Generating function for level correlations, semiclassical evaluation

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conventional semiclassics

$$\langle \rho(E)\rho(E') - \overline{\rho}^2 \rangle \propto \sum_{a,b} F_a F_b^* \mathrm{e}^{\mathrm{i}(S_a - S_b)/\hbar}$$

contributions from

* $b=a$ (and $b=\mathrm{T}a$ for TRI)

* a,b identical up to reconnections in encounters

gives non-oscillatory part in full

but misses oscillatory part

reminder: RMT

$$R(\varepsilon) = \Re\begin{cases} \dfrac{-1 + e^{i2\varepsilon}}{2} & \text{unitary} \\ (-1 + \ldots) + (\dfrac{1}{2\varepsilon^4} + \ldots)e^{i2\varepsilon} & \text{orthogonal} \end{cases}$$

$$\varepsilon = (E - E')2\pi\overline{\rho}$$

generating function

$$\left\langle \frac{\det(E+\varepsilon_C-H)\det(E-\varepsilon_D-H)}{\det(E+\varepsilon_A-H)\det(E-\varepsilon_B-H)} \right\rangle = Z(\varepsilon_A,\varepsilon_B,\varepsilon_C,\varepsilon_D)$$

$$\det \;=\; \exp\ \mathrm{tr}\ \ln$$

$$\frac{\partial^2}{\partial\varepsilon_A\partial\varepsilon_B} Z\Big|_{\varepsilon_A=\varepsilon_B=\varepsilon_C=\varepsilon_D=\varepsilon^+} \propto \left\langle \mathrm{tr}\frac{1}{E+\varepsilon^+-H}\,\mathrm{tr}\frac{1}{E-\varepsilon^+-H} \right\rangle = C(\varepsilon^+)$$

semiclassical evaluation

$$\left[\det(E^+ - H)\right]^{\pm 1} \propto \exp \pm \int^{E^+} dE' \,\mathrm{tr}\, \frac{1}{E'-H}$$

$$\propto \exp\left\{ \pm i\pi \overline{N}(E^+) \pm \sum_a F_a e^{iS_a(E^+)/\hbar} \right\}$$

$$\propto \exp\{\pm i\pi \overline{N}(E^+)\} \sum_A F_A (\pm 1)^{n_A} e^{iS_A(E^+)/\hbar}$$

$$\det(E^- - H) = [\det(E^+ - H)]^*$$

Riemann-Siegel lookalike

allows for real *E*, enforcing convergence and reality

$$\det(E-H) \sim \exp\{-\mathrm{i}\pi \overline{N}(E)\} \sum_{A}^{T_A < T_H/2} F_A \, (-1)^{n_A} \, \mathrm{e}^{\mathrm{i}S_A(E)/\hbar} \qquad + \text{c.c.}$$

rigorous for finite matrices, respects unitarity, modelled after Riemann's ζ

not available for inverse determinants

$$
\begin{aligned}
Z \sim \Big\langle & \exp\{+\mathrm{i}\pi\overline{N}(E+\varepsilon_A)\}\sum_A F_A\, e^{\mathrm{i}S_A(E+\varepsilon_A)/\hbar} \\
& \times \exp\{-\mathrm{i}\pi\overline{N}(E-\varepsilon_B)\}\sum_B F_B^*\, e^{-\mathrm{i}S_B(E-\varepsilon_B)/\hbar} \\
& \times \Big\{ \exp\{-\mathrm{i}\pi\overline{N}(E+\varepsilon_C)\}\sum_C^{T_C<T_H/2} F_C\,(-1)^{n_C}\, e^{\mathrm{i}S_C(E+\varepsilon_C)/\hbar} \\
& \times \exp\{+\mathrm{i}\pi\overline{N}(E-\varepsilon_D)\}\sum_D^{T_D<T_H/2} F_D^*\,(-1)^{n_D}\, e^{-\mathrm{i}S_D(E-\varepsilon_D)/\hbar} \\
& + \text{c.c.} \Big\} \Big\rangle = Z^{(1)} + Z^{(2)}
\end{aligned}
$$

$$
Z^{(2)}(\varepsilon_A,\varepsilon_B,\varepsilon_C,\varepsilon_D) = Z^{(1)}(\varepsilon_A,\varepsilon_B,-\varepsilon_D,-\varepsilon_C)
$$

