Shot Noise Suppression in Single and Multiple Chaotic Cavities: the Role of Diffraction, Disorder and Symmetries

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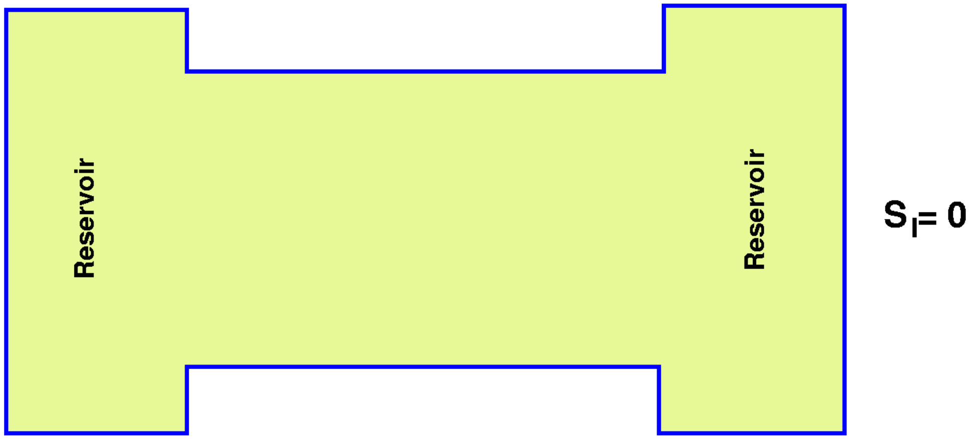
Summary

- **Brief introduction to shot noise in mesoscopic devices**
- **Numerical evaluation of the Fano factor in chaotic cavities and diffusive conductors**
- **Conductance and Fano factor of cascaded barriers and constrictions: quantum-semiclassical discrepancy**
- **The cascaded barrier system as a testbed to understand the origin of the discrepancy**
- **Analytical results for 2 and 3 barriers**
- **Conclusions**

Shot noise in mesoscopic devices



$$S_I = 4\frac{e^2}{h}|eV|\mathrm{Tr}\left[t^\dagger t(I - t^\dagger t)\right]$$

$$S_I = 4\frac{e^2}{h}|eV|\sum_i T_i(1 - T_i)$$

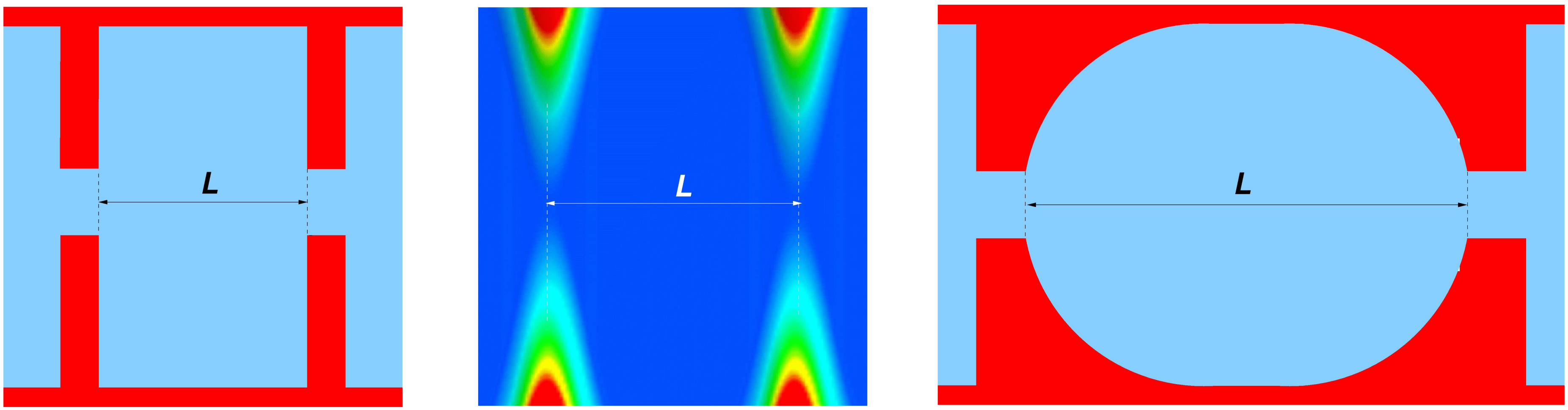
Since, from the Landauer-Büttiker formula,

