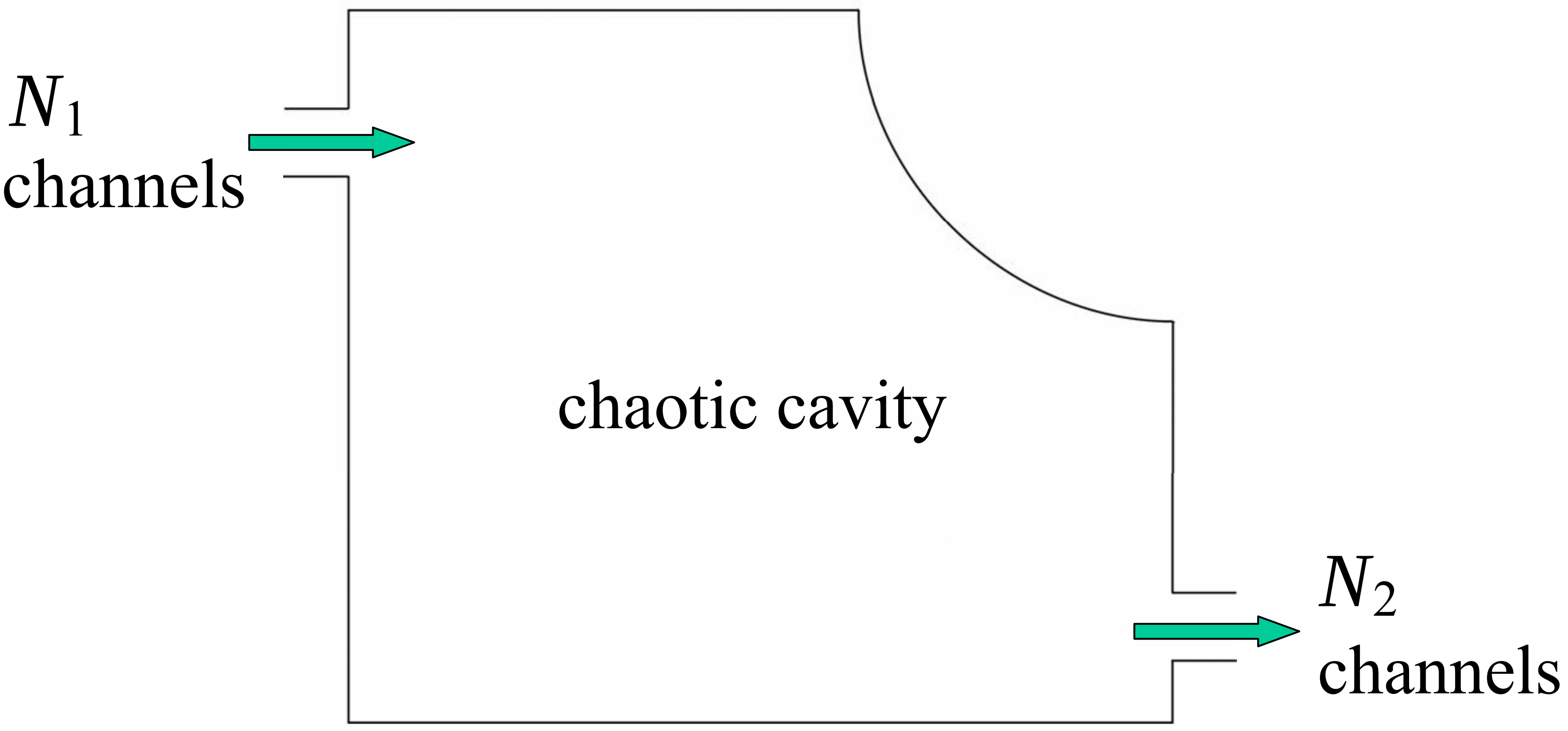
Transport through chaotic cavities: RMT reproduced from semiclassics

P.B., joint work with

Stefan Heusler, Sebastian Müller
and Fritz Haake

Universität Duisburg-Essen

Transport problem



$$N = N_1 + N_2$$

S, t, and r-matrices

Inside leads:

$$\psi = C\sin(k^{\perp}y)\exp(ik^{\parallel}x),$$ $$E = \hbar^2\frac{(k^{\perp})^2+(k^{\parallel})^2}{2m}$$ fixed.