Weyl symmetry

$$\overline{N}(E \pm \varepsilon) = \overline{N}(E) \pm \varepsilon, \qquad S(E \pm \varepsilon) = S(E) \pm \tau\varepsilon$$

$$Z^{(1)} = e^{i(\varepsilon_A+\varepsilon_B-\varepsilon_C-\varepsilon_D)/2} \times \sum_{A,B,C,D}^{(T_C,T_D<T_H/2)} \langle F_A F_B^* F_C F_D^* (-1)^{n_C+n_D} \times e^{i(S_A(E)-S_B(E)+S_C(E)-S_D(E))} \times e^{i(\varepsilon_A\tau_A+\varepsilon_B\tau_B-\varepsilon_C\tau_C-\varepsilon_D\tau_D)} \rangle$$

contributions only from terms where orbits in A and C are repeated in either B or D, identically (diagonal appr)

or at least up to reconnections in encounters (bunches)

diagonal approximation

p.o.'s enter as if uncorrelated

average $\langle\bullet\rangle$ ``sees'' p.o. sum $\sum_a F_a \mathrm{e}^{\mathrm{i}S_a(E^+)/\hbar}$ as Gaussian random variable, due to central limit theorem

Gaussian average most conveniently done in starting expression where four p.o. sums appear in exponent

$$Z^{(1)}_{\mathrm{diag}} = \mathrm{e}^{\mathrm{i}(\varepsilon_A+\varepsilon_B-\varepsilon_C-\varepsilon_D)} \left\langle \mathrm{e}^{X} \right\rangle_{\mathrm{diag}}$$

$$X = \sum_a F_a \mathrm{e}^{\mathrm{i}S_a(E)/\hbar+\mathrm{i}\varepsilon_A\tau_a} + \sum_b F_b^* \mathrm{e}^{-\mathrm{i}S_b(E)/\hbar+\mathrm{i}\varepsilon_B\tau_b} - \sum_c F_c \mathrm{e}^{\mathrm{i}S_c(E)/\hbar+\mathrm{i}\varepsilon_C\tau_c} - \sum_d F_d^* \mathrm{e}^{\mathrm{i}S_d(E)/\hbar+\mathrm{i}\varepsilon_D\tau_d}$$

$$\left\langle \mathrm{e}^{X} \right\rangle_{\mathrm{diag}} = \exp\left(\langle X^2 \rangle_{\mathrm{diag}}\right)$$

$$\langle X^2 \rangle_{\mathrm{diag}} = \left\langle \sum_a |F_a|^2 \left(\mathrm{e}^{\mathrm{i}(\varepsilon_A+\varepsilon_B)\tau_a} - \mathrm{e}^{\mathrm{i}(\varepsilon_A+\varepsilon_D)\tau_a} \right) \right\rangle - \left\langle \sum_c |F_c|^2 \left(\mathrm{e}^{\mathrm{i}(\varepsilon_C+\varepsilon_B)\tau_c} - \mathrm{e}^{\mathrm{i}(\varepsilon_C+\varepsilon_D)\tau_c} \right) \right\rangle$$

HOdA: $$\sum_a |F_a|^2 e^{i\varepsilon\tau_a} \sim -\ln(i\,\varepsilon) + \text{const}$$

$$Z_{\text{diag}}^{(1)} = e^{i(\varepsilon_A+\varepsilon_B-\varepsilon_C-\varepsilon_D)} \frac{(\varepsilon_A+\varepsilon_D)(\varepsilon_C+\varepsilon_B)}{(\varepsilon_A+\varepsilon_B)(\varepsilon_C+\varepsilon_D)}$$

$$Z_{\text{diag}}^{(2)} = e^{i(\varepsilon_A+\varepsilon_B+\varepsilon_C+\varepsilon_D)} \frac{(\varepsilon_A-\varepsilon_C)(-\varepsilon_D+\varepsilon_B)}{(\varepsilon_A+\varepsilon_B)(-\varepsilon_D-\varepsilon_C)}$$

$$C_{\text{diag}}(\varepsilon) = \frac{1}{2(i\varepsilon)^2} - \frac{e^{2i\varepsilon}}{2(i\varepsilon)^2}$$