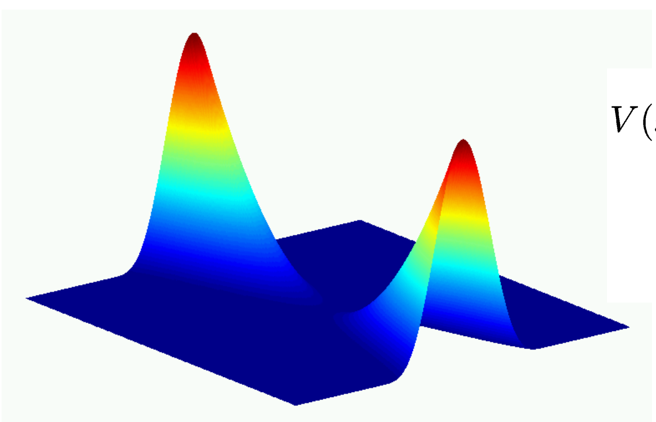
$$|I| = G|V| = 2\frac{e^2}{h}|V|\sum_i T_i,$$

$$\gamma = \frac{\sum_i T_i(1 - T_i)}{\sum_i T_i}.$$

- Shot noise is the result of the granularity of charge
- In the case of independent electrons, we obtain Schottky's result: ***S***=2***qI***
- In a perfect quantum wire ***S***=0
- In general, shot noise in quantum devices is linked to the transmission eigenvalues (M. Buettiker, Phys. Rev. Lett. **65**, 2901 (1990))
- The ratio of the actual noise power spectral density to that of full shot noise is defined "Fano factor"

Cavities with different geometries



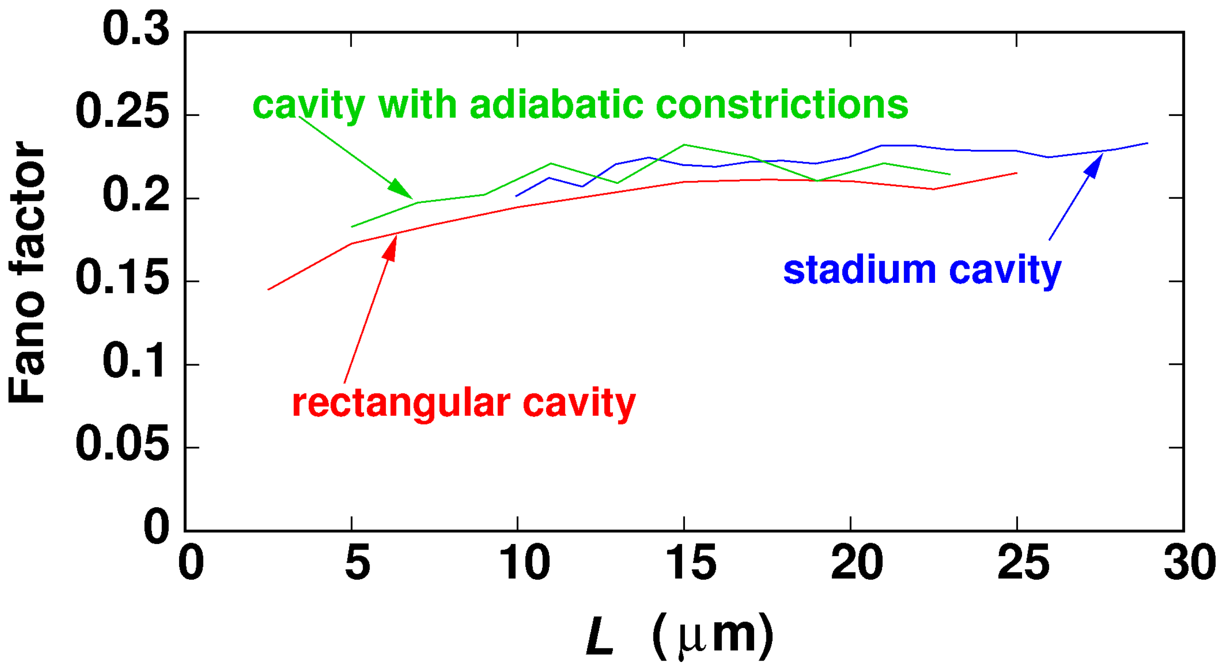


$$V(x,y) = \frac{E_1}{2}\left[1+\cos\left(\frac{2\pi x}{L_x}\right)\right] + E_2 \sum_{\pm}\left(\frac{y-y_{\pm}(x)}{\Delta}\right)^2 \theta[\pm(y-y_{\pm}(x))]$$

$$y_{\pm}(x) = \pm\frac{L_y}{4}\left[1-\cos\left(\frac{2\pi x}{L_x}\right)\right]$$