In-(1) and out- (2) channels (*w* = width):

$$k_i^{\perp} = \frac{m_i\pi}{w_i}, \quad m_i = 1\ldots N_i \quad (i = 1,2)$$

In- and out- states in the transport problem:

$$\Psi^{(1)} = \psi_{m_1}^{(1)} + \sum_{l_1=1}^{N_1} r_{m_1 l_1}\psi_{l_1}^{(1)*}, \qquad \Psi^{(2)} = \sum_{l_2=1}^{N_2} t_{m_1 l_2}\psi_{l_2}^{(2)}$$

S-matrix:

$$S = \begin{pmatrix} r & t \\ t' & r' \end{pmatrix}$$

Transport properties

- Conductance $\sim\langle \mathrm{Tr}\; tt^\dagger \rangle$
- Conductance variance $\sim\left\langle (\mathrm{Tr}\; tt^\dagger)^2 \right\rangle - \langle \mathrm{Tr}\; tt^\dagger \rangle^2$
- Shot noise $\sim\langle \mathrm{Tr}\; tt^\dagger - \mathrm{Tr}\, tt^\dagger\, tt^\dagger \rangle$
- …

(Averaging done over energy interval)

Conductance, random-matrix prediction

$$G = \begin{cases} \dfrac{N_1 N_2}{N} \\ \\ \dfrac{N_1 N_2}{N+1} = \dfrac{N_1 N_2}{N} - \dfrac{N_1 N_2}{N^2} + \dfrac{N_1 N_2}{N^3} - \cdots \end{cases}$$

unitary case
(with magnetic field,
no time-reversal invariance)

orthogonal case
(without magnetic field,
time-reversal invariance)

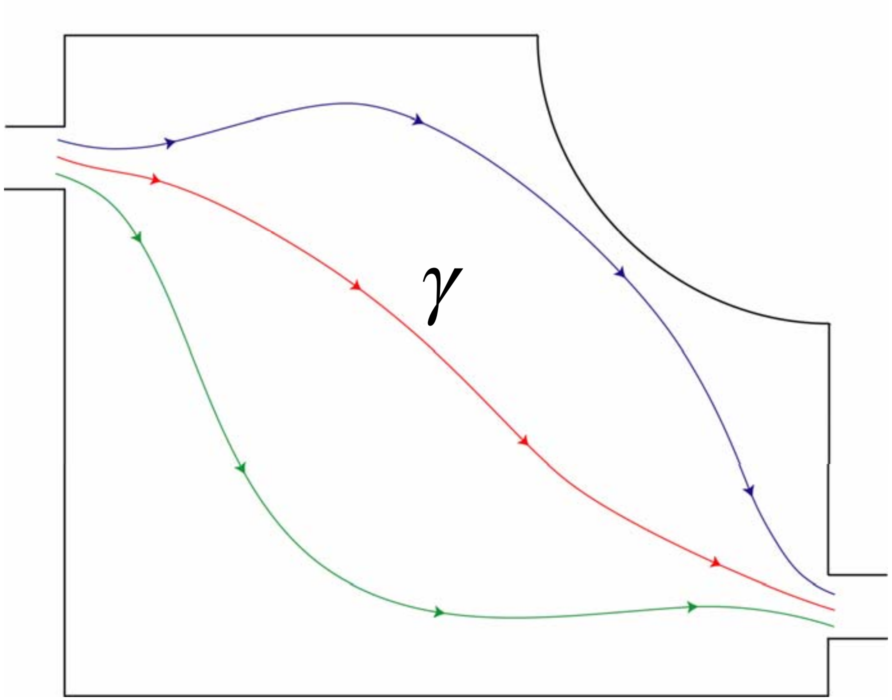
why true for individual systems?

Semiclassical approach

Van Vleck: $t_{m_1 m_2} \propto \sum_\gamma A_\gamma e^{iS_\gamma/\hbar}$

In- channel Out- channel

Entrance and exit angles fixed by channel numbers



Semiclassical approach

Conductance

$$G = Tr(t^{\dagger}\, t) = \frac{1}{T_H}\left\langle \sum_{\gamma,\gamma'} A_\gamma A^{*}_{\gamma'}\, e^{i(S_\gamma - S_{\gamma'})/\hbar} \right\rangle$$

↑

Heisenberg time $T_H \propto \hbar^{-1}$

Need pairs of trajectories with small action difference

Diagonal approximation

identical trajectories $\gamma = \gamma'$

$$G_{\text{diag}} = \frac{1}{T_H} \left\langle \sum_\gamma |A_\gamma|^2 \right\rangle$$

$$= \frac{N_1 N_2}{T_H} \int_0^\infty dT\, e^{-\frac{N}{T_H} T} = \frac{N_1 N_2}{N}$$

