Birth of the Ehrenfest time

SOVIET PHYSICS JETP VOLUME 28, NUMBER 6

QUASICLASSICAL METHOD IN THE THEORY OF SUPERCONDUCTIVITY

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It is shown that replacement of quantum-mechanical averages by the average values of the corresponding classical quantities over all trajectories with a prescribed energy is not valid in the general case. The dependence of the penetration depth on the field is found without making any assumptions about the weakness of the interaction between the electrons and the field of the impurities; the case of very dirty films is also considered.

An estimate of the validity of the resulting formulas may be obtained from the condition that an initial displacement of a particle of the order of its wavelength hp_0^{-1} must lead at a moment of time t to a displacement which is smaller than the interaction radius $\sim\sigma^{1/2}$:

$$\lambda^2 Y(t) \ll \sigma \quad \text{or} \quad h^2 e^{t/t_0} \ll \sigma p_0^2. \qquad (31)$$

$$h^2 e^{t/t_0} \ll \sigma p^2$$

Quantum Chaos
in
Mesoscopic Superconductivity

Philippe Jacquod
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C. Beenakker (Leiden)
M. Goorden (Delft)
H. Schomerus (Lancaster)
J. Weiss (Arizona)

Outline

- Mesoscopic superconductivity - Andreev reflection
- Density of states in ballistic Andreev billiards
- Transport through ballistic Andreev interferometers
- Symmetries of multi-terminal transport in presence of superconductivity

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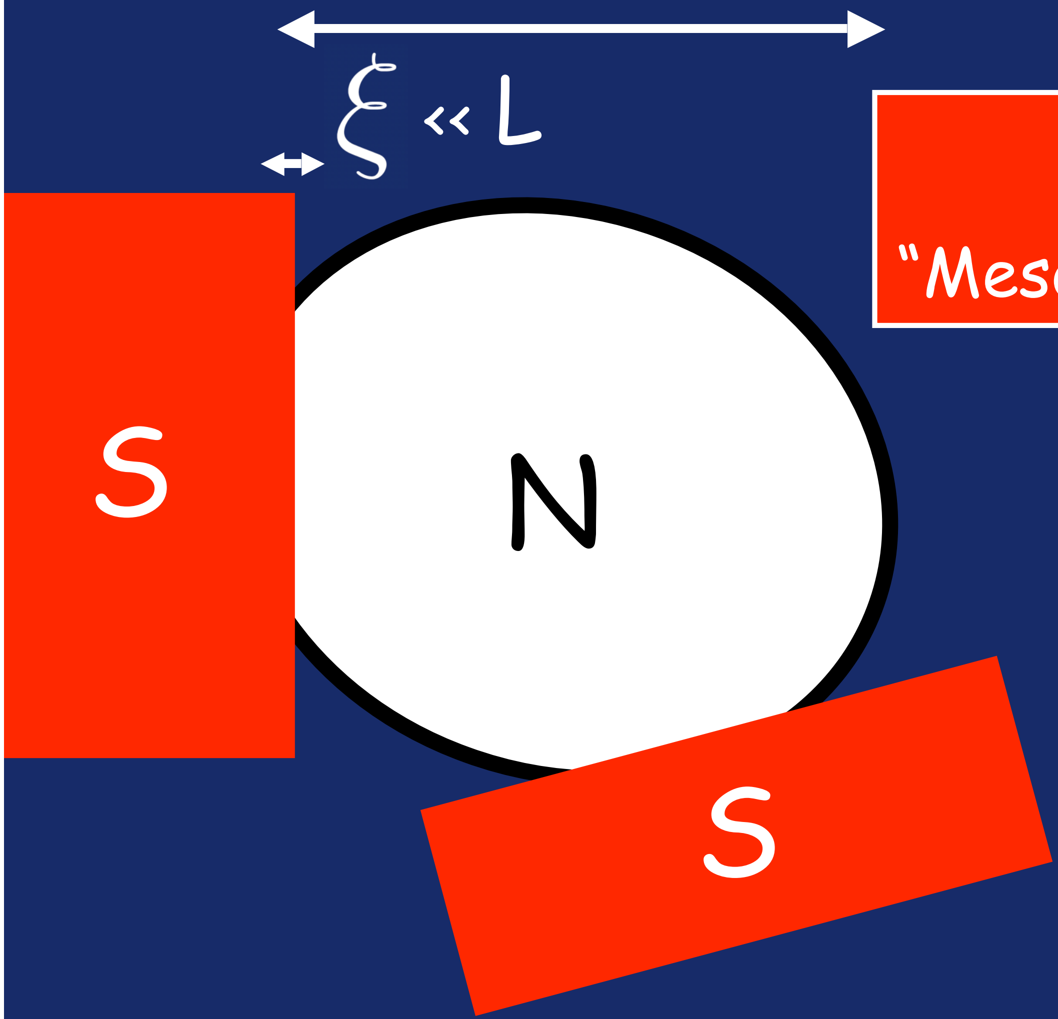
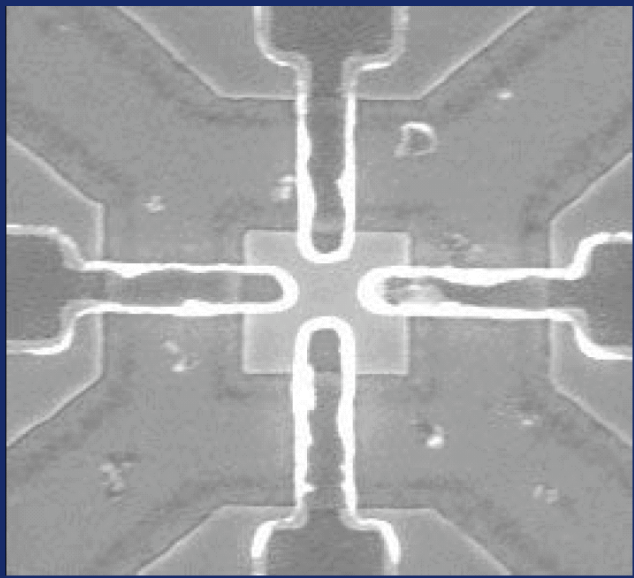
Mesoscopic Superconductivity

Mesoscopic metal (N) in contact with superconductors (S)



S invades N

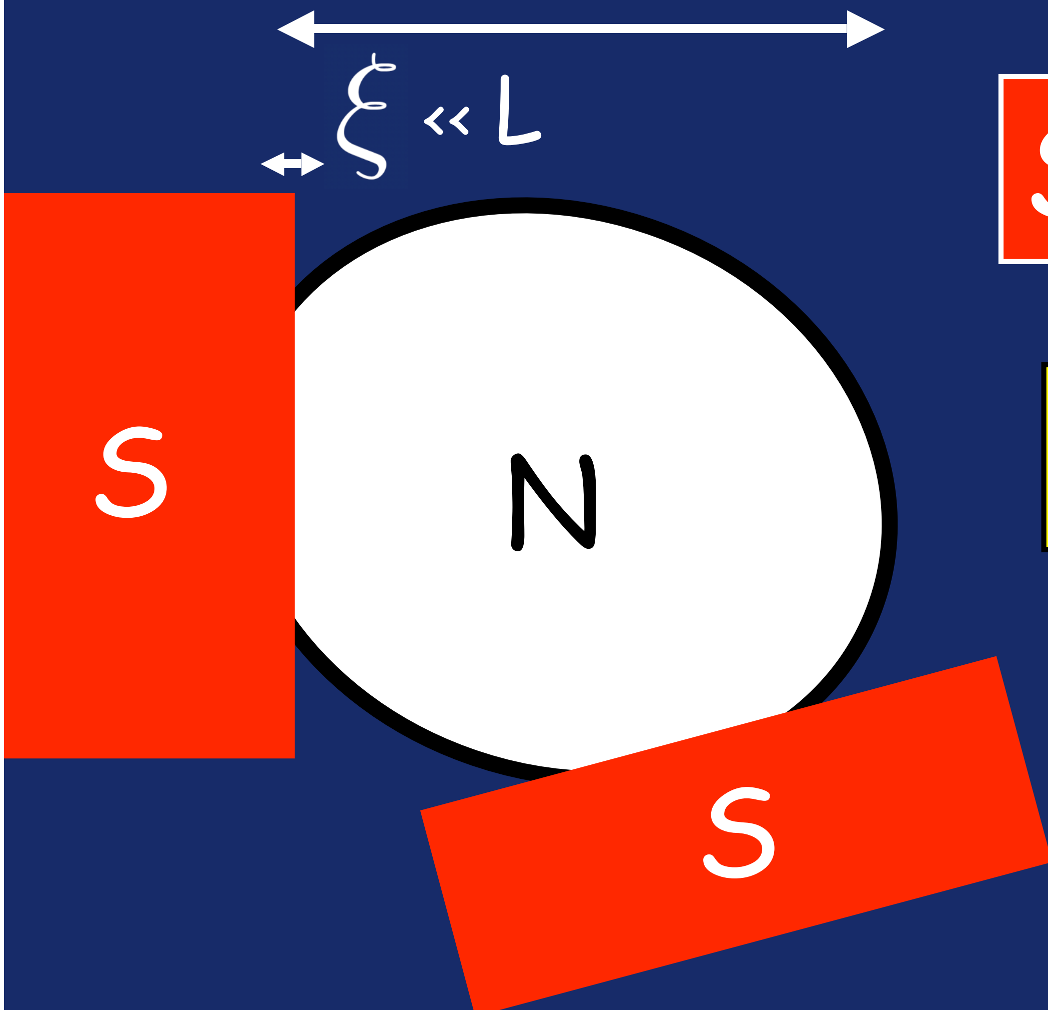
"Mesoscopic proximity effect"



Device by AT Filip, Groningen

Mesoscopic Superconductivity

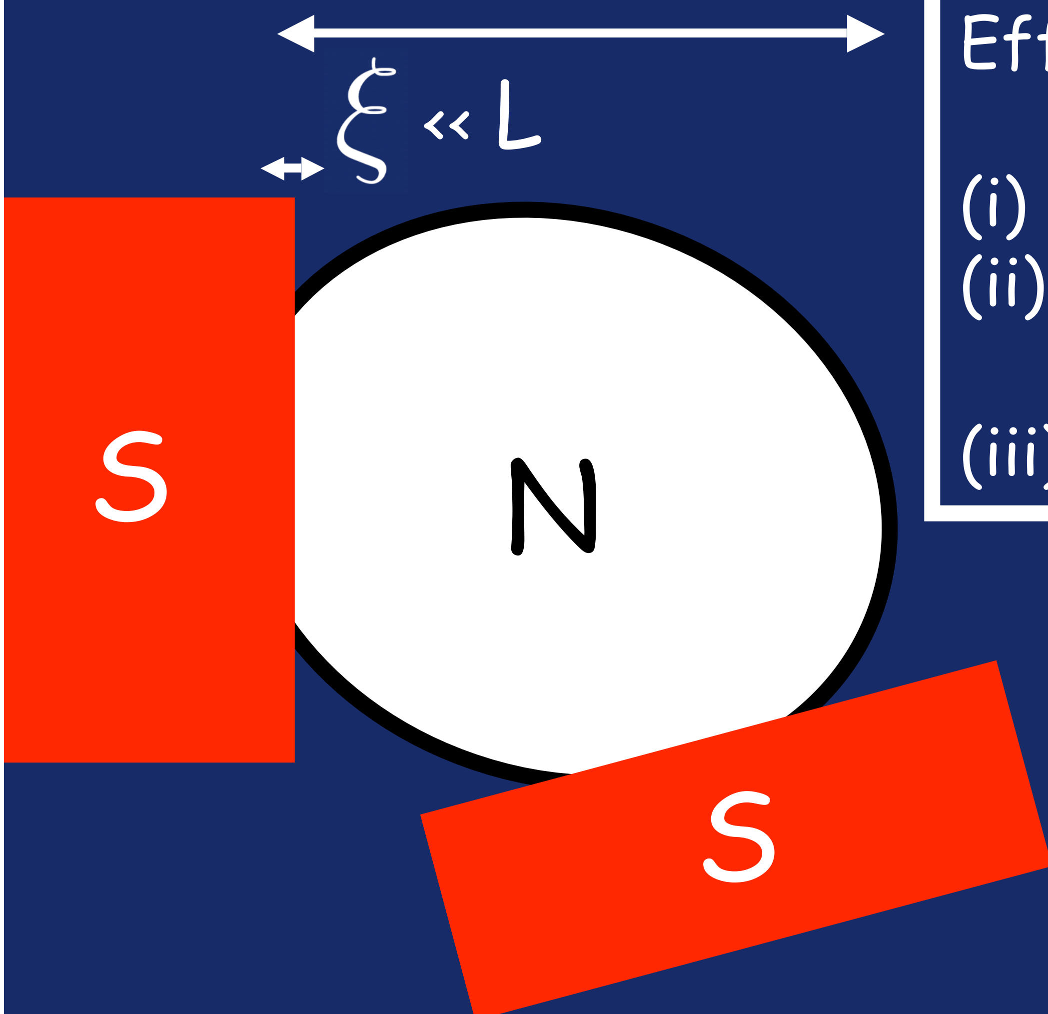
Mesoscopic metal (N) in contact with superconductors (S)



S invades N

But how ??