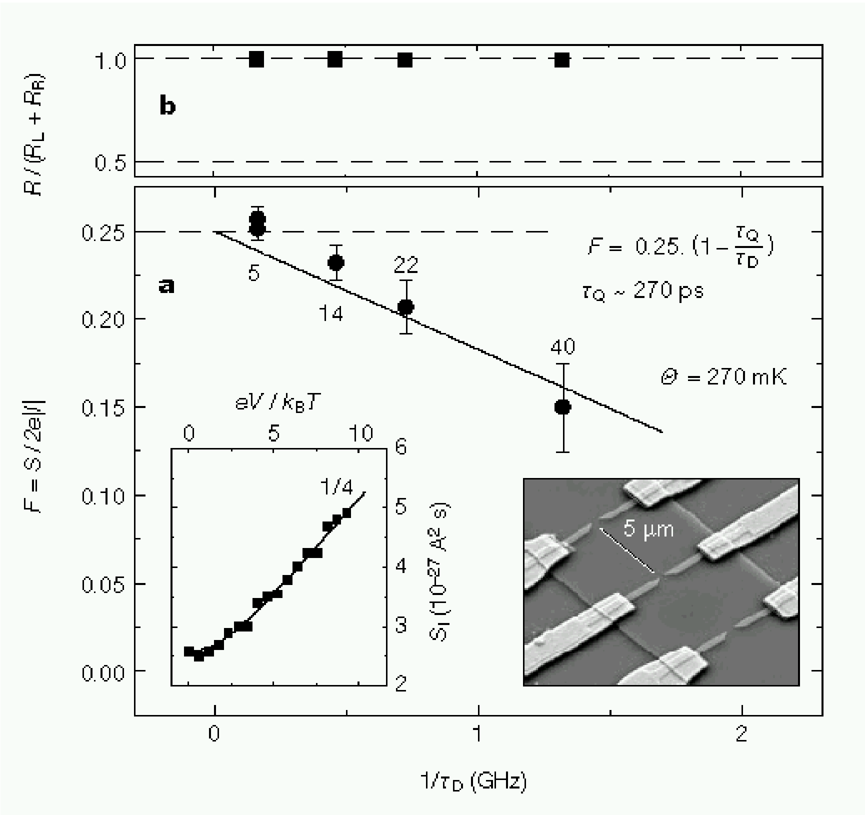
Influence of cavity shape

Contrary to what appears in much of the literature on the subject, our previous work has led to the conclusion (in agreement with the results by Aigner, Rotter et al.) that the shape of the cavity has no significant influence on the shot noise suppression factor