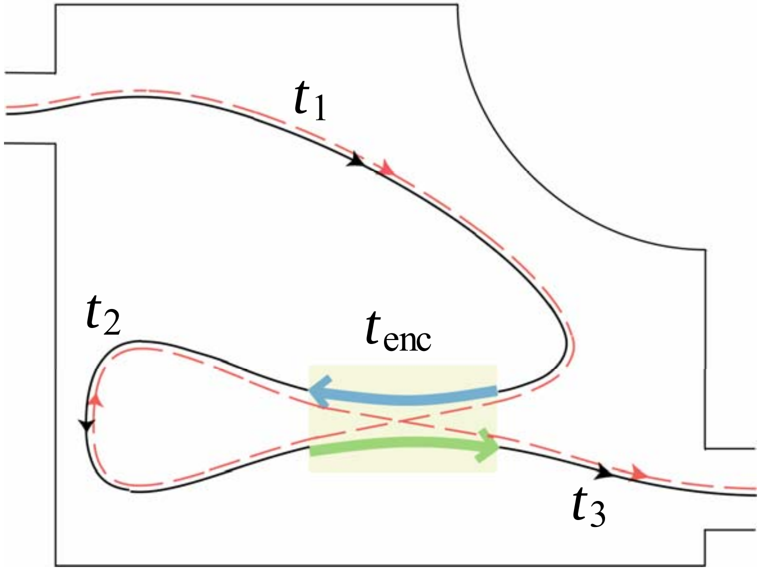
probability of staying inside up to time T

$\frac{N}{T_H}$ = escape rate

Higher orders

- Created by pairs of trajectories-partners composed of same pieces traversed in different order / with different sense
- Switches of motion: encounters
- *l*-encounter: avoided crossing in phase space of *l* stretches of same trajectory, or trajectory and its time reversed, or of different trajectories
- 2-encounter, viewed in configuration space: small-angle crossing / narrow avoided crossing

Richter / Sieber pairs



- dwell time $T = t_1 + t_2 + t_3 + 2t_{enc}$
- if no escape on first stretch, no escape on second stretch either

➡ survival probability

$$e^{-\frac{N}{T_H}(t_1+t_2+t_3+t_{enc})} > e^{-\frac{N}{T_H}T}$$

Encounters hinder escape into leads.

Richter / Sieber pairs

Pairs characterized by

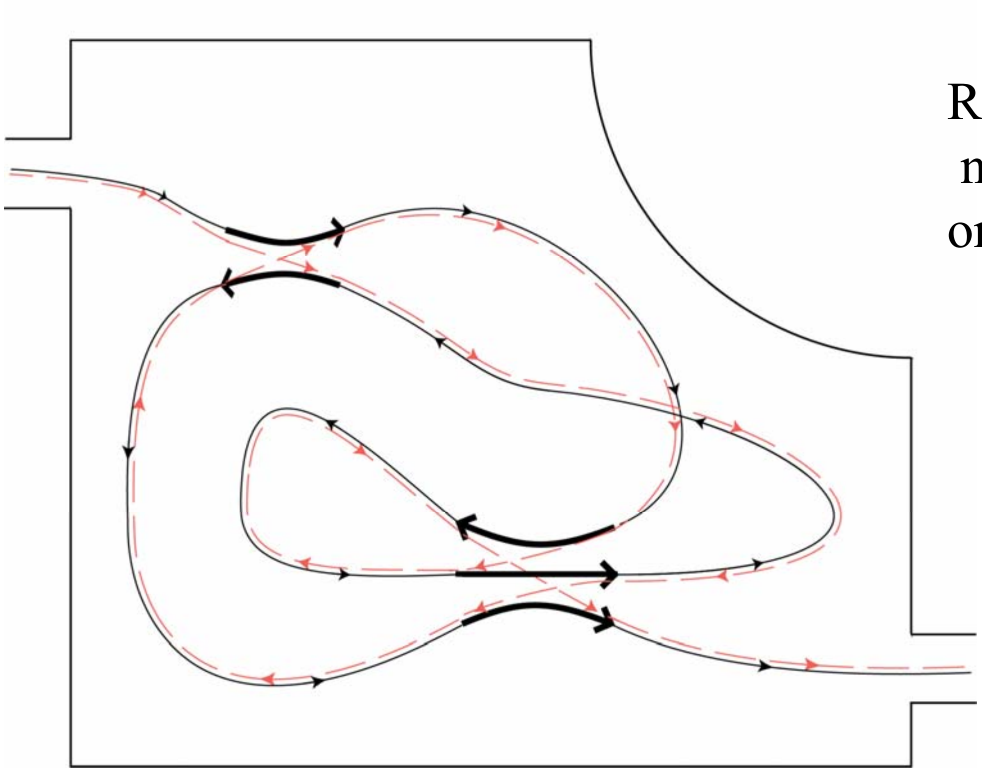
- link durations $0 < t_1,\ t_2,\ t_3 < \infty$
- phase-space separations inside encounter *s*,*u*