Mesoscopic Superconductivity



Effect of S in N depends on:

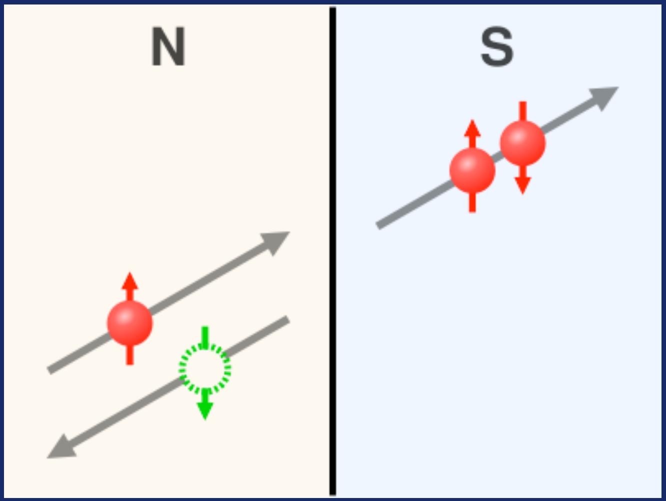
(i) Electronic dynamics in N

(ii) Symmetry of S state
(s- or d-wave; S phases…)

(iii) τ_E/τ_D

Andreev reflection

- $(e, E_F+\varepsilon) \rightleftharpoons (h, E_F-\varepsilon)$



(fig taken from Wikipedia)

- Reflection phase :

$$\delta S_{\mathrm{A}} = -\arccos[\varepsilon/\Delta] \mp \phi \simeq -\pi/2 \mp \phi$$

- Angle mismatch : Snell's law

$$k_+ \sin\theta_+ = k_- \sin\theta_-$$

$$\theta_- - \theta_+ \approx \frac{\varepsilon}{E_{\mathrm{F}}} \tan\theta_+$$

S phase
+ : h->e
- : e->h

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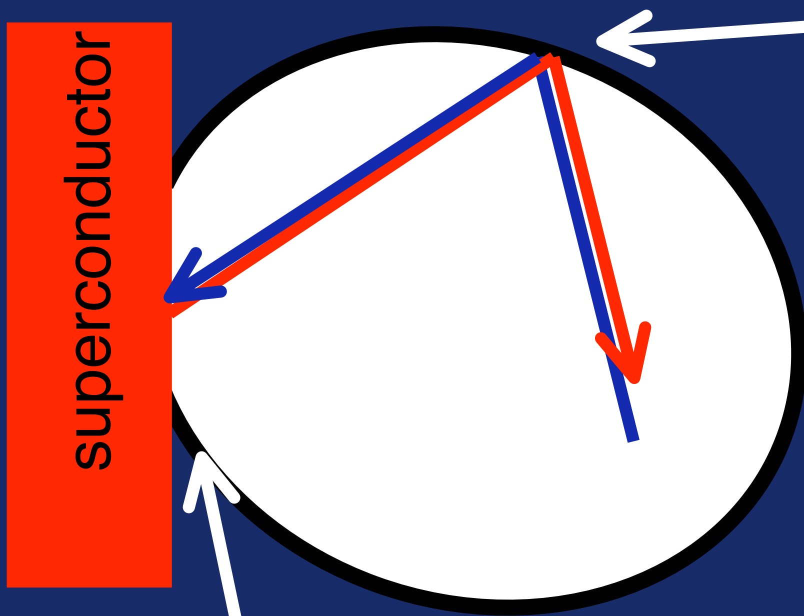
- Symmetries of charge transport in presence of superconductivity

PJ, H. Schomerus, and C. Beenakker, PRL '03
M. Goorden, PJ, and C. Beenakker, PRB '03; PRB '05

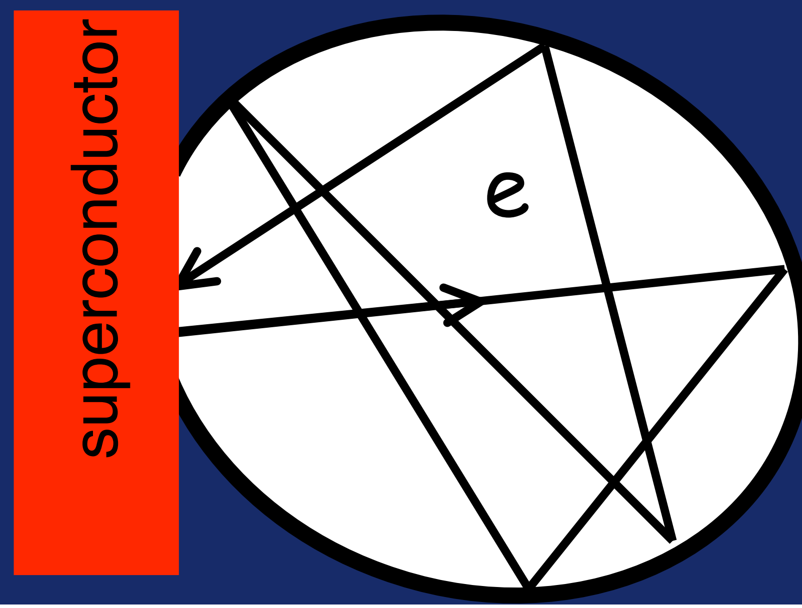
Andreev billiards: classical dynamics



At NI interface:
Normal reflection

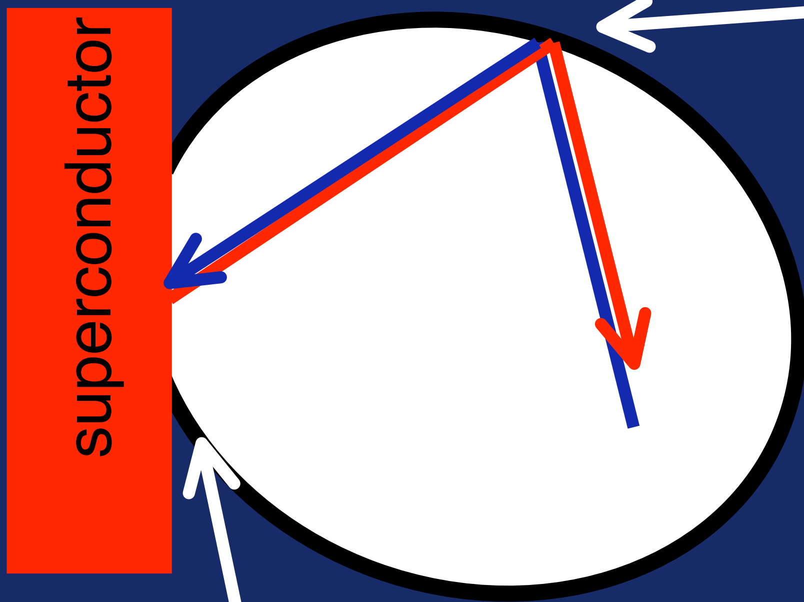
Note #1: Billiard is chaotic
⇒ all trajectories become periodic!

At NS interface:
Andreev reflection



Kosztin, Maslov, Goldbart '95

Andreev billiards: classical dynamics



At NI interface:
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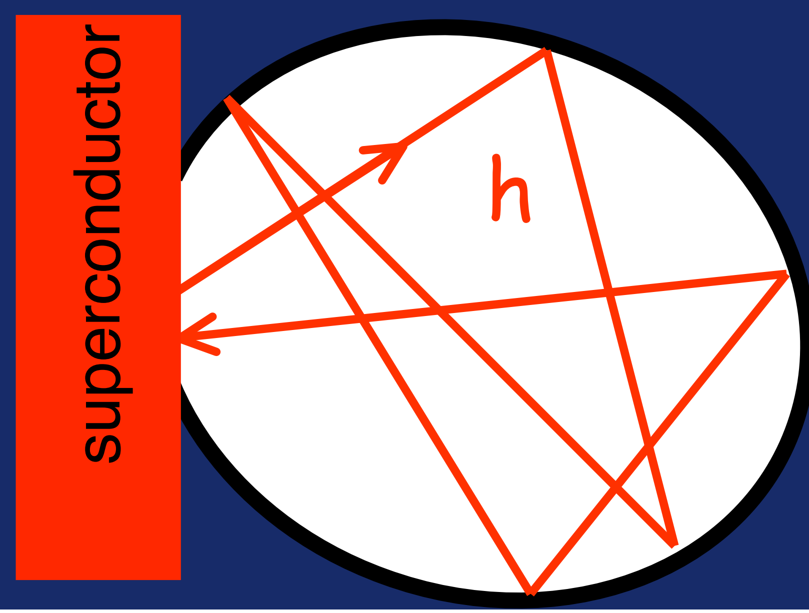
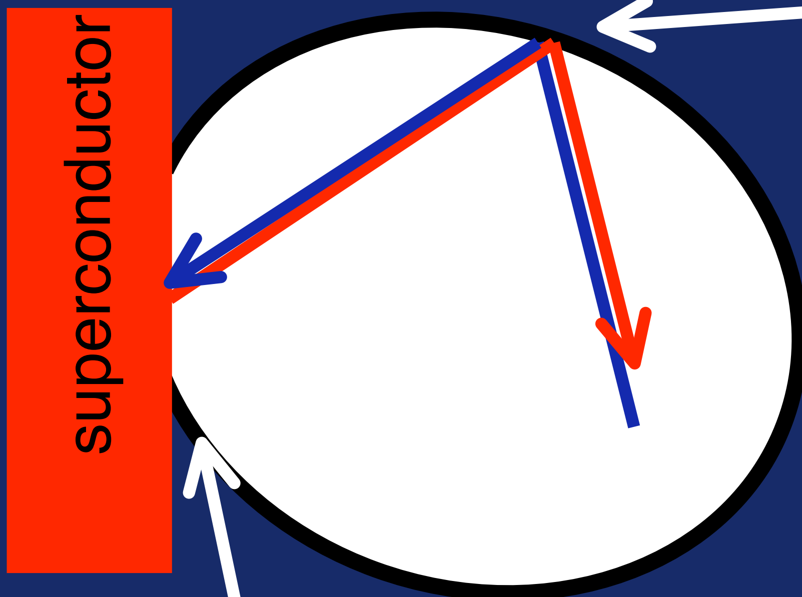


Kosztin, Maslov, Goldbart '95

Andreev billiards: classical dynamics

At NI interface:
Normal reflection



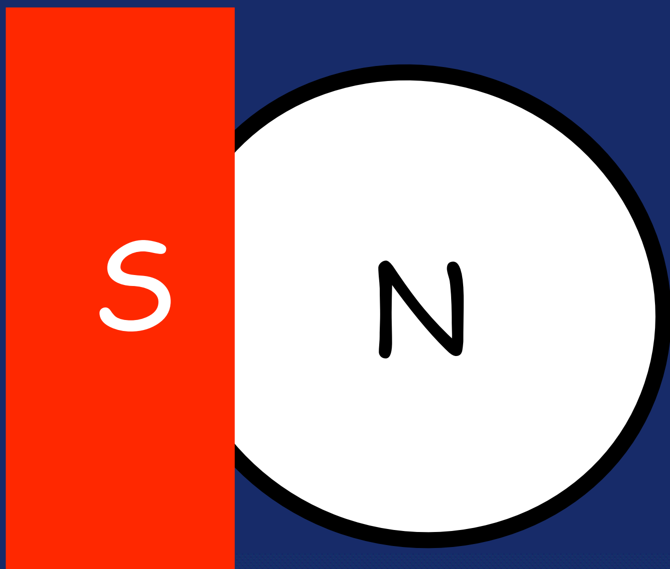
Note #2: Action on P.O.

