NETWORK MODELS

FOR THE QUANTUM HALL EFFECT

AND ITS GENERALISATIONS

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Outline

- **Network models**

  Quantum lattice models for single-particle systems with disorder

- **Symmetry classes**

  Discrete symmetries and additions to Wigner-Dyson classification

- **Random-bond Ising model and network model**

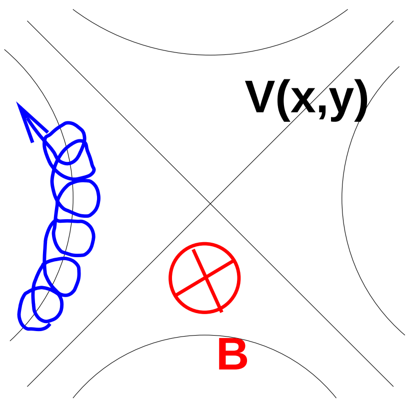
  Classical spin order and quantum delocalisation

- **Classical percolation and network model**

  Classical and quantum delocalistion

Network Models

Motivation

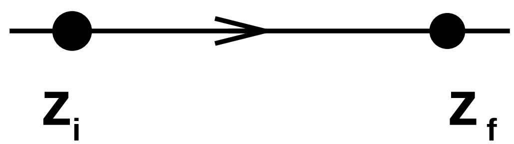


charged particle

in magnetic field

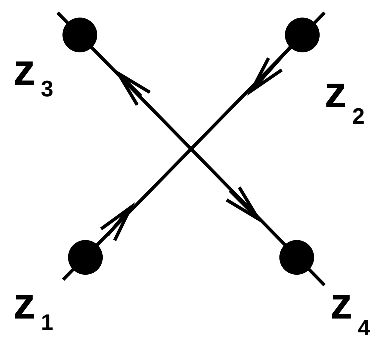
Ingredients

Links:



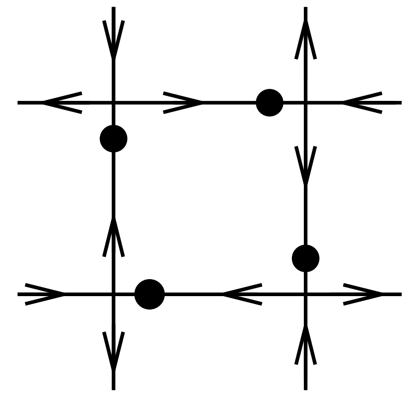
$$z_f = e^{i\phi} z_i$$

Nodes:



$$\begin{pmatrix} z_3 \\ z_4 \end{pmatrix} = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

2D lattice



Evolution operator

$$W = W_1 W_2$$

W_1**: links**

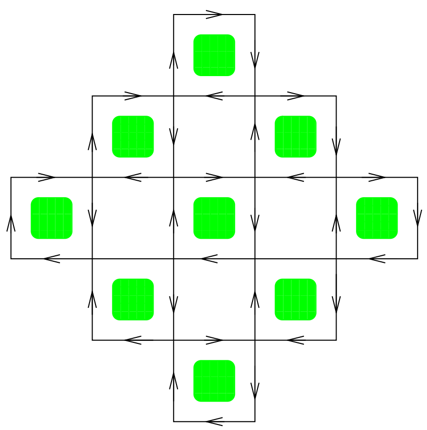
W_2**: nodes**

Two-dimensional U(1) model

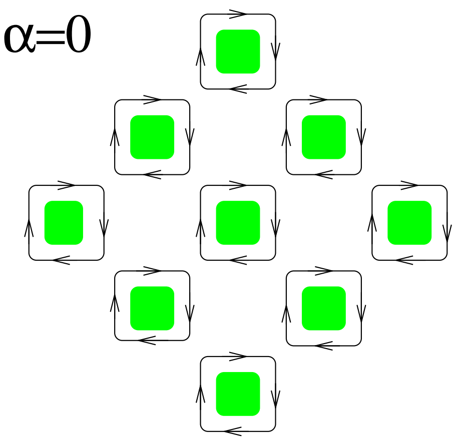
Random link phases + uniform scattering angle α at nodes