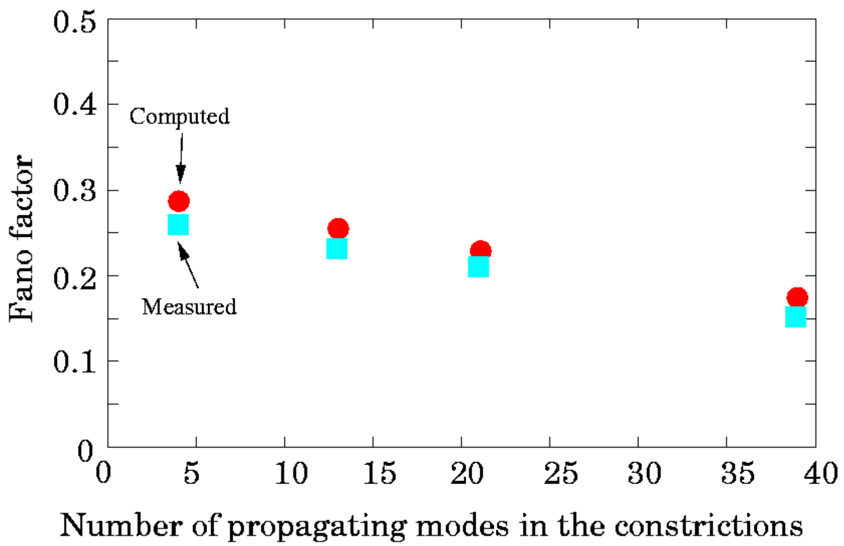


number of modes propagating in each constriction: 40

Noise vs. aperture width



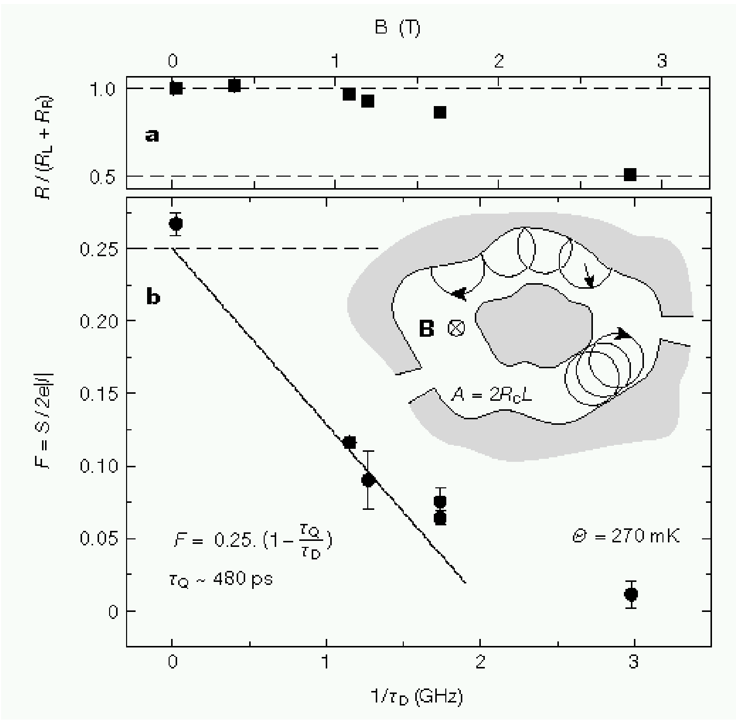
S. Oberholzer et al., Nature 415, 765 (2002)



Although based on a very simple hard-wall model, these results are already in reasonable agreement with the experimental data.

P. Marconcini et al., Europhys. Lett. 73, 574 (2006)

Noise as a function of magnetic field



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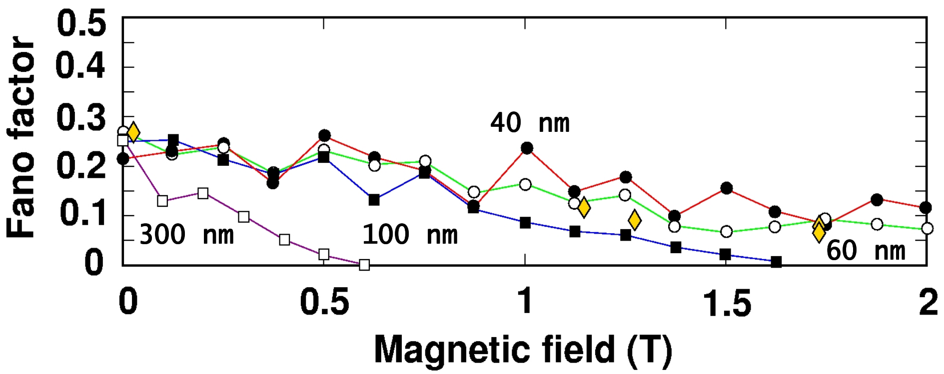
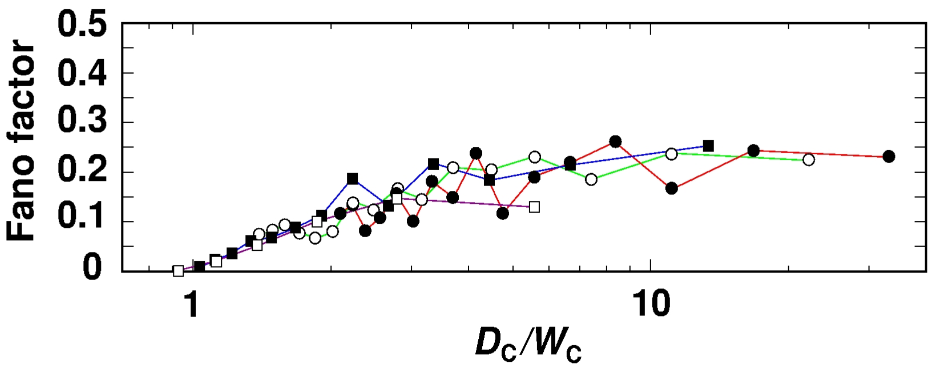
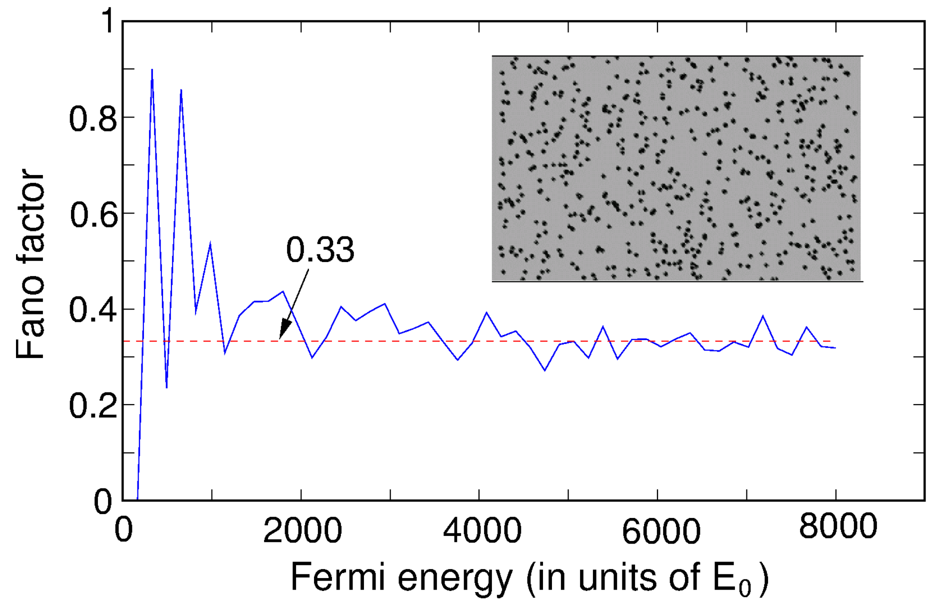