$$\delta S_\gamma = 2\varepsilon T_\gamma - \pi\hbar$$

Andreev reflection phase

At NS interface:
Andreev reflection

Andreev billiards: semiclassical quantization



All orbits are periodic
-> Bohr-Sommerfeld

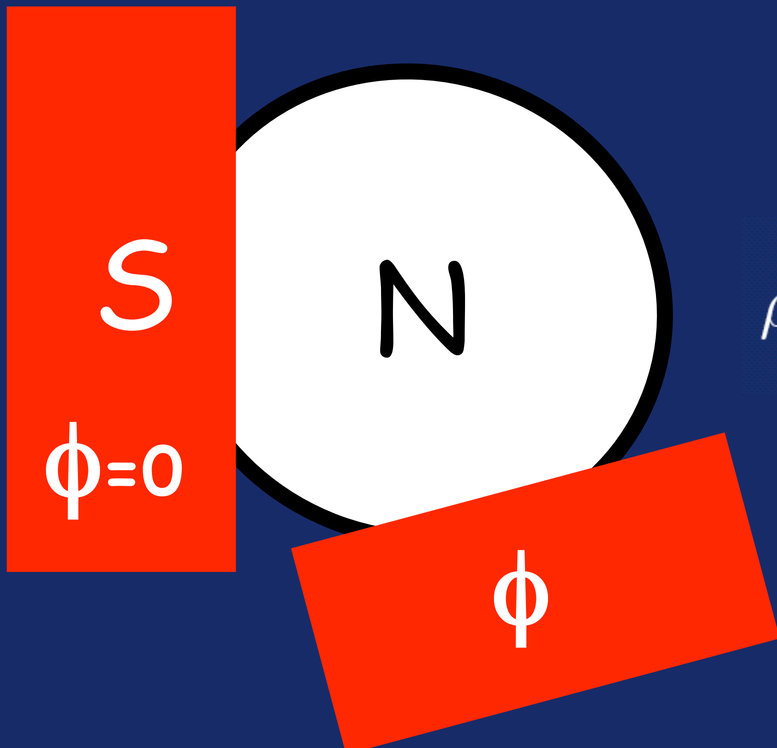
Andreev reflection phase

$$\rho_{\mathrm{BS}}(E) = N_{\mathrm{S}} \int_0^\infty dT\, P(T) \times 2 \sum_n \delta\left(E - [n + \frac{1}{2}]\frac{\pi\hbar}{T}\right)$$

Distribution of return times to S
chaos-> exp. Suppression at E=0
regular->algebraic / others

See also: Melsen et al. '96;Ihra et al. '01; Zaitsev '06

Andreev billiards: semiclassical quantization



All orbits are periodic
-> Bohr-Sommerfeld

$$\rho_{\mathrm{BS}}(E) = N_{\mathrm{S}} \int_0^\infty dT P(T)$$

$$\times \sum_n \left[\delta\left(E - [n + \frac{1}{2}] \frac{\pi\hbar}{T}\right) + \frac{1}{2}\delta\left(E - [n + \frac{1}{2} + \frac{\phi}{2\pi}] \frac{\pi\hbar}{T}\right) + \frac{1}{2}\delta\left(E - [n + \frac{1}{2} - \frac{\phi}{2\pi}] \frac{\pi\hbar}{T}\right) \right]$$

$|\phi| = \pi$: DoS has peak at E=0 !!

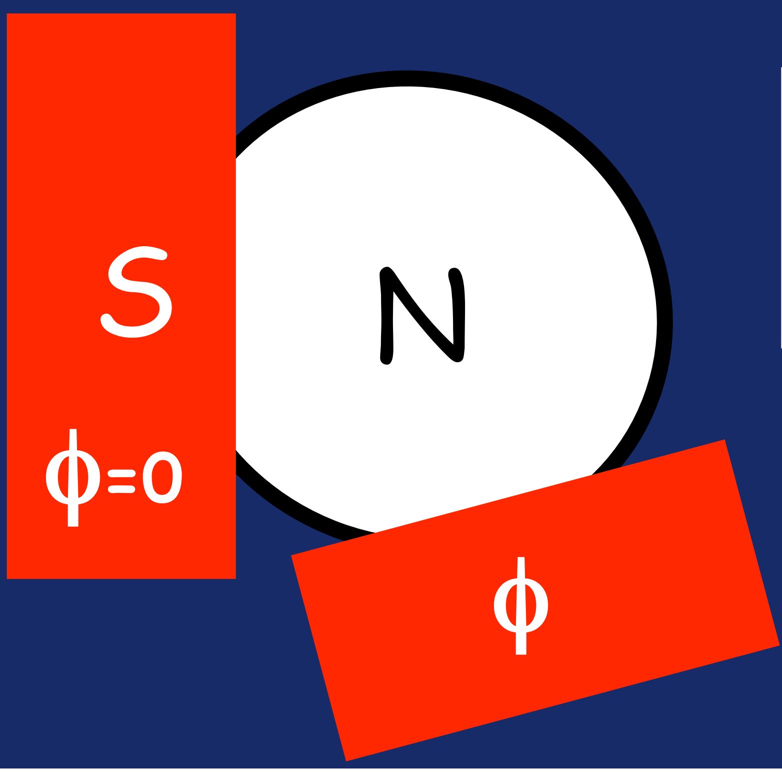
All trajs touching both contribute to n=0 term

Goorden, PJ, Weiss '08

Andreev billiards: semiclassical quantization

Bohr-Sommerfeld for "chaotic" systems

$$\rho_{\mathrm{BS}}(u) = \frac{2\pi}{\delta}\frac{\cosh\left(\frac{\phi}{2u}\right)}{\sinh\left(\frac{\pi}{u}\right)u^2}\left(\pi\cosh\left(\frac{\phi}{2u}\right)\coth\left(\frac{\pi}{u}\right) - \phi\sinh\left(\frac{\phi}{2u}\right)\right)$$



$$\lim_{u\to 0}\lim_{\phi\to\pi}\rho_{\mathrm{BS}}(u) = \frac{2\pi}{\delta}\delta(u)$$

$u=E/E_T$

Goorden, PJ, Weiss '08

Andreev billiards: random matrix theory

N = MxM RMT Hamiltonians
S -> particle-converting projectors

$$\mathcal{H} = \begin{pmatrix} H & -\pi W W^T \\ -\pi W W^T & -H^* \end{pmatrix}$$

$$W_{mn} = \delta_{mn} \left(\frac{M\delta}{\pi^2} \right)^{1/2}$$

$$m = 1, 2, \ldots M, \qquad n = 1, 2, \ldots 2N_S$$

CONSTANT DOS EXCEPT:

⇒ hard gap at 0.6 E_T for $\phi=0$

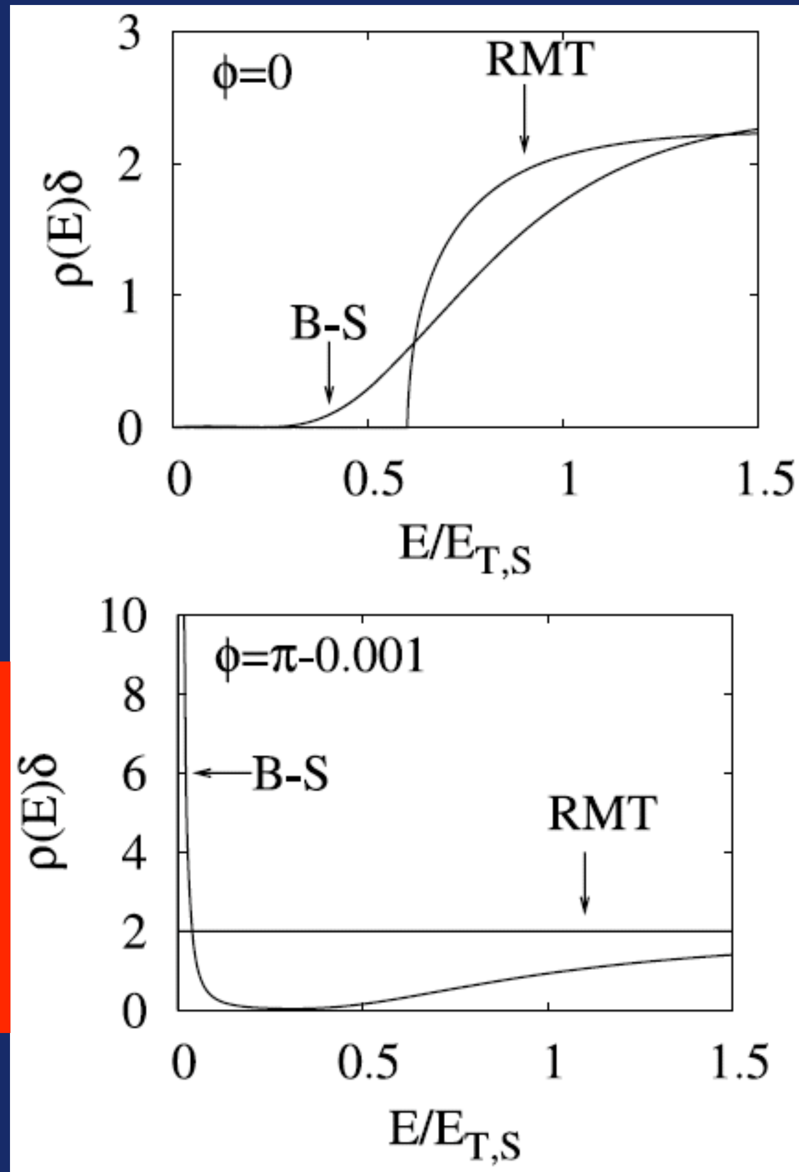
⇒ linear "gap" of size δ for $\phi= \pi$

(class C1 with DoS: $\rho(E) = \frac{\pi}{\delta} \int_0^{2\pi E/\delta} dt J_0(t) J_1(t)/t$)

Melsen et al. '96, '97; Altland+Zirnbauer '97

Andreev billiards: RMT vs. B-Sommerfeld

At $\phi=0$: the "gap problem"
?: which theory is right ?
?: which theory is wrong ?



At $\phi=\pi$: macroscopic peak (semiclassics) vs. minigap (RMT)
?: which theory is right ?
?: which theory is wrong ?

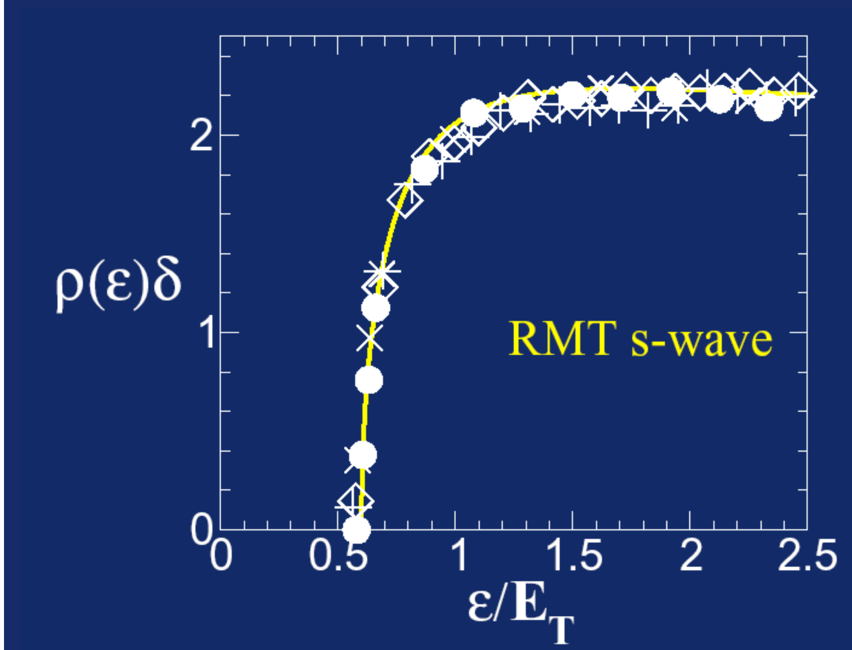
Andreev billiards - Solution to the "gap problem"