Delocalisation transition as α varied

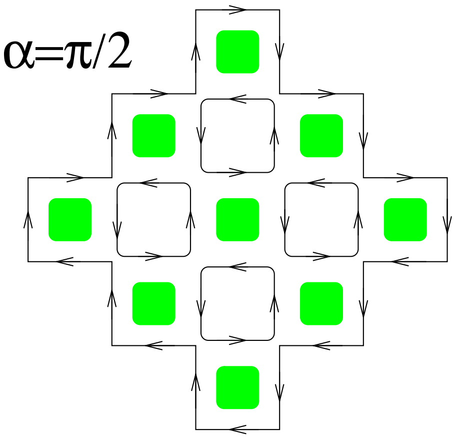
Model



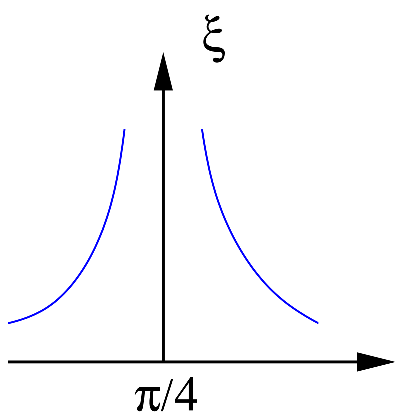
Chalker and Coddington, 1988



Limiting cases



Localisation length

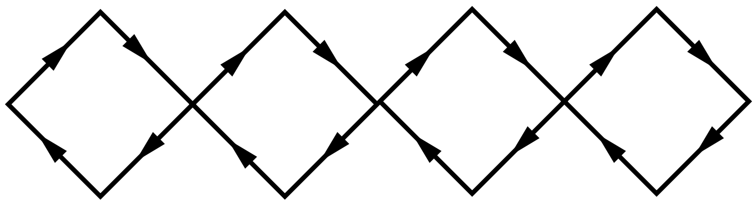


$$\xi \sim |\alpha - \pi/4|^{-\nu}$$

$$\nu \simeq 2.3$$

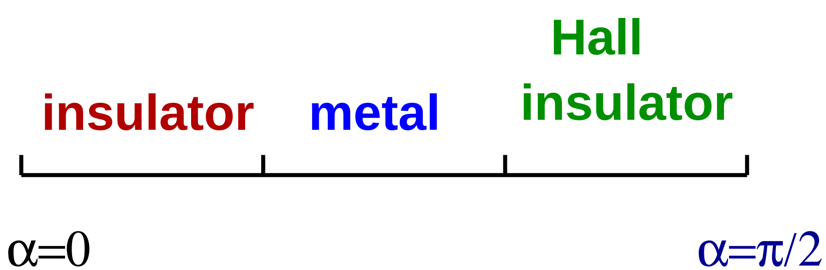
U(1) model in other geometries

One dimension

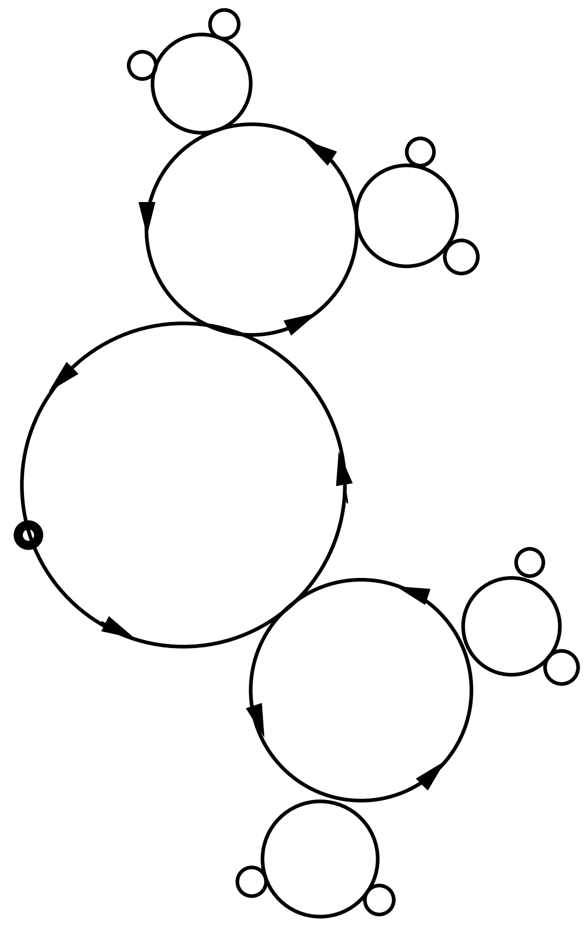


$$\xi = 1/\ln(\csc\alpha)$$

Three dimensions



Cayley tree



Symmetry Classes

Dyson random matrix ensembles

Orthogonal

with time-reversal symmetry

Symplectic

with time-reversal symmetry

and Kramers degeneracy

Unitary

without time-reversal symmetry

Additional symmetry classes

Altland and Zirnbauer 1997

Hamiltonian H **with discrete symmetry**