Integration gives $$G_{\mathrm{RS}} = \frac{1}{T_H}\left\langle \sum_{(\gamma,\gamma')_{\mathrm{RS}}} A_\gamma A^*_{\gamma'} e^{i\,\Delta S_{\gamma\gamma'}/\hbar}\right\rangle$$

$$= \frac{N_1 N_2}{T_H}\int dt_1\, dt_2\, dt_3\, ds\, du\, \underbrace{\frac{1}{\Omega\, t_{\mathrm{enc}}}}_{\text{“ergodic density of encounters”}}\, \underbrace{e^{-\frac{N}{T_H}(t_1+t_2+t_3+t_{\mathrm{enc}})}}_{\text{survival probability}}\, e^{i\Delta S/\hbar}$$

$$= \frac{N_1 N_2}{T_H}\underbrace{\left(\frac{T_H}{N}\right)^3}_{\text{three links}}\underbrace{\left(-\frac{N}{T_H^{\,2}}\right)}_{\text{one encounter}} = -\frac{N_1 N_2}{N^2}$$

Higher orders in 1/N



Reconnections may
not lead to periodic
orbits splitting off

Survival probability

$$e^{-\frac{N}{T_H}(\Sigma_{\text{loops}}\ t_{\text{loop}} + \Sigma_{\text{enc}}\ t_{\text{enc}})} > e^{-\frac{N}{T_H}T}$$

Diagrammatic rules for trajectory pairs

- $1/N$ for every link
- $(-N)$ for every encounter
- Multiply by the number of in- and out-channels $(N_1 N_2)$
- Sum over all families (topological versions) of trajectory pairs.

Higher orders in 1/N

Each family of trajectory pairs contributes

$$\frac{(-1)^{\#\text{encounters}} N_1 N_2}{N^{\#\text{loops}-\#\text{encounters}}}$$

Summation gives

$$G = \begin{cases} \dfrac{N_1 N_2}{N} & \text{with magnetic field (not TR invariant)} \\ \dfrac{N_1 N_2}{N} - \dfrac{N_1 N_2}{N^2} + \dfrac{N_1 N_2}{N^3} - \ldots = \dfrac{N_1 N_2}{N+1} & \text{without magnetic field (time-reversal invariant)} \end{cases}$$

in agreement with RMT

Shot noise

Fluctuations of current through a cavity:

$$P \sim Tr\left(t^{\dagger}t - t^{\dagger}t\ t^{\dagger}t\right)$$

Semiclassically

$$\mathrm{Tr}\ \ tt^{\dagger}\ tt^{\dagger} = \sum_{m_1,n_1}^{N_1}\sum_{m_2,n_2}^{N_2}\sum_{pqrt} A_p A_q^* A_r A_t^* \exp\left(i\,\frac{S_p+S_r-S_q-S_t}{\hbar}\right)$$