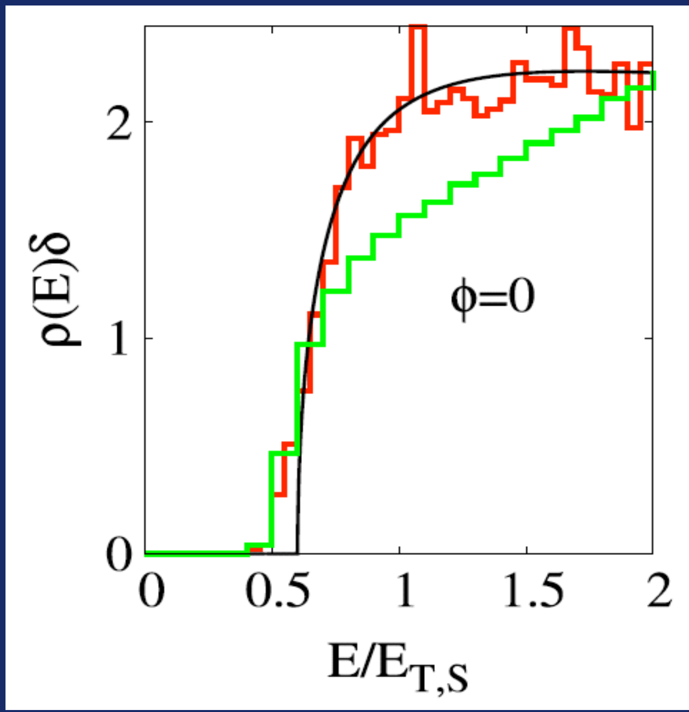
$\tau_E \ll \tau_D$

Universal, RMT regime

$\tau_E \gtrsim \tau_D$

Deep semiclassical regime





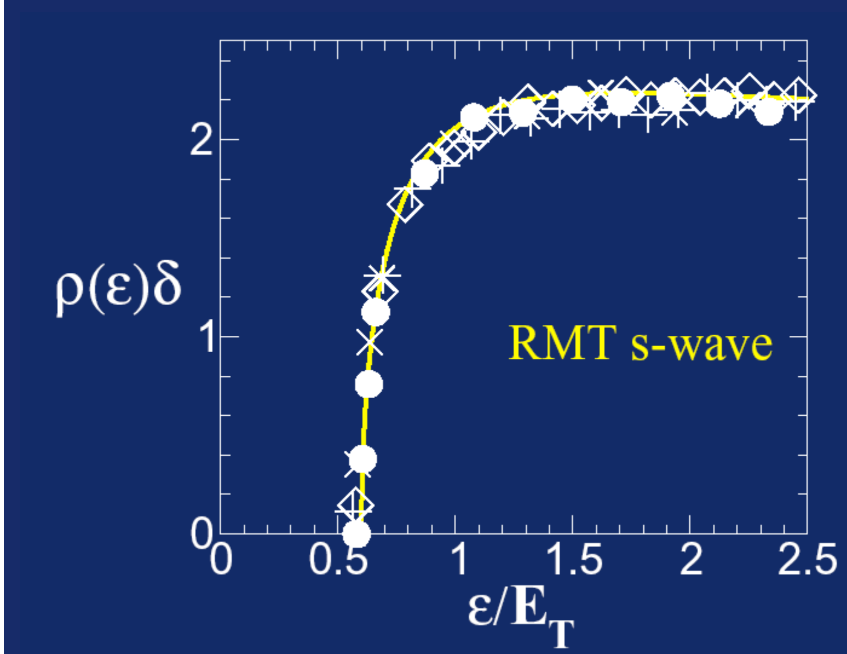
Note: numerics on "Andreev kicked rotator", PJ Schomerus and Beenakker '03

See also: Lodder and Nazarov '98; Adagideli and Beenakker '02; Vavilov and Larkin '03

Andreev billiards - Solution to the "gap problem"

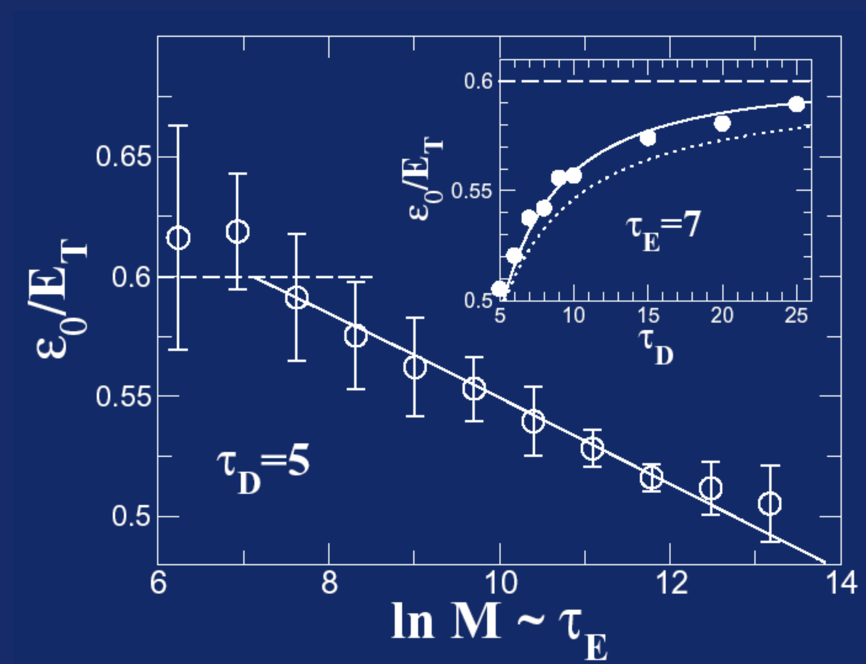
$\tau_E \ll \tau_D$

Universal, RMT regime:
Gap at Thouless energy



$\tau_E \gtrsim \tau_D$

Deep semiclassical regime:
Gap at Ehrenfest energy



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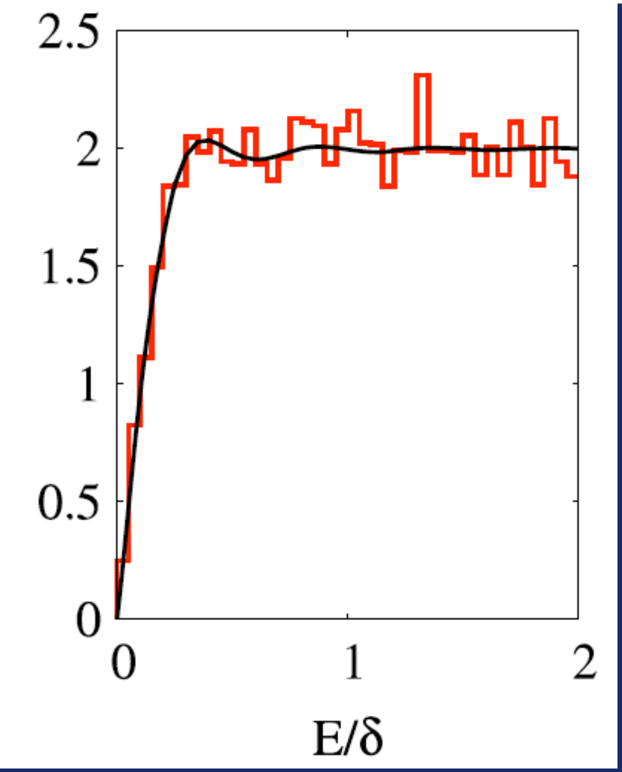
Andreev billiards: DoS at $\phi=\pi$

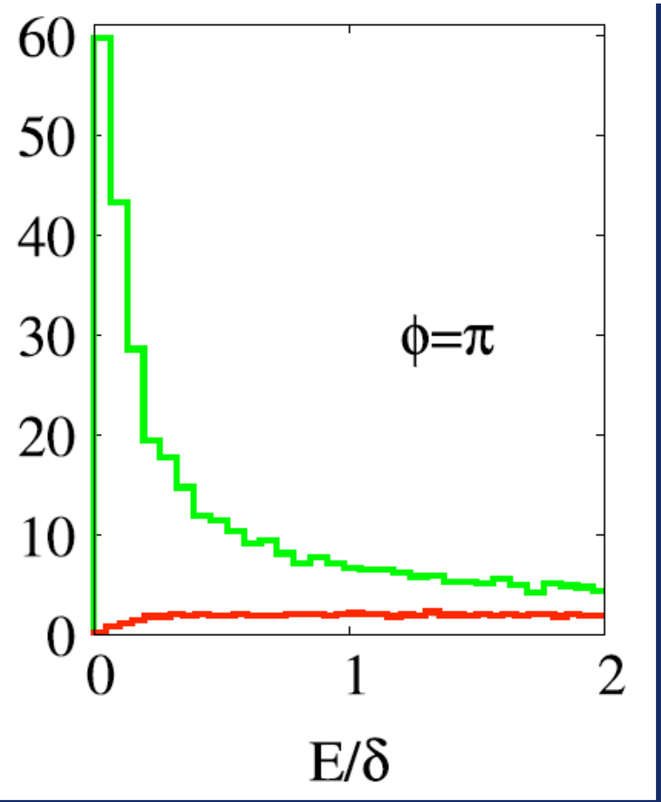
$\tau_E \ll \tau_D$

Universal, RMT regime:
Minigap at level spacing



$\tau_E \gtrsim \tau_D$

Deep semiclassical regime:
Large peak around E=0 !



Goorden, PJ and Weiss '08.

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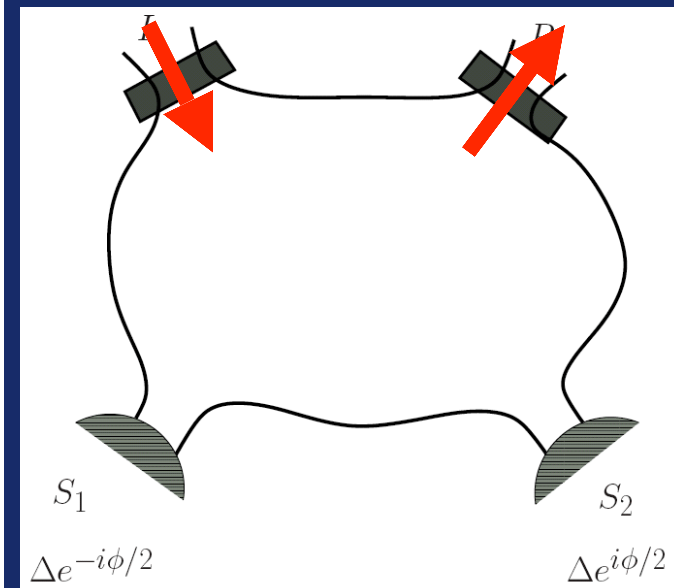
M. Goorden, PJ, and J. Weiss, PRL '08, Nanotechnology '08

Transport through Andreev interferometers



Lambert '93 formula

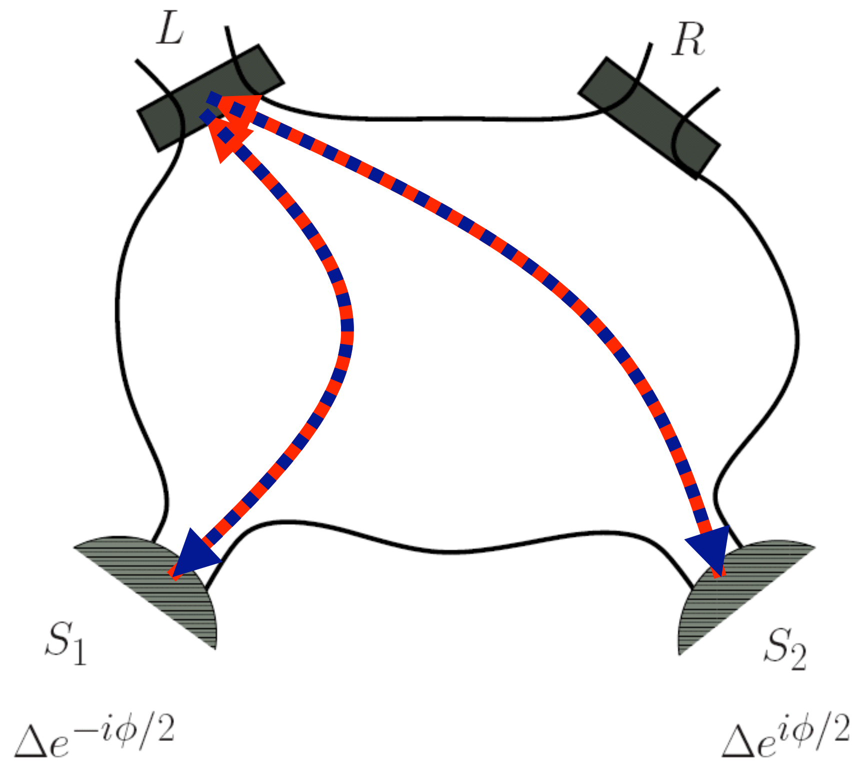
$$G/G_0 = T^{ee}_{RL} + T^{he}_{RL} + 2\frac{T^{he}_{LL}T^{he}_{RR} - T^{he}_{LR}T^{he}_{RL}}{T^{he}_{LL} + T^{he}_{RR} + T^{he}_{LR} + T^{he}_{RL}}$$

Average conductance for $N_L = N_R$

$$\langle G \rangle / G_0 = \langle T^{ee}_{RL} \rangle + \langle T^{he}_{LL} \rangle$$

- New, Andreev reflection term
- Gives classically large interference contributions

Transport through Andreev interferometers



At ε=0, any pair of Andreev reflected trajectories contributes to T_{LL}^{he} in the sense of a SPA !

