underline{u})$ for encounters,

$$\left\langle \sum_{\sigma,\sigma'=-s}^{s} \left| S^{21}_{\alpha_2\alpha_1'} \right|^2 \right\rangle_{\Delta E} - \frac{2s+1}{N_1+N_2}$$

$$\approx \left\langle \sum_{\gamma} \sum_{\underline{v}} N(\underline{v}) \int \cdots \int d^{L-V}u \; d^{L-V}s \; w_T^{s.o.}(\underline{u},\underline{s}) \, |B_\gamma|^2 \, e^{\frac{i}{\hbar}\Delta S_\gamma} \right\rangle_{\Delta E}$$

where:

- $\underline{v} = (v_2, v_3, \ldots)$
- $N(\underline{v})$: # $(\gamma, \tilde{\gamma})$ with encounter structure $\underline{v}$
- $\Delta S_\gamma \approx \sum_j s_j u_j$

Density $w_T^{s.o.}(\underline{u},\underline{s})$ of encounters :

ergodicity of spin-orbit dynamics ($\rightsquigarrow$ sum rule) implies

$$w_T^{s.o.}(\underline{u},\underline{s}) = w_T(\underline{u},\underline{s}) \cdot M_{\gamma\tilde{\gamma}}$$

$$M_{\gamma\tilde{\gamma}} := \int_{SU(2)} \cdots \int_{SU(2)} \mathrm{tr}\left[\pi_s\left(g_L \cdots g_2 \, g_{k_2}^{+\eta_2} \cdots g_{k_L}^{+\eta_L} \right) \right] dg_2 \cdots dg_L$$

$$= (2s+1) \left(\frac{(-1)^{2s}}{2s+1} \right)^{L-V}$$

Proof uses diagrams of encounters and induction on the number of 2-encounters.

Calculation based on

- $$\int_{SU(2)} \mathrm{tr}\left(\pi_s(g_1 h g_2 h)\right) dh = \frac{(-1)^{2s}}{2s+1} \mathrm{tr}\, \pi_s(g_1 g_2^{-1})$$

- $$\int_{SU(2)} \int_{SU(2)} \mathrm{tr}\, \pi_s\left(h_1 g_1 h_2^{-1} g_2 h_1^{-1} g_3 h_2 g_4\right) dg_2\, dg_1$$

$$= \frac{1}{(2s+1)^2} \mathrm{tr}\, \pi_s\left(g_4 g_3 g_2 g_1\right)$$

Result:

Transmission coefficient (for $s \in \{ \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots \}$)

$$T = \left\langle \sum_{\alpha_2 \alpha_1'} | S^{21}_{\alpha_2 \alpha_1'} |^2 \right\rangle_{\Delta E}$$

$$\sim (2s+1) \frac{N_1 N_2}{N_1 + N_2} \sum_{k=0}^{\infty} \frac{1}{(N_1+N_2)^k} \frac{1}{(2s+1)^k}$$

$$= \frac{(2s+1)^2}{(2s+1)(N_1+N_2) - 1} \underset{s=1/2}{=} \frac{4}{2(N_1+N_2) - 1} = T_{CSE}$$

Apply magnetic field: time-reversal invariance broken

$\rightsquigarrow$ only diagonal contribution survives

$$T_{t.r.} - T_{b.t.r.} = \frac{N_1 N_2}{N_1 + N_2} \frac{2s+1}{(2s+1)(N_1+N_2) - 1} > 0$$

weak anti-localisation

Remark: $s \in \{0, 1, 2, \dots\} \Rightarrow$ weak localisation

Conclusions

- semiclassics of spin-orbit coupling $\Rightarrow$ classical spin-orbit dynamics

- linear response, Kubo, Landauer $\Rightarrow$ scl. calculation of transmission (reflection)

- speration of spin-orbit contribution through sum rules ($\hat{=}$ ergodicity) of spin-orbit dynamics

- calculation to all orders yields T_{CSE}
  $\Rightarrow$ weak anti-localisation