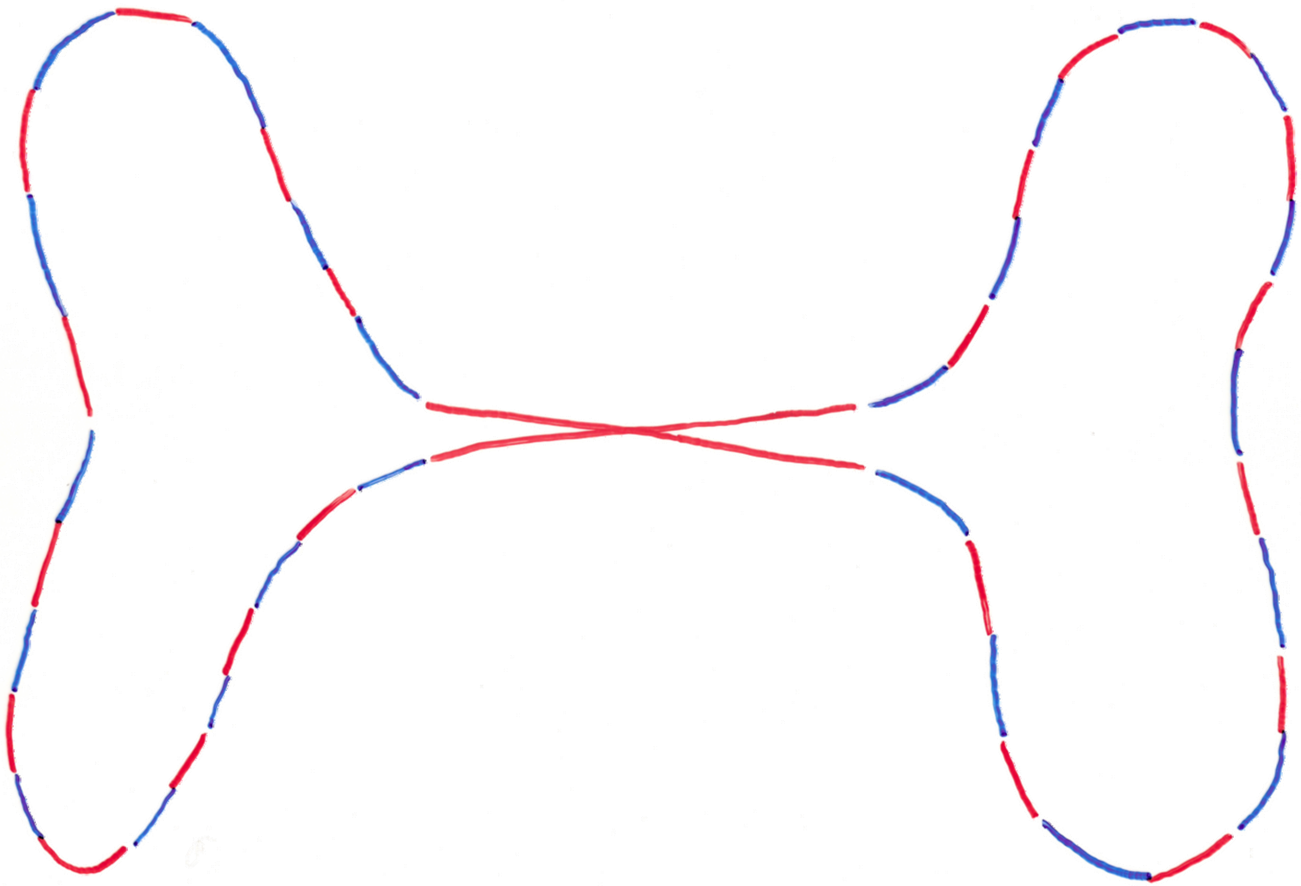
off-diagonal terms from orbit bunches

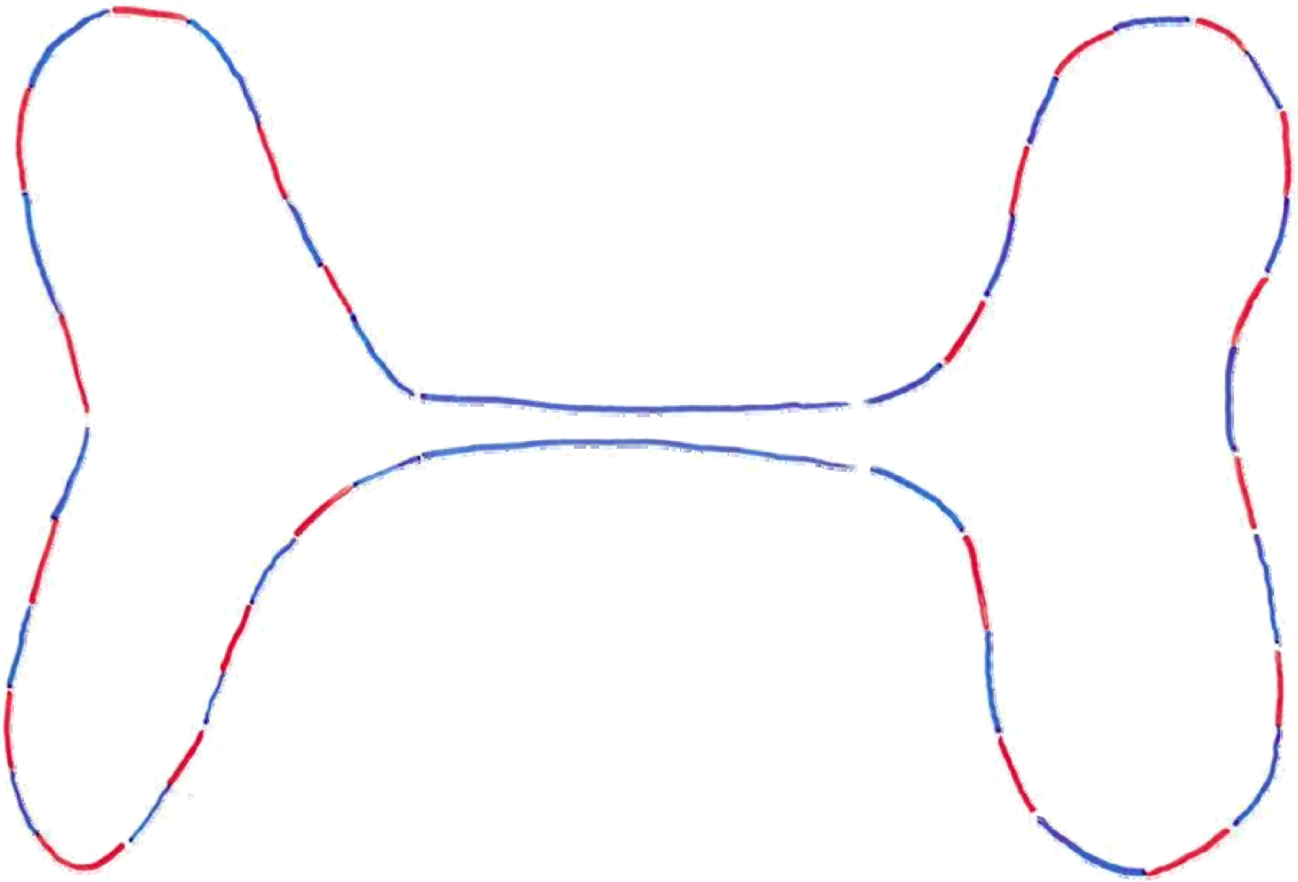
in chaotic dynamics, long p. o.'s do not arise as mutually independent entities but rather in closely packed bunches

under weak resolution of configuration space,
bunch looks like single orbit

orbits in bunch practically identical, apart
from reconnections within self-encounters

all orbits in bunch generated from single one
by reshuffling stretches within self-encounters,
so as to differently connect practically unchanged links





``bunch’’ of 2 orbits: nearly same links,
differently connected by the two encounter stretches;
action difference can be arbitrarily small

respectful bows to Martin & Klaus, and their “disordered precursors”