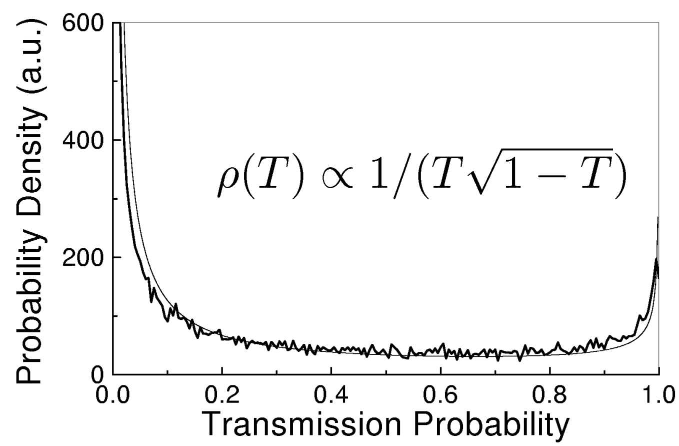


P. Marconcini et al., Europhys. Lett. 73, 574 (2006)

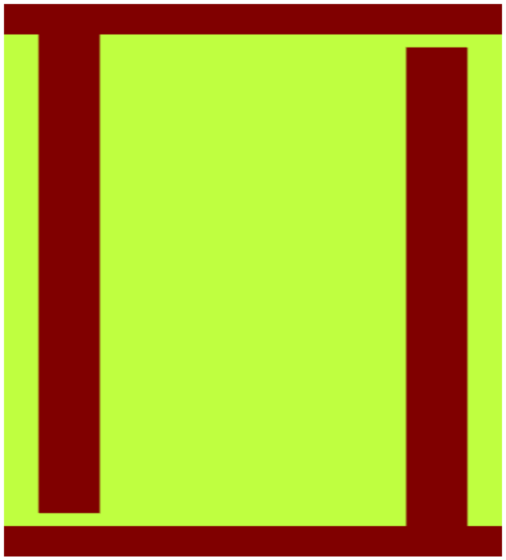
Shot noise in diffusive conductors

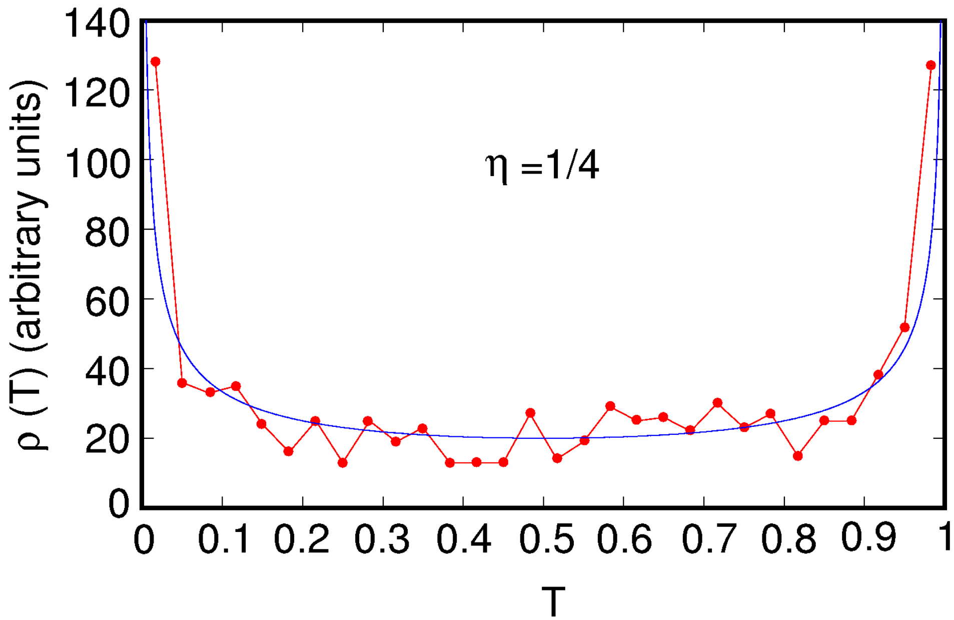
- **Beenakker and Buettiker showed, by means of Random Matrix theory, that in diffusive conductors shot noise is suppressed down to 1/3 of its full value (**PRB **46**, 1889 (1992)**)**
- **Shot noise suppression down to 1/3 is the result of the bimodal distribution of the transmission eigenvalues**
- **Numerical calculation with randomly located hard-wall or soft-wall obstacles exactly agree with the results obtained from random matrix theory**