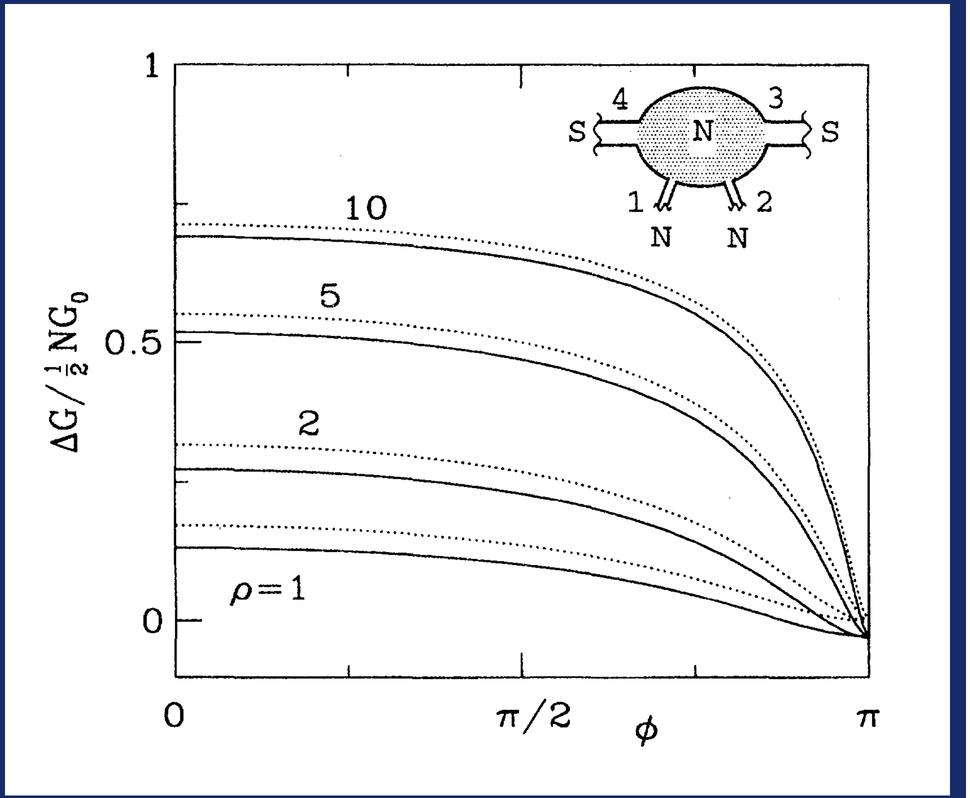
These pairs give classically large positive coherent backscattering at ϕ=0, vanishing for ϕ=π

$$\langle G \rangle / G_0 = \langle T_{RL}^{ee} \rangle + \langle T_{LL}^{he} \rangle$$

Transport through Andreev interferometers

No tunnel barrier :
Coherent backscattering is
-O(N)
-positive, increases G

This is (obviously) not
related to the DoS in
the Andreev billiard



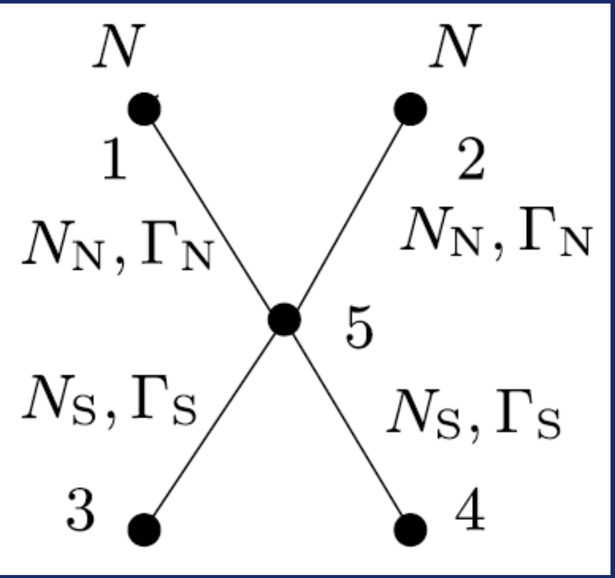
!! INTRODUCE TUNNEL BARRIERS
TUNNELING CONDUCTANCE ~ DOS !!

Beenakker, Melsen and Brouwer '95

Tunneling transport through Andreev interferometers

$$\langle G\rangle/G_0 = \langle T^{ee}_{RL}\rangle + \langle T^{he}_{LL}\rangle$$

Plan a) : extend circuit theory to tunneling



$$\langle G\rangle/G_0 = \begin{cases} \Gamma_\mathrm{N}^2 N_\mathrm{N} \left(1 + \dfrac{N_\mathrm{N}}{N_\mathrm{S,eff}} \dfrac{1+\cos\frac{\phi}{2}}{\cos\frac{\phi}{2}}\right)\Big/4, & \text{if } \delta\phi \gg \Gamma_\mathrm{N}, \\ \Gamma_\mathrm{N} N_\mathrm{N}/2 + \mathcal{O}(\delta\phi^2), & \text{if } \delta\phi \ll \Gamma_\mathrm{N}. \end{cases}$$

Goorden, PJ and Weiss '08; inspired by : Nazarov '94; Argaman '97.

Tunneling transport through Andreev interferometers

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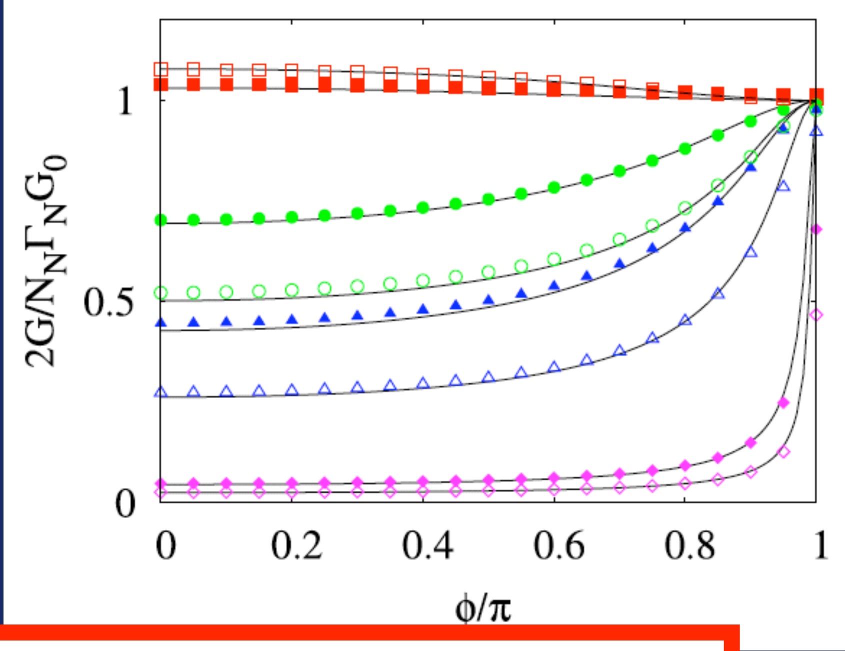

$$\langle G\rangle/G_0 = \begin{cases} \Gamma_N^2 N_N \left(1 + \frac{N_N}{N_{S,\mathrm{eff}}} \frac{1+\cos\frac{\phi}{2}}{\cos\frac{\phi}{2}}\right) \Big/ 4, & \text{if } \delta\phi \gg \Gamma_N, \\ \Gamma_N N_N/2 + \mathcal{O}(\delta\phi^2), & \text{if } \delta\phi \ll \Gamma_N. \end{cases}$$

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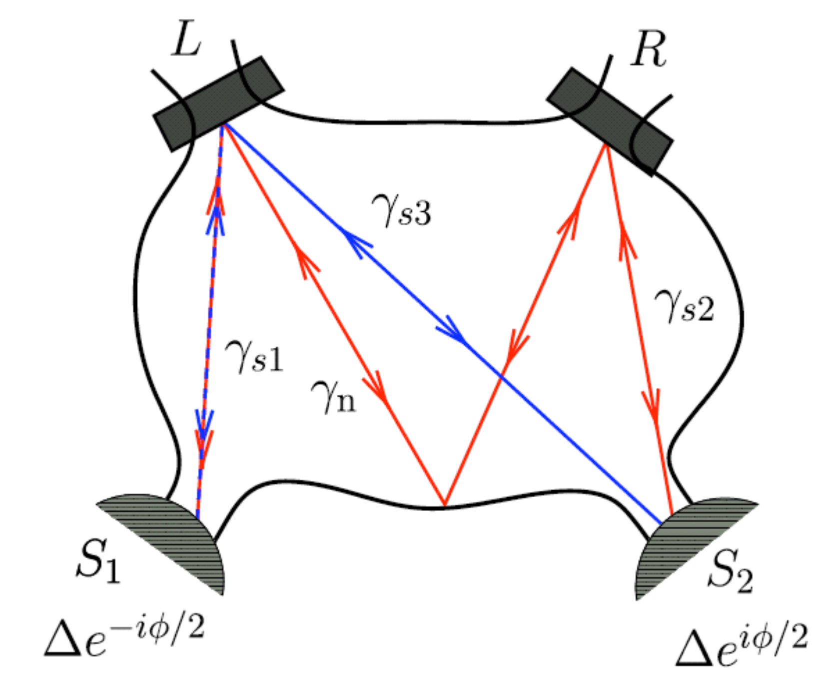
Tunneling transport through Andreev interferometers

$$\langle G\rangle/G_0 = \langle T^{ee}_{RL}\rangle + \langle T^{he}_{LL}\rangle$$

Plan b) : semiclassics

"Macroscopic Resonant Tunneling"

red line	contribution to T^{ee}_{RL}
blue line	contribution to T^{he}_{LL}



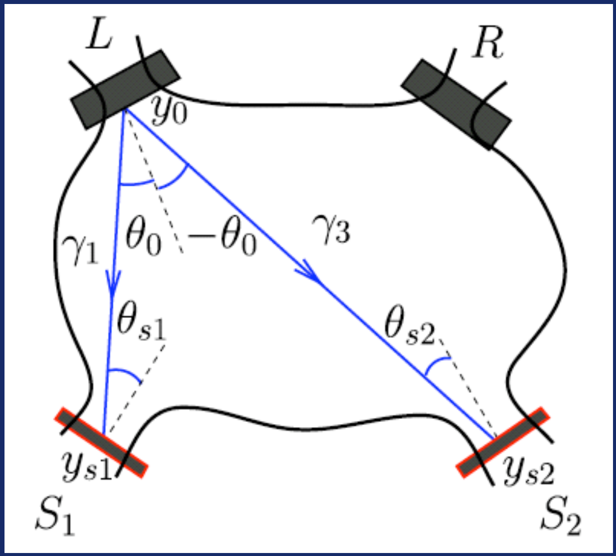
Why "macroscopic" ?
A: O(N) effect !

Goorden, PJ and Weiss '08.