Energy levels in pairs $\pm E$

$$X^{-1} H^* X = -H$$

Given $H\psi = E\psi$ **, define** $\tilde{\psi} = X\psi^*$**.**

Then $H\tilde{\psi} = -E\tilde{\psi}$**.**

'Class D' $X = 1$

'Class C' $X = i\sigma_y$

Generalisations of network models

Amplitudes $z_i \longrightarrow$ **n-component vector**

Link phases $e^{i\phi} \longrightarrow n \times n$ **unitary matrices** U

Without further restrictions: U(n) model

not time-reversal invariant, so member of unitary symmetry class

With discrete symmetries:

Class D: $H^* = -H$ **so link phases** $U \sim e^{iH}$ **are *real***

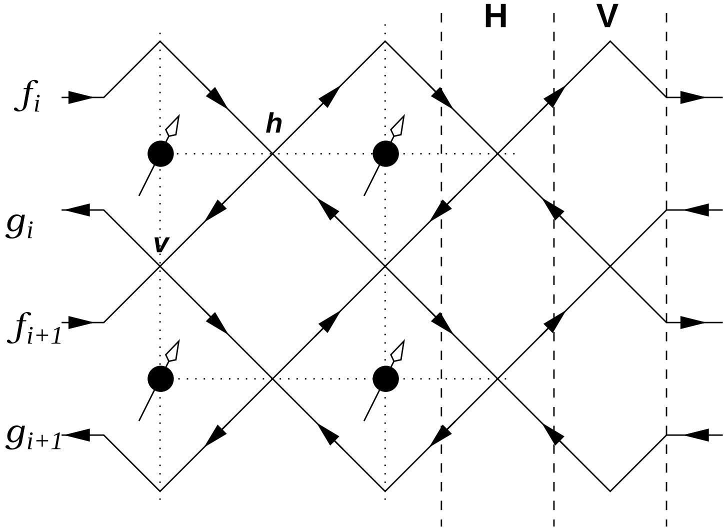
O(n) model **related to random bond Ising model**

Class C: $\sigma_y H^* \sigma_y = -H$ **so link phases** $\in$ **Sp(n), with Sp(2)** $\sim$ **SU(2)**