Empty cavity

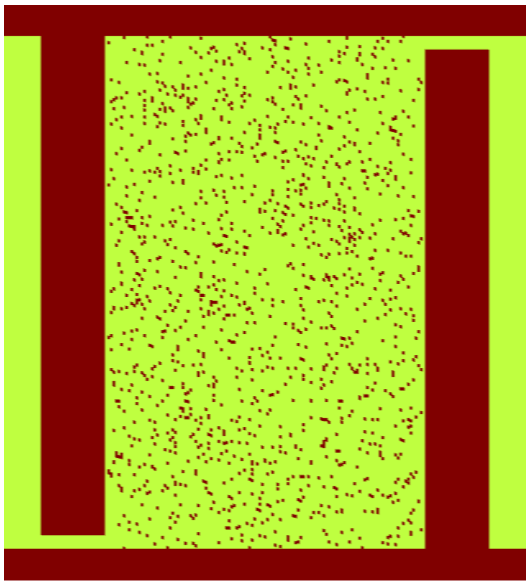


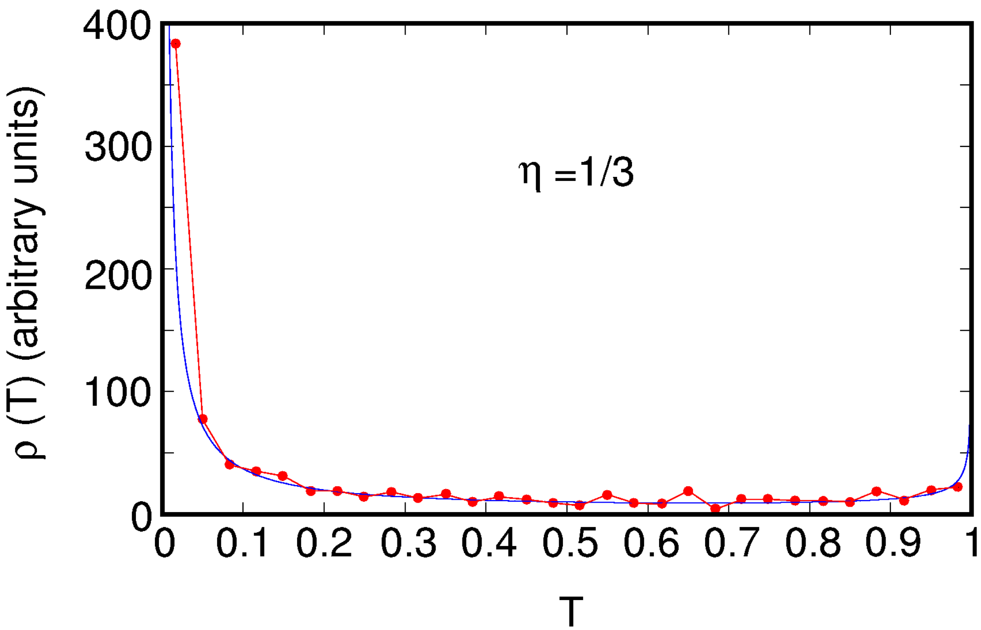


For an empty cavity with constrictions much narrower than the cavity width, the transmission eigenvalues have a symmetric bimodal distribution, given by