l-encounter: *l* orbit stretches mutually close

l links

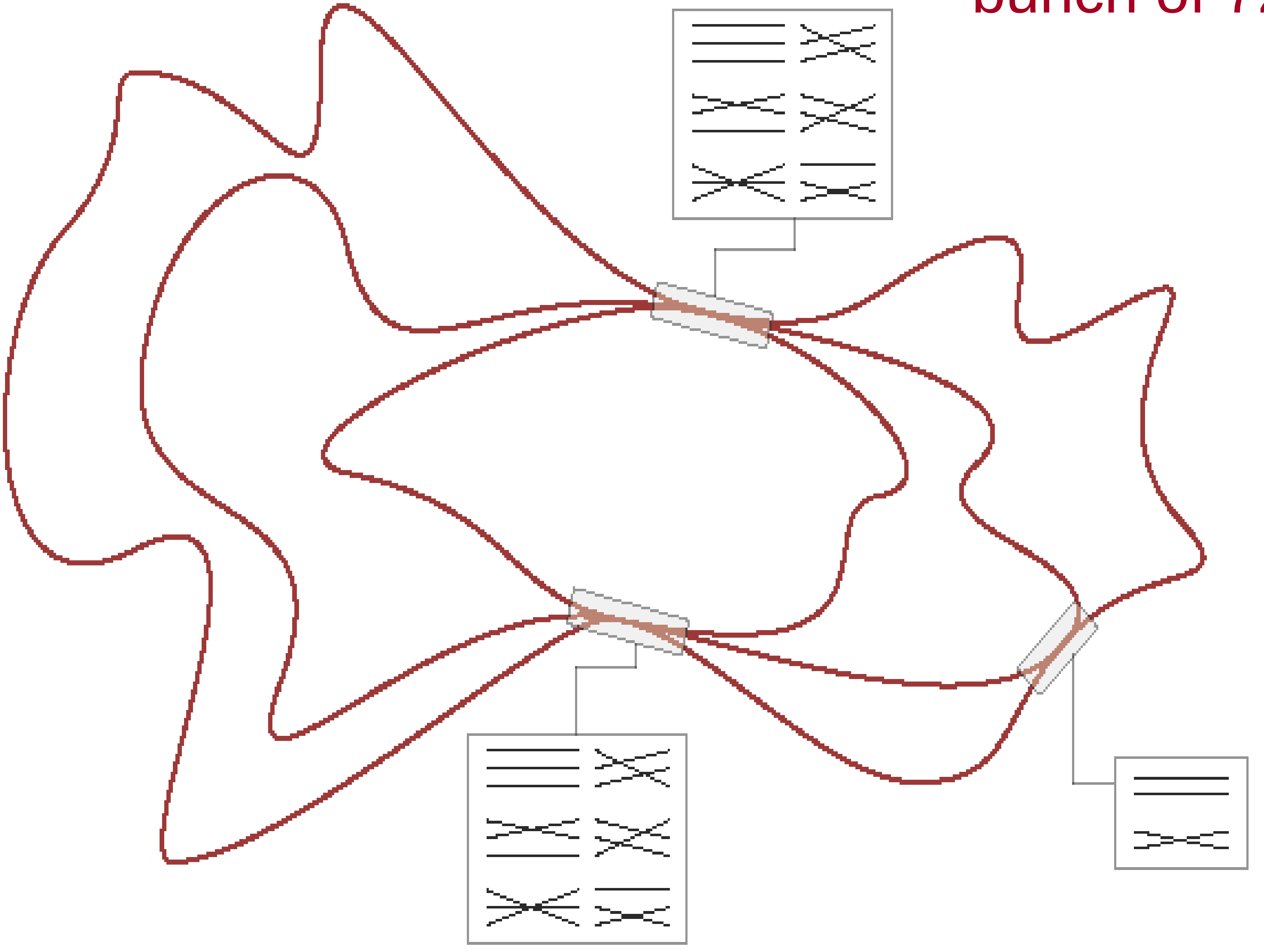
encounter stretches can connect
links in *l*! different ways

⇓

bunch of *l*! (pseudo-)orbits, nearly same
``rigid'' links; action differences arbitrarily small

bunch of 12

bunch of 72



orbit bunches yield asymptotic expansion of $Z^{(1)}$ in $1/\varepsilon$

then $Z^{(2)}$ from $Z^{(1)}$ with Riemann Siegel

differentiation of $Z^{(1)} + Z^{(2)}$ gives correlator *R(e)*,

in agreement with RMT; in particular, no addition to diagonal approximation for unitary class

n-th order term from bunches of *R(e)* from bunches with $n = L - V + 2$, where

V = # encounters (vertices),
L = # links

scene thus set for Sebastian Müller

concluding remarks:

oscillatory terms through generating function and Riemann-Siegel

close correspondence to sigma model of RMT

asymptotic series for osc and non-osc terms there arise from perturbative treatment of two saddle points for integral over matrix manifold,

Feynman diagrams correspond to orbit bunches, vertices to encounters, links to propagator lines