SU(2) model **related to classical percolation**

Ising model and network model

Relation between models

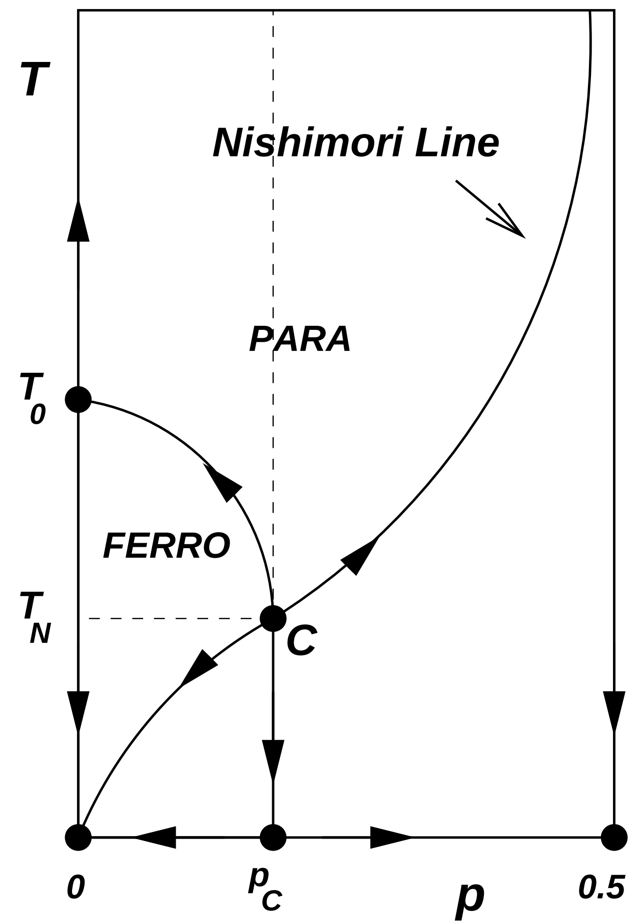


$$H_{\text{Ising}} - \sum_{\langle ij \rangle} J_{ij}\sigma_i\sigma_j$$

$$J_{ij} = \begin{cases} +J & \text{probability} \quad p \\ -J & \text{probability} \quad (1-p) \end{cases}$$

$$\sin\alpha_{ij} = \tanh 2\beta J_{ij}$$

Phase diagram



Cho and Fisher, 1998; Gruzberg, Read and Ludwig, 2001; Merz and Chalker 2002.

SU(2) network model and classical random walks

Feynman path expansion for Green function $G(\zeta) = (1 - \zeta W)^{-1}$

$$[G(\zeta)]_{r_1,r_2} = \sum_{\text{n−step paths}} \zeta^n A_{\text{path}}$$

with weight $A_{\text{path}} \sim \prod_{\text{links}} U_{\text{link}} \left\{ \begin{array}{c} \cos(\alpha) \\ \pm\sin(\alpha) \end{array} \right\}^n$

SU(2) Averages

$$\langle U^n \rangle = \begin{cases} 1 & n = 0 \\ -1/2 & n = \pm 2 \\ 0 & \text{otherwise} \end{cases}$$

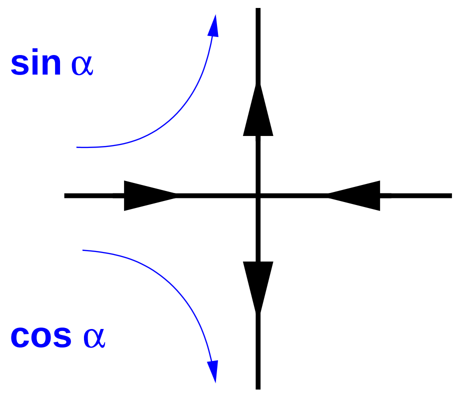
– keep only paths that cross each link 0 or 2 times.

Gruzberg, Ludwig and Read, 1999; Beamond, Cardy and Chalker, 2002

Quantum to classical mapping

Disorder-average for quantum system $\longrightarrow$ average over classical paths