$$\rho(T) \propto 1/\sqrt{T(1-T)}$$

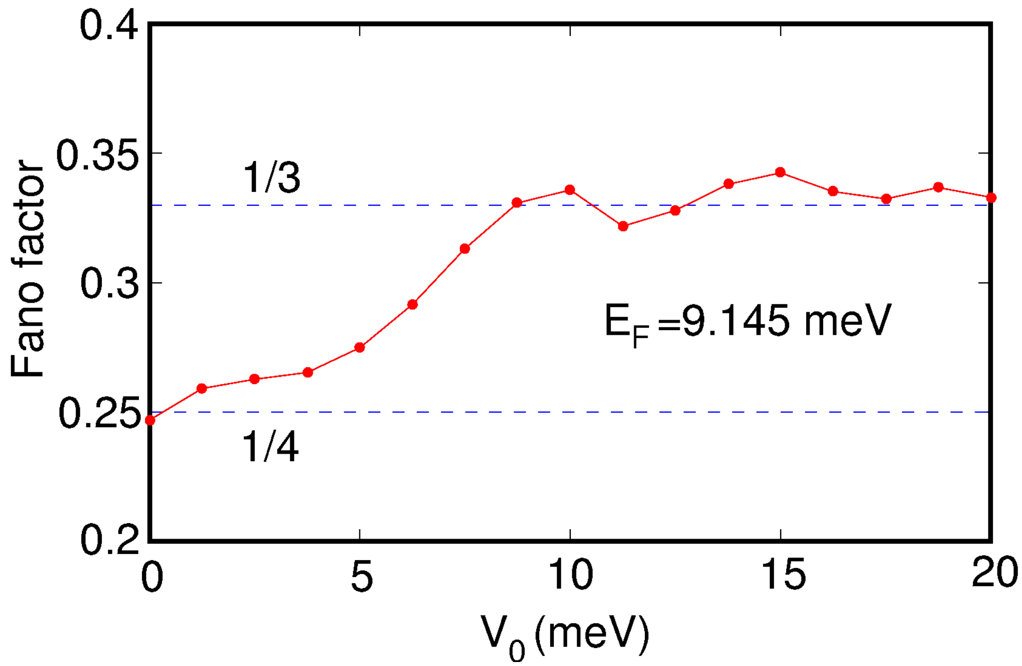
Cavity with scatterers





For a cavity with strong disorder we find a distribution of transmission eigenvalues corresponding to that of a purely diffusive conductor, and the Fano factor is 1/3

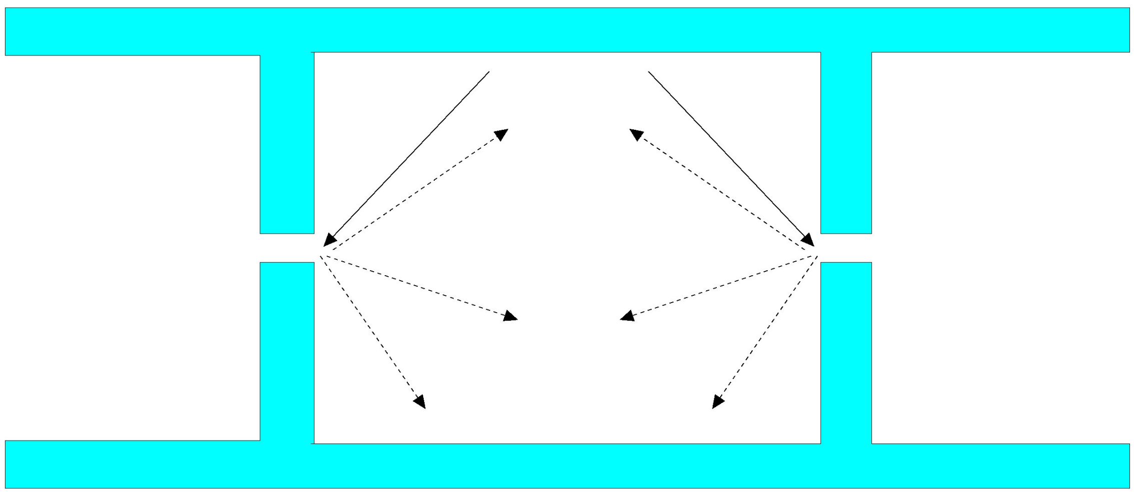
Fano factor as a function of disorder strength



As the strength of the disorder is increased, we observe a smooth transition of the Fano factor from 0.25 to 0.33