Tunneling transport through Andreev interferometers

Plan b) : semiclassics

"Macroscopic Resonant Tunneling"



Calculate transmission

$$T_{ji}^{\beta\alpha} = \frac{1}{2\pi\hbar}\int_i y_0 \int_j y_0' \sum_{\gamma 1,\gamma 2} A_{\gamma 1} A_{\gamma 2}^* \exp[i\delta S/\hbar]$$

on blue trajectories (i.e. for T_{LL}^{he})

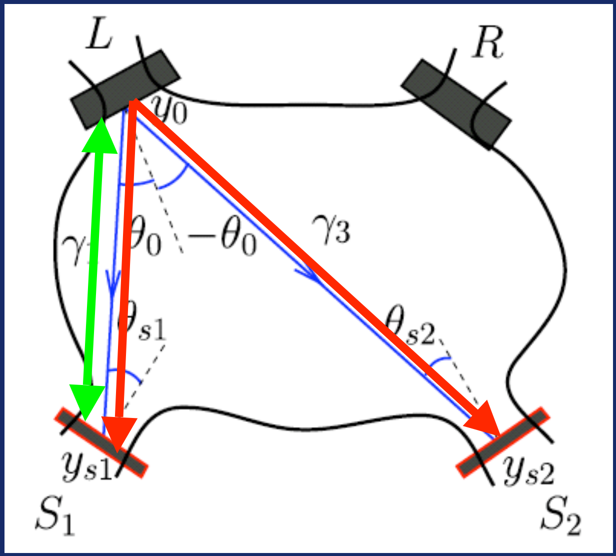
$$\gamma_I^{(p)} = \gamma_{s1}^{(e)} + \gamma_{s1}^{(h)} + p \times \left[\gamma_{s3}^{(h)} + \gamma_{s3}^{(e)} + \gamma_{s1}^{(e)} + \gamma_{s1}^{(h)}\right]$$

"primitive traj." **"Andreev loop travelled p times"**

Goorden, PJ and Weiss '08.

Tunneling transport through Andreev interferometers

Plan b) : semiclassics



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Tunneling transport through Andreev interferometers

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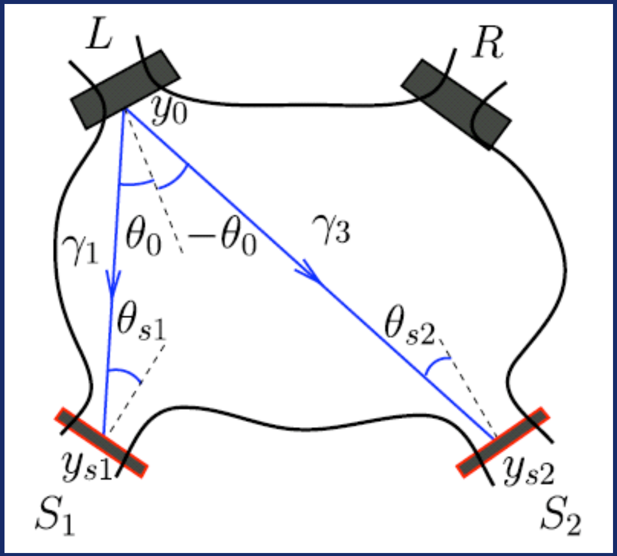

"Macroscopic Resonant Tunneling"

Calculate transmission

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on blue trajectories with action phase and stability

$$S_{\gamma,I} = p(-\pi + \phi + E\, t_{\ell,\mathrm{I}}) + 2\, E\, t_{\gamma_{s1}} - (\pi/2 - \phi/2)$$

$$A_\gamma = B_\gamma\, t_i t_j \prod_k [r_k]^{l_\gamma(k)}$$

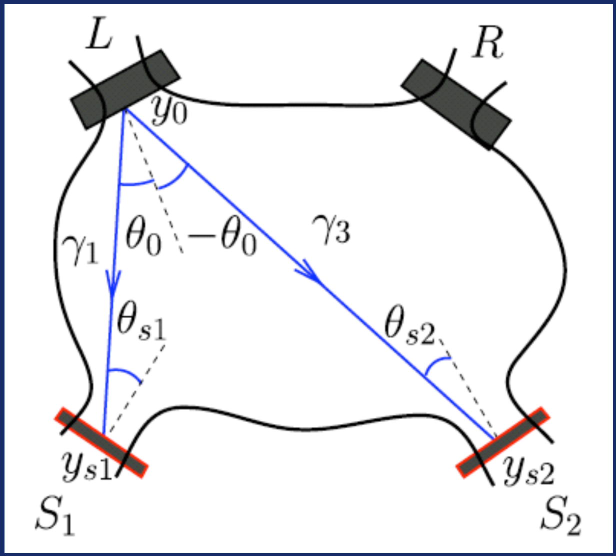
Stability of trajectory

Sequence of transmissions and reflections at tunnel Barriers (Whitney '07)

Tunneling transport through Andreev interferometers

Plan b) : semiclassics

"Macroscopic Resonant Tunneling"



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One key observation :
Andreev reflections refocus the dynamics
for Andreev loops shorter than Ehrenfest time

- ***Stability does not depend on p !***
- ***Stability is determined only by*** $\gamma_{s3}^{(e)} + \gamma_{s1}^{(e)}$

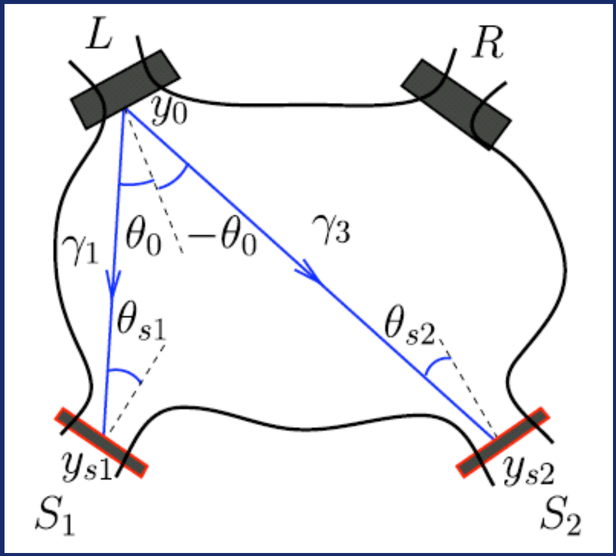
Tunneling transport through Andreev interferometers

Plan b) : semiclassics

"Macroscopic Resonant Tunneling"

Calculate transmission

$$T_{ji}^{\beta\alpha} = \frac{1}{2\pi\hbar}\int_i y_0 \int_j y_0' \sum_{\gamma 1,\gamma 2} A_{\gamma 1} A^*_{\gamma 2} \exp[i\delta S/\hbar]$$



->Pair all trajs. (w. different p's) on $\gamma_1 + \gamma_3$
->Substitute

$$\sum_{\gamma 1,\gamma 2} A_{\gamma 1} A^*_{\gamma 2}\,[\ldots]_{\gamma 1,\gamma 2} \longrightarrow \Gamma_{\rm N}^2 \sum_{\gamma=\text{primitive}} B_\gamma^2 \sum_{p,p'=0}^{\infty} (1-\Gamma_{\rm N})^{a(p+p')+b}\,\Gamma_{\rm S}^{p+p'+c}\,[\ldots]_{\gamma,p,p'}$$

Determine B_γ as for normal transport
~classical transmission probabilities

Tunneling transport through Andreev interferometers

Plan b) : semiclassics

"Macroscopic Resonant Tunneling"

$$\langle T_{LL}^{he}\rangle_{\rm r} = \frac{\pi\Gamma_{\rm N}^2 N_{\rm N}}{4}\left(\frac{N_{\rm S}}{2\Gamma_{\rm N}N_{\rm N}+2\Gamma_{\rm S}N_{\rm S}}\right)^2$$

$$\times\left(1-(1+\tau_{\rm E}/\tau_{\rm D,s})\exp[-\tau_{\rm E}/\tau_{\rm D,s}]\right)$$ Measure of trajs.

$$\times\frac{\Gamma_{\rm S}}{1-2\,\Gamma_{\rm S}\,(1-\Gamma_{\rm N})\cos[\pi-\phi]+\Gamma_{\rm S}^2\,(1-\Gamma_{\rm N})^2}$$ Resonant tunneling

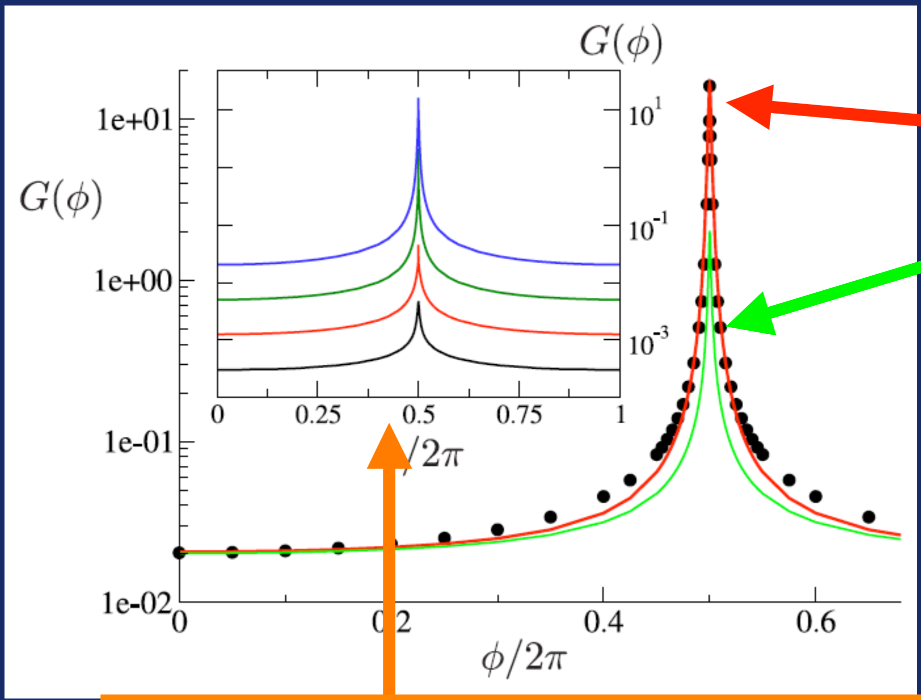
$$\langle T_{RL}^{ee}\rangle_{\rm r} = \frac{\pi^2\Gamma_{\rm N}^2 N_{\rm N}^2}{8N_{\rm S}}\left(\frac{N_{\rm S}}{2\Gamma_{\rm N}N_{\rm N}+2\Gamma_{\rm S}N_{\rm S}}\right)^3$$

$$\times\left(1-(1+\tau_{\rm E}/\tau_{\rm D,s}+\tau_{\rm E}^2/2\tau_{\rm D,s}^2)\exp[-\tau_{\rm E}/\tau_{\rm D,s}]\right)$$ Measure of trajs.

$$\times\frac{1+\Gamma_{\rm S}^2\,(1-\Gamma_{\rm N})^2}{1-2\,\Gamma_{\rm S}\,(1-\Gamma_{\rm N})^2\cos[\pi-\phi]+\Gamma_{\rm S}^2\,(1-\Gamma_{\rm N})^4}$$ Resonant tunneling

Tunneling transport through Andreev interferometers

Plan c) : numerics



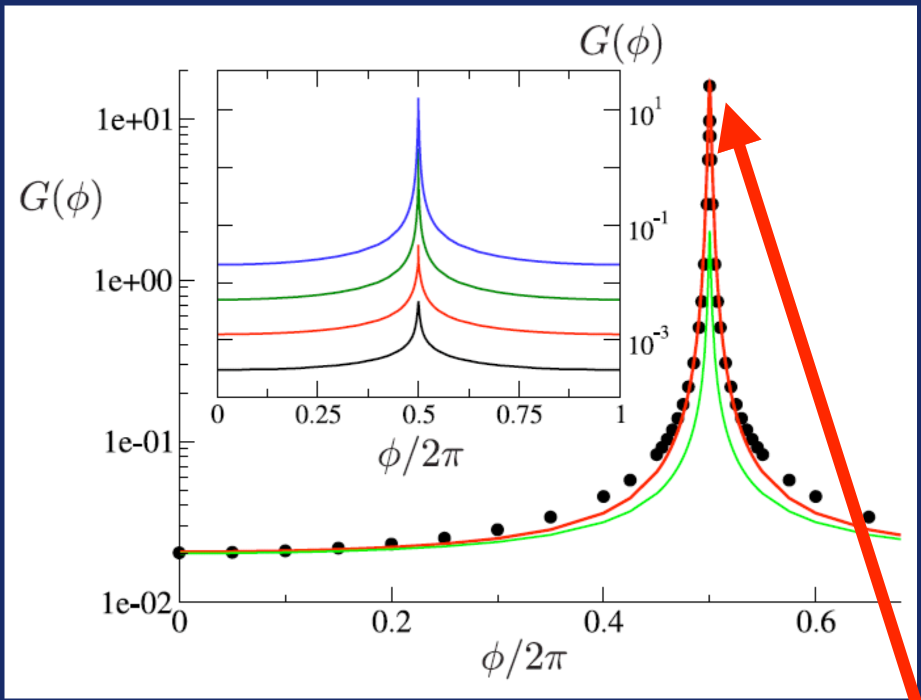
Order of magnitude enhancement from universal (green) to MRT (red)

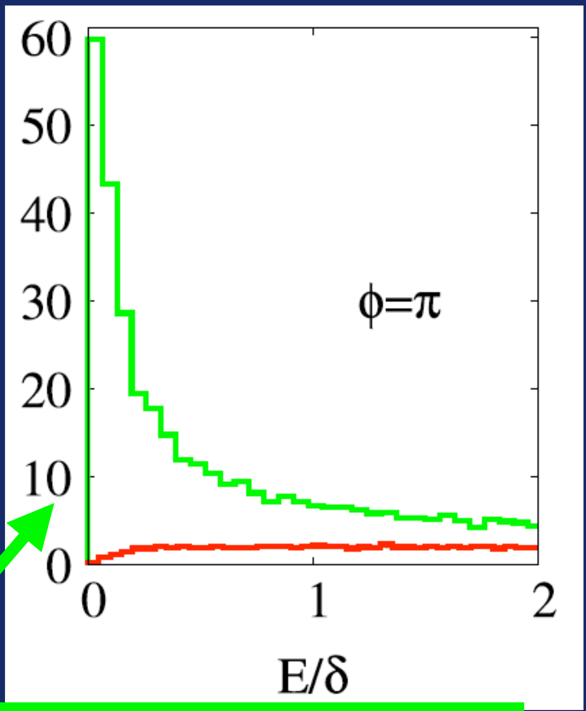
Effect increases as $k_F L$ increases
Peak-to-valley ratio goes from Γ to Γ^2

Goorden, PJ and Weiss PRL '08, Nanotechnology '08.