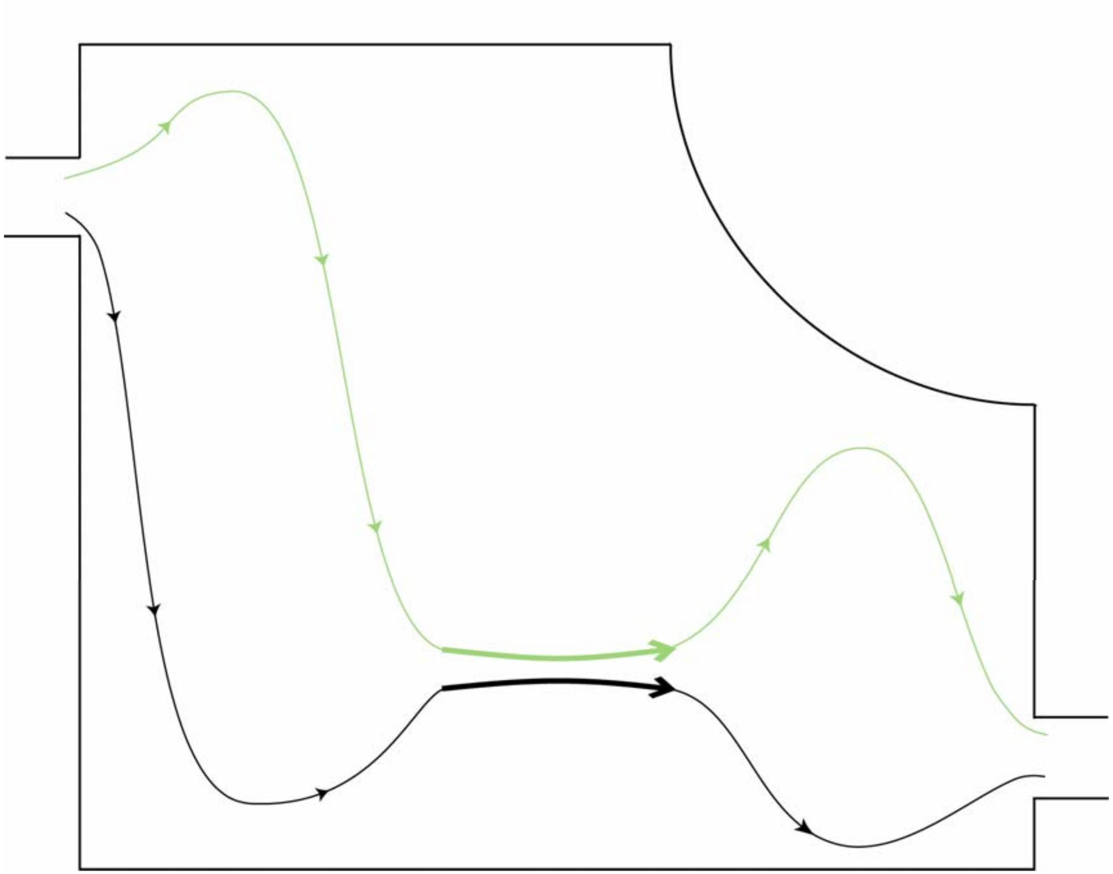
Sum over quadruplets of classical trajectories *p, q, r, t ,* connecting channels m_1m_2 , n_1m_2 , n_1n_2 , m_1n_2