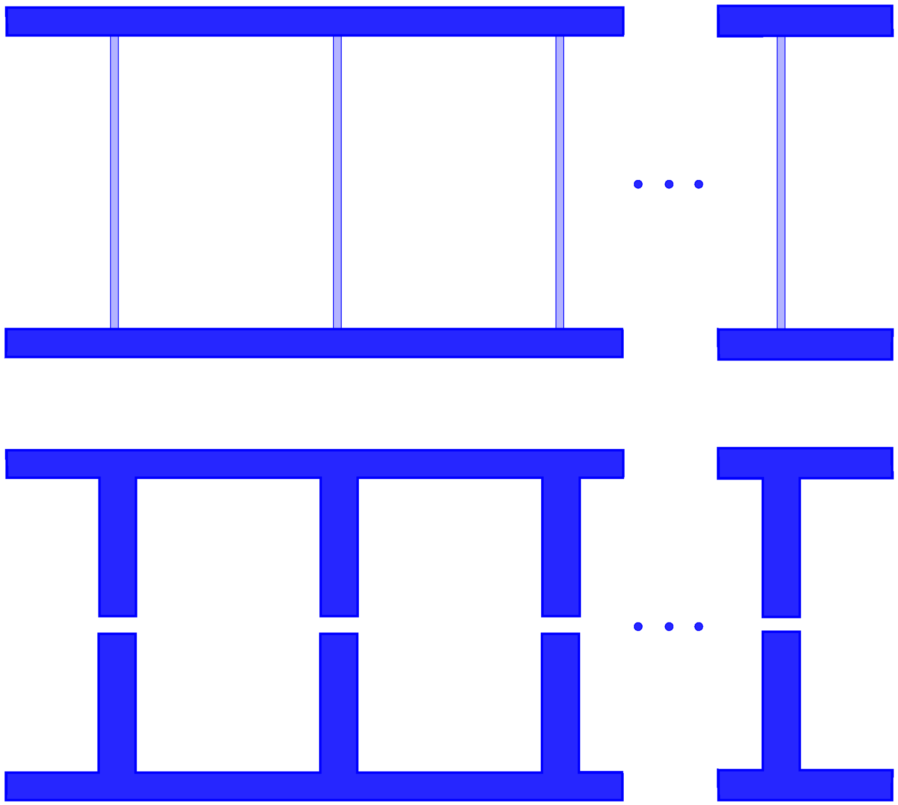
Interpretation of the results

- This is further evidence that the Fano factor 0.25 in mesoscopic cavities is to be attributed to scattering and the associated mode mixing at the constrictions



- The actual classically chaotic shape of the cavity or the presence of disorder are not needed to achieve a Fano factor of 0.25

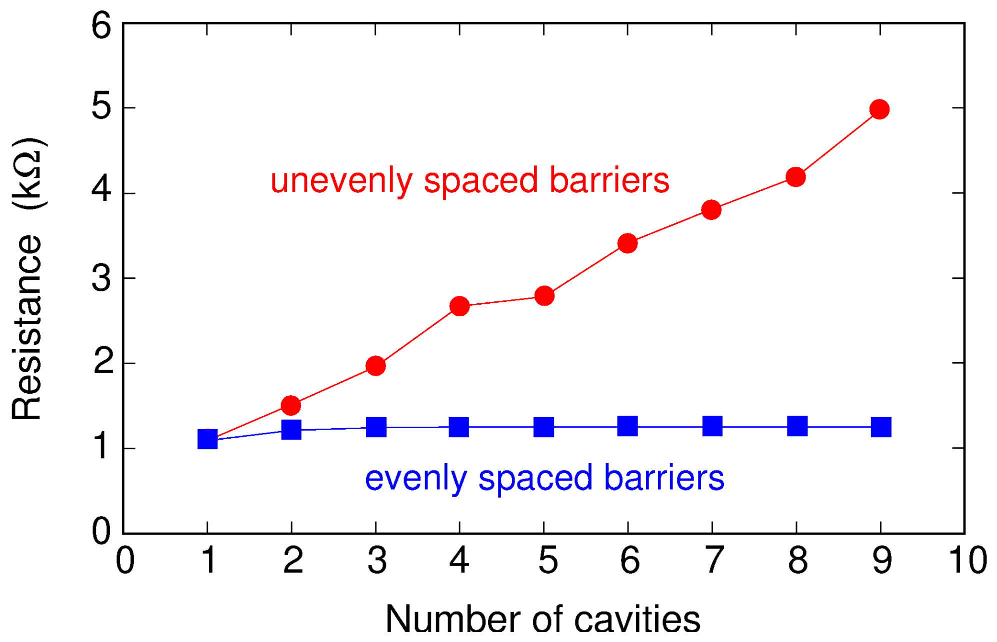
Cascaded barriers and constrictions



- We have studied the conductance and noise behavior of two types of structures: a series of barriers and a series of cascaded constrictions
- We have obtained numerical (and for some cases analytical) results with a quantum-mechanical model, and we compare them with those from semiclassical approaches