Tunneling transport through Andreev interferometers

Plan c) : numerics





Tunneling through ~10-15 levels
i.e. half of those in the peak in the DoS

"TUNNELING THROUGH LEVELS AT ε=0"

Gorden, PJ and Weiss PRL '08, Nanotechnology '08.

Outline

- Mesoscopic superconductivity - Andreev reflection
- Density of states in ballistic Andreev billiards
- Transport through ballistic Andreev interferometers
- Symmetries of charge transport in presence of superconductivity

J. Weiss and PJ, in progress

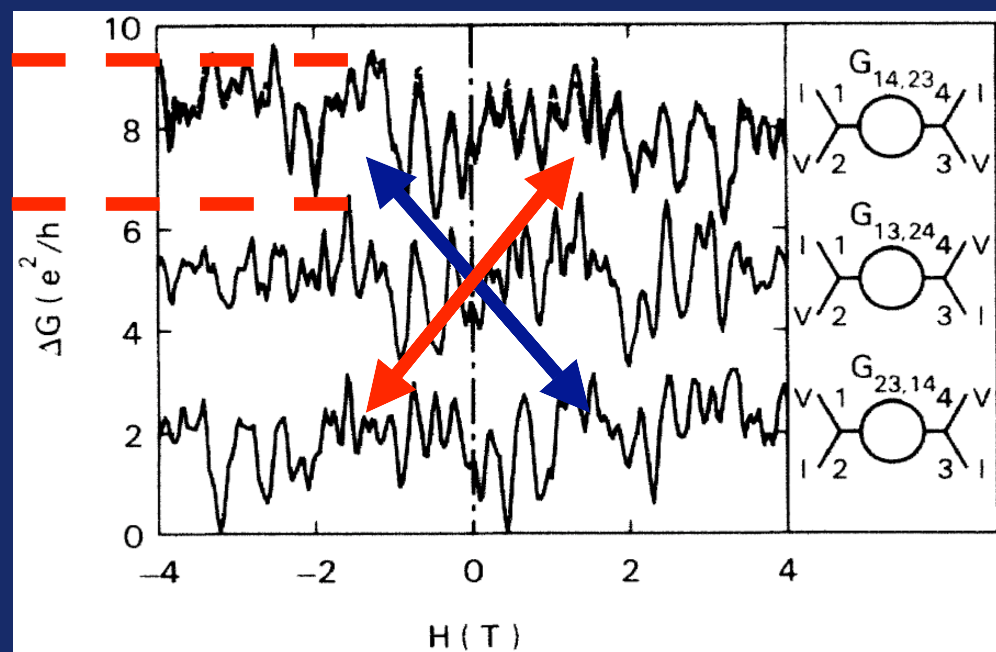
Symmetry of multi-terminal transport

NORMAL METAL:

Two-terminal measurement $G(H)=G(-H)$

Four-terminal measurement $G_{ij;kl}(H)= G_{kl;ij}(-H)$

$O(e^2/h)$



Onsager, Casimir…
Buttiker '86
Benoit et al '86

VOLUME 81, NUMBER 2 PHYSICAL REVIEW LETTERS 13 JULY 1998

Phase Dependent Thermopower in Andreev Interferometers

Jonghwa Eom, Chen-Jung Chien, and Venkat Chandrasekhar

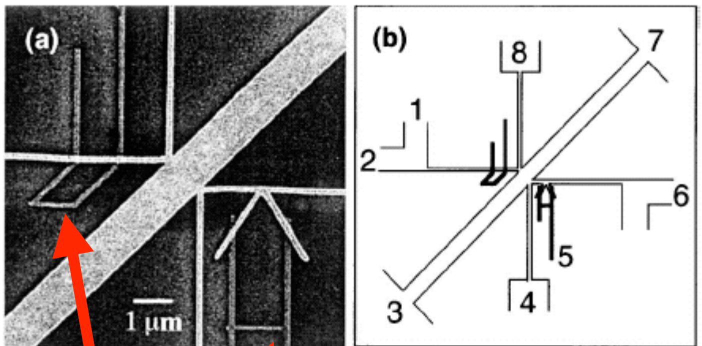
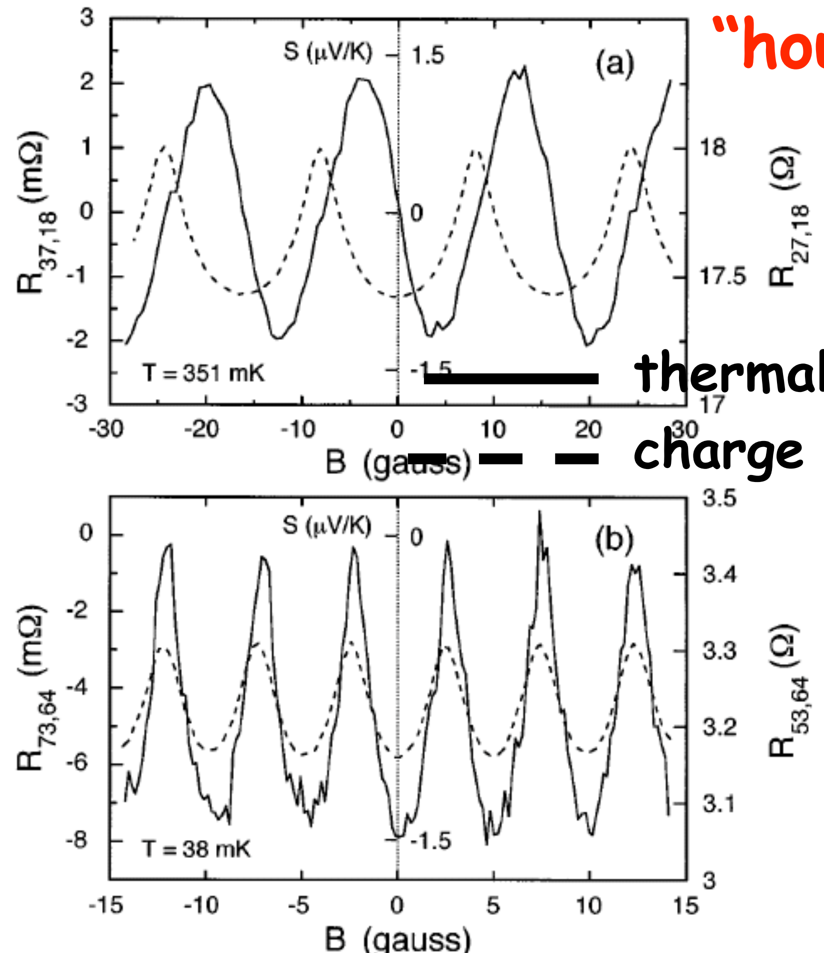


FIG. 1. (a) Scanning electron micrograph of a typical sample. (b) Schematic of the sample structure. Fat solid lines represent Al wires, while thin solid lines outline the Au film. The two hybrid loops are labeled as "parallelogram" (left) and "house" (right). The contacts are labeled with Arabic numerals.

S



"house"

thermal

charge

"parallelogram"

Symmetry of multi-terminal transport with superconductivity

VOLUME 76, NUMBER 24 PHYSICAL REVIEW LETTERS 10 JUNE 1996

Sample-Specific Conductance Fluctuations Modulated by the Superconducting Phase

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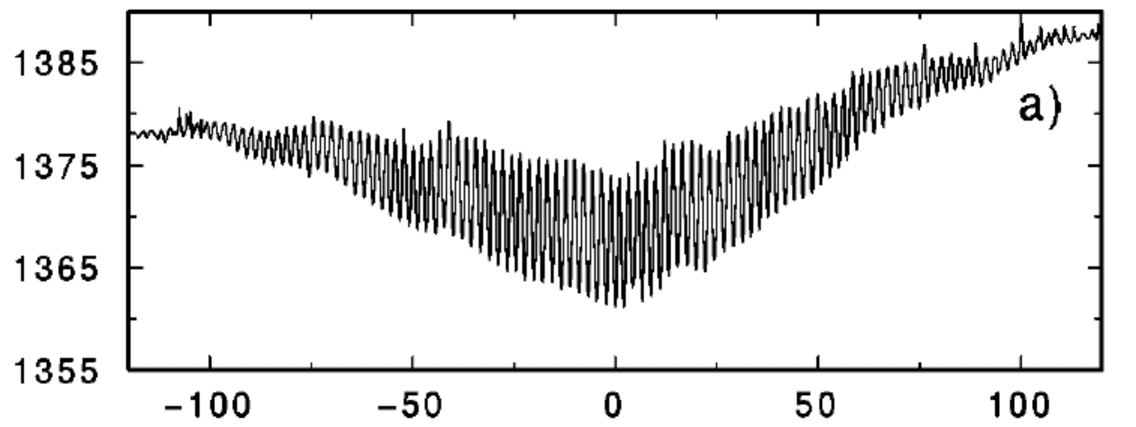
Interuniversity Micro Electronics Centre, Kapeldreef 75, B-3030 Leuven, Belgium
(Received 20 October 1995)

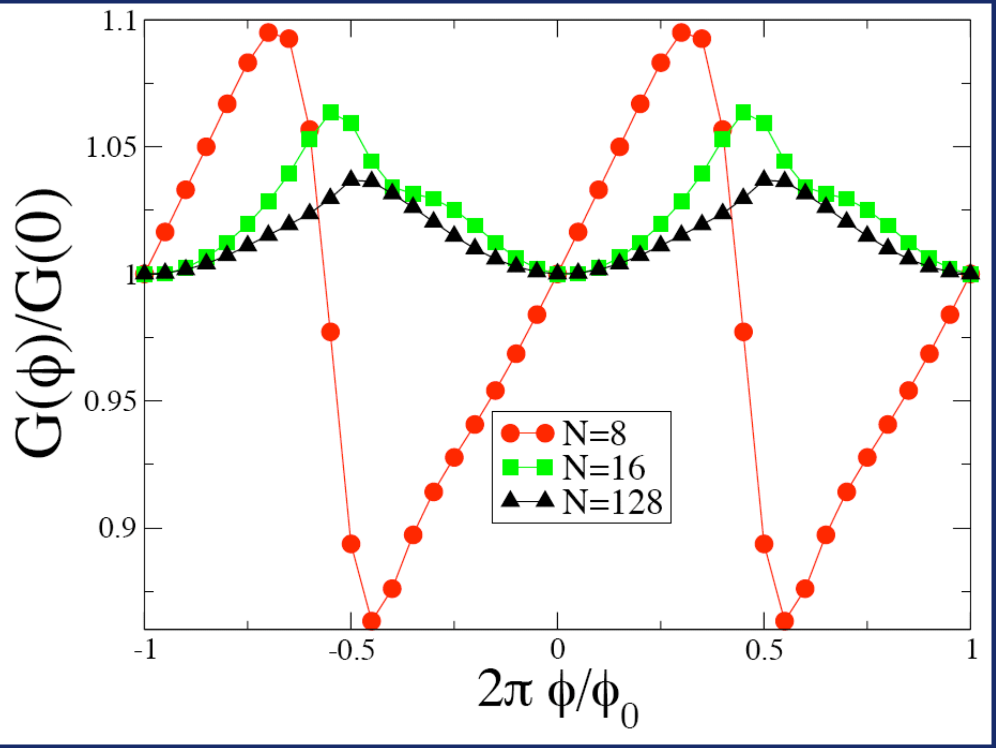


Symmetry of multi-terminal transport with superconductivity

Numerics :

❖No particular symmetry
❖AB-Amplitude is O(N)

➢G looks more and more symmetric as N grows



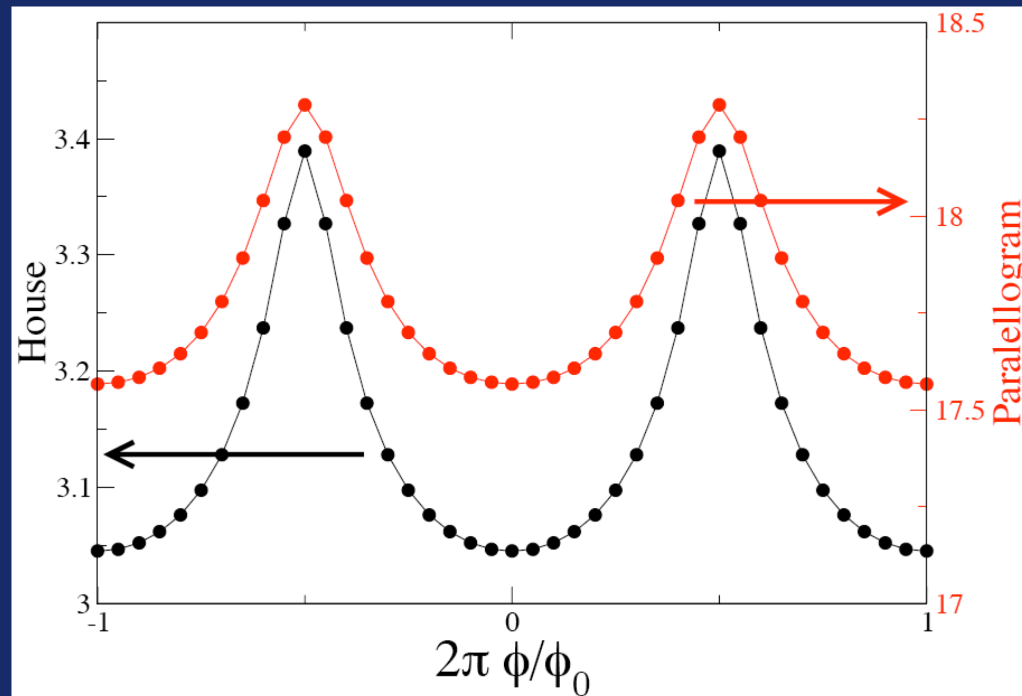
Exps.: <G>=1500 / 7700
δG= 60 / 300

Unreachable numerically - use circuit theory!

Symmetry of multi-terminal transport with superconductivity

Nazarov's circuit theory:

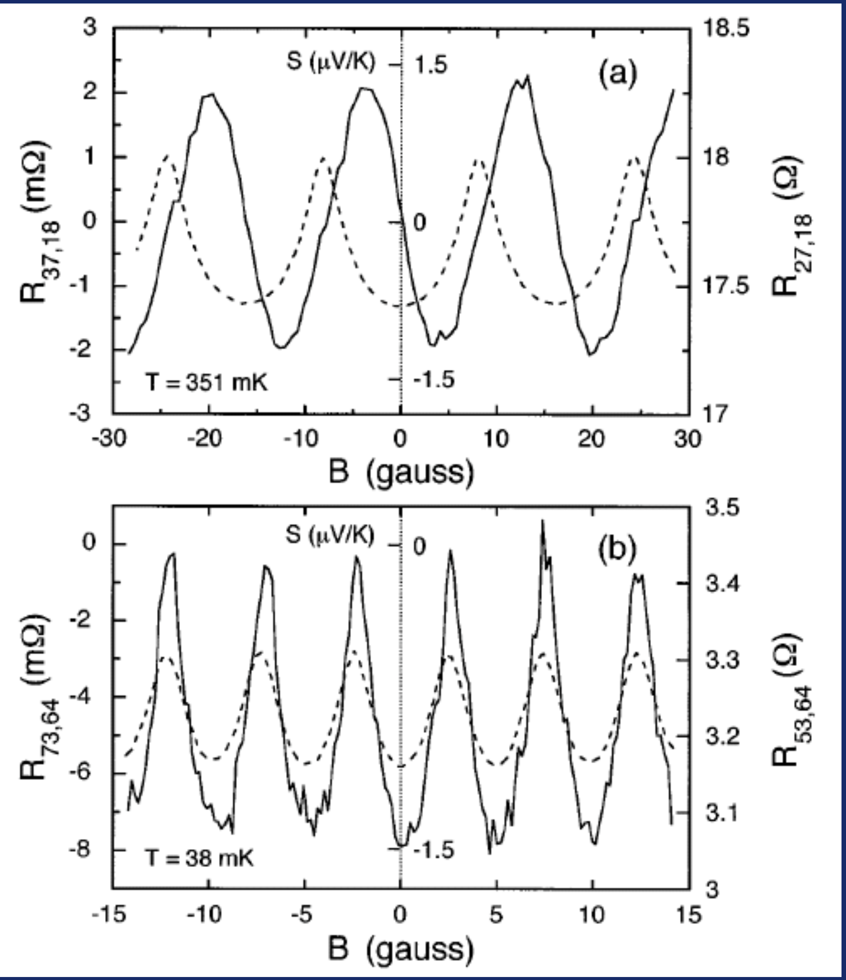
- ❖Valid for N>>1
- ❖Neglects "weak loc" effects

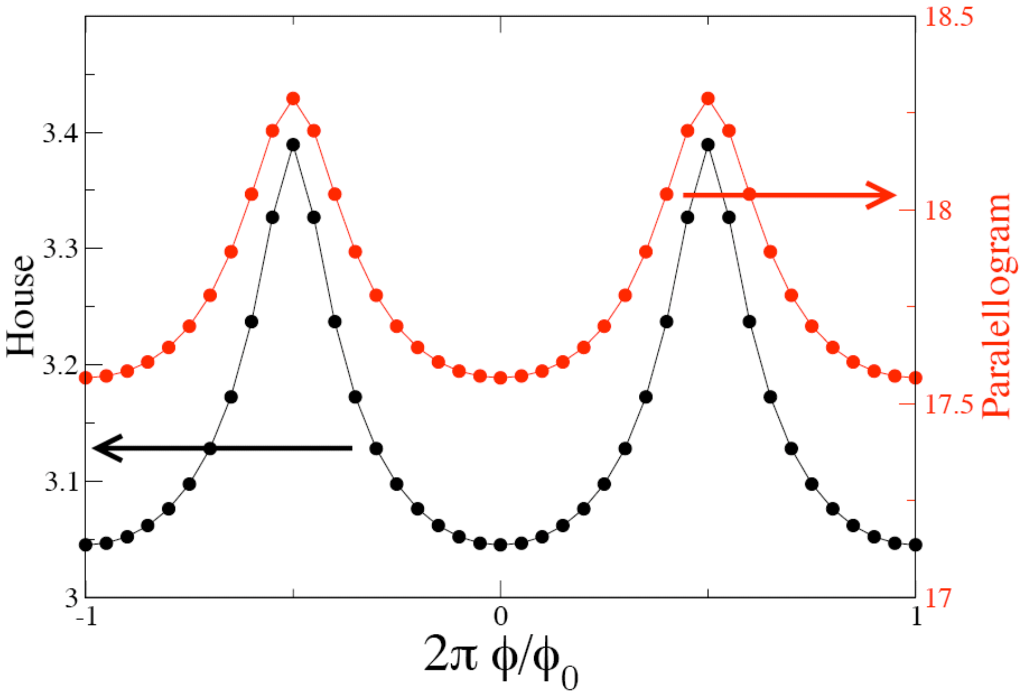


- ➢symmetric 4-terminal "charge" conductance
- ➢AB oscillations O(N)
- ➢Minimum at ϕ=0
- ➢Ratio $\delta R/\langle R\rangle$ is in good agreement with exps

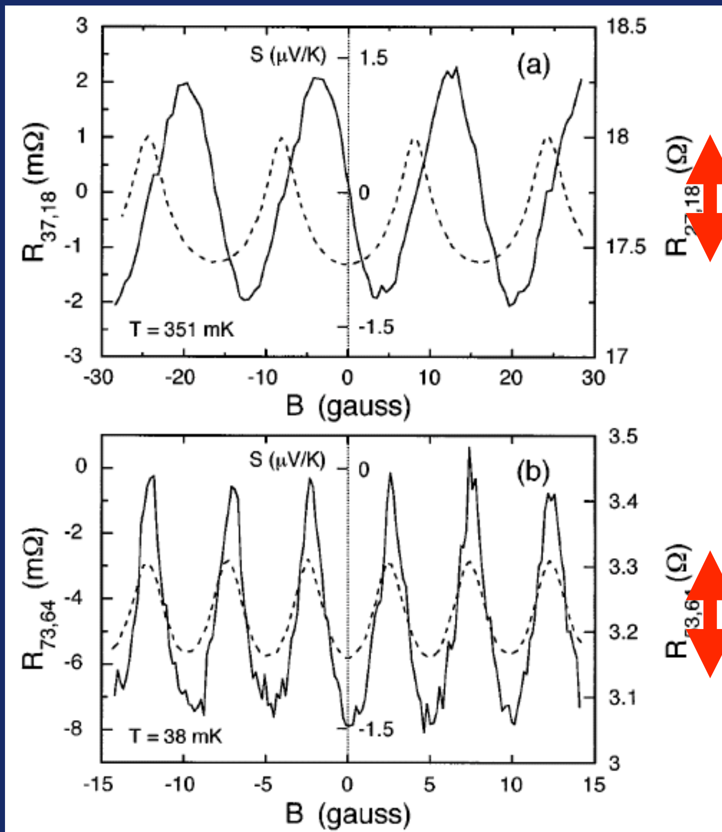
C.Th.: Nazarov '94; Argaman '97.

Symmetry of multi-terminal transport with superconductivity





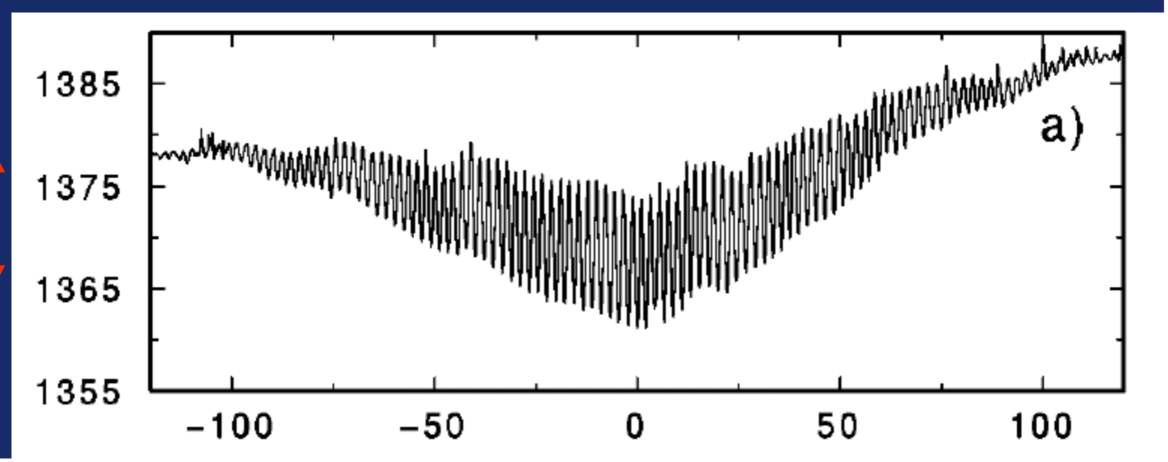
Nazarov '94; Argaman '97.



<G> =1600
dG =70

<G> =7700
dG =300

<G> =18
dG < 1



Future perspectives

- Proximity effect with exotic superconductivity