Contributing quadruplets must have

$$S_p + S_r \approx S_q + S_t$$

i.e. the pairs (*q, t)* and *(p, r)* must be *partners*

Leading term



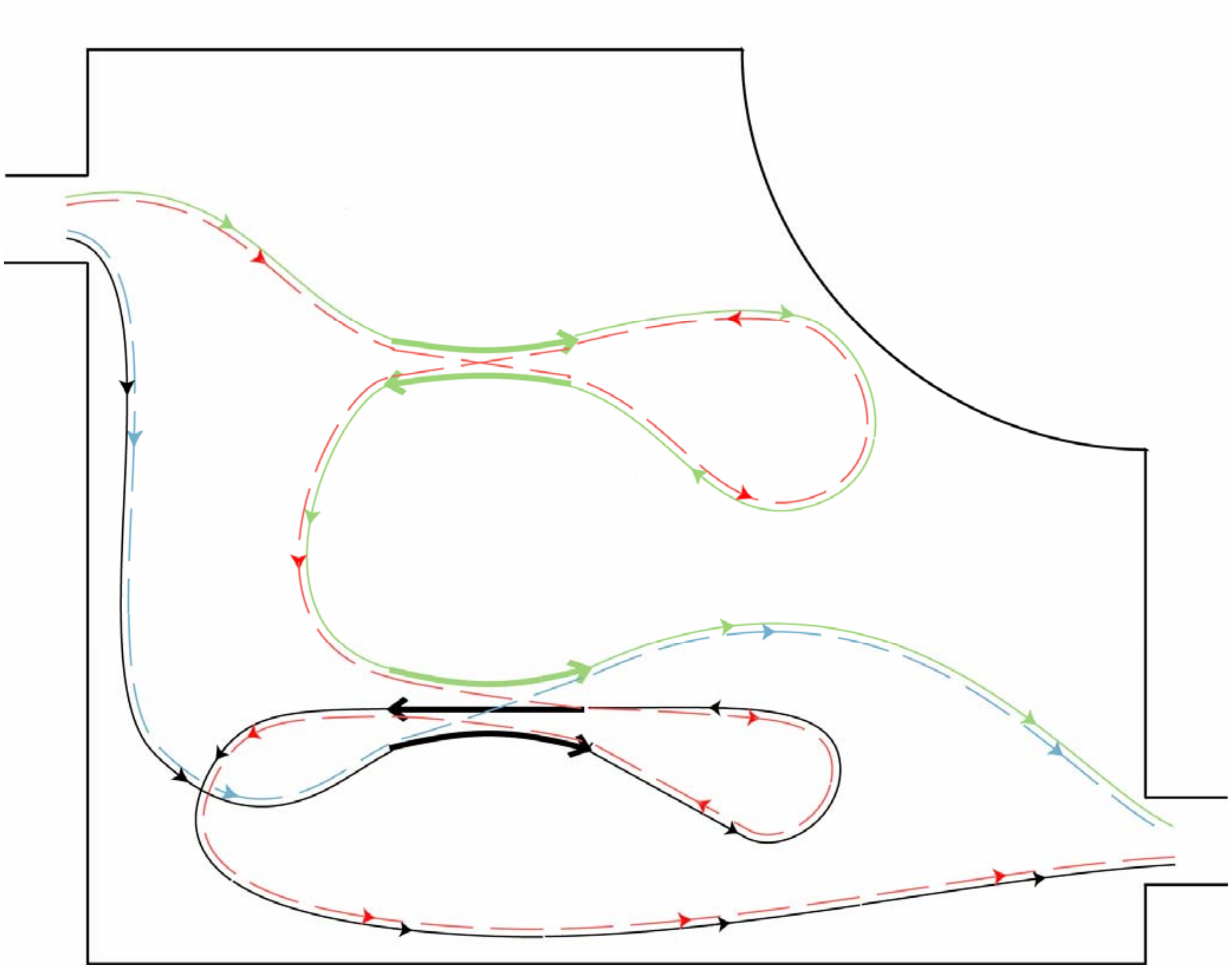
Schanz,
Puhlmann,
Geisel 03

Using diagrammatic rules (1/N per link, –N per encounter),

$$P = -\frac{1}{N^4}\times(-N) \quad \times N_1^2 N_2^2$$

Shot noise

Example for higher orders:



Shot noise

semiclassical prediction

$$P = \begin{cases} \dfrac{N_1^2 N_2^2}{N(N^2-1)} & \text{unitary case} \\ \\ \dfrac{N_1(N_1+1)N_2(N_2+1)}{N(N+1)(N+3)} & \text{orthogonal case} \end{cases}$$

O(N) and O(1) agree with RMT

higher orders first obtained semiclassically
(later confirmed in RMT)

GOE/GUE crossover

Weak magnetic field B: trajectories unchanged.
Additional action:

$$\Theta = \frac{eB}{2mc}\int \underset{\Uparrow}{L_z}(t)dt \qquad ;$$

Angular momentum

$$e^{i(S_\gamma - S_{\gamma'})/\hbar} \rightarrow e^{i(S_\gamma - S_{\gamma'} + \Theta_\gamma - \Theta_{\gamma'})/\hbar}$$

Magnetic phase on elements of γ and γ’ traversed in same direction cancels, in opposite directions is doubled.

Diagrammatic rules under crossover

- For a loop changing direction : $N^{-1}(1+\xi)^{-1}$
- preserving direction : N^{-1}
- For an encounter with μ stretches changing direction : $-N\,(1+\mu^2\xi\,)$

$\xi \propto (B/\hbar)^2$ is the crossover parameter.

(Per one in- and one out-channel; for each topologic family)

GOE/GUE crossover

$$G = \frac{N_1 N_2}{N}\left(1 - \frac{1}{N(1+\xi)} + \frac{1}{N^2(1+\xi)^2} - \frac{4\xi^4+3\xi^3+13\xi^2+2\xi+1}{N^3(1+\xi)^5} + \ldots\right)$$

$$\xi \propto (B/\hbar)^2 \quad \rightarrow \quad \begin{cases} 0 & \text{orthogonal case} \\ \infty & \text{unitary case} \end{cases}$$

Coincides with RMT (Weidenmüller *e.a.*, 1995)

Wigner delay time (Cuipers, Sieber 2007)

- Approach: similar, leads to RMT results.
- Equivalence proven of delay time representation as sums over trajectory pairs and periodic orbits of open resonator

Conclusions

- Diagrammatic rules found leading to RMT results for all examined transport properties. Based on: a) partnership of trajectories differing in encounters; b) increase of dwell time in orbits with encounters
- Applicability limited by Ehrenfest-time corrections (case N>>1) and diffraction effects (N~1)