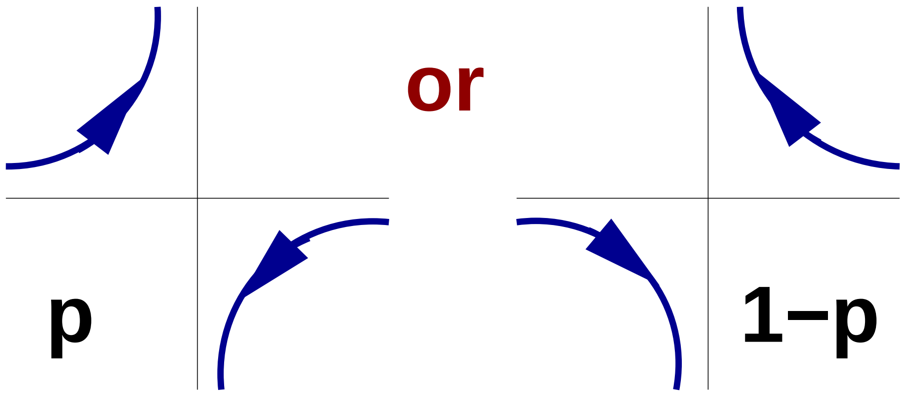
Quantum



Quantum amplitudes

+ random SU(2) phases

Classical



Classical probabilities

$p = \sin^2(\alpha) \quad 1 - p = \cos^2(\alpha)$

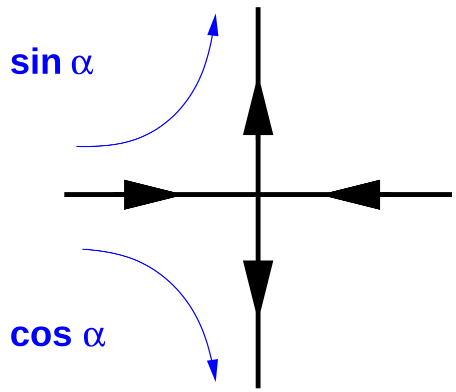
Application

Eigenphase density of evolution operator, W: $\rho(\theta) = \frac{1}{2\pi}\left[1 - \sum_n p_n \cos(2n\theta)\right]$

where W has eigenvalues $e^{i\theta}$ and p_n is classical return probability after n steps

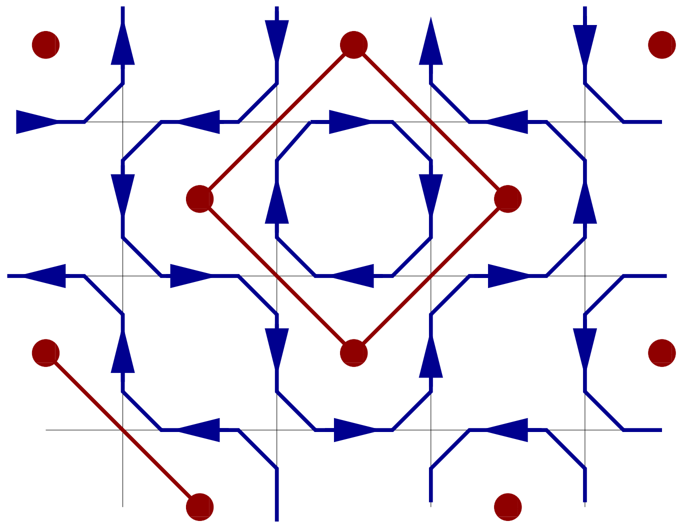
SU(2) network model and percolation

Quantum



Quantum amplitudes
+ random SU(2) phases

Classical



Classical probabilities

$p = \sin^2(\alpha), 1 - p = \cos^2(\alpha)$

Consequences: $\xi_{Quantum} \sim |\alpha - \pi/4|^{-4/3}$

and $\rho(\theta) \sim |\theta|^{1/7}$ **at** $\alpha = \pi/4$

Summary

Network models

Single quantum particle moving on lattice with randomness

Distinct phases for $\alpha \to 0$ and $\alpha \to \pi/2$

– distinguished by nature of edge states, separated by critical point

Symmetry classes via link phases: U(n), O(n) and Sp(n)

Discrete symmetries define classes additional to Wigner-Dyson ones

Mappings to problems from classical statistical physics

Random bond Ising model and O(1) model

Classical percolation and SU(2) model

Selected references

U(1) model

J. T. Chalker and P. D. Coddington, J. Phys C 21, 2665 (1988)

Review

B. Kramer, T. Ohtsuki and S. Kettemann, Phys. Rep. 417, 211 (2005)

O(1) model and Ising model

I. A. Gruzberg, N. Read, and A. W. W. Ludwig, Phys. Rev. 63, 104422 (2001)

F. Merz and J. T. Chalker, Phys. Rev. B65, 54424 (2002)

Classical percolation and SU(2) model

I. A. Gruzberg, A. W. W. Ludwig, and N. Read, Phys. Rev. Lett. 82, 4254 (1999)

E. Beamond, J. L. Cardy, and J. T. Chalker, Phys. Rev. B65, 214301 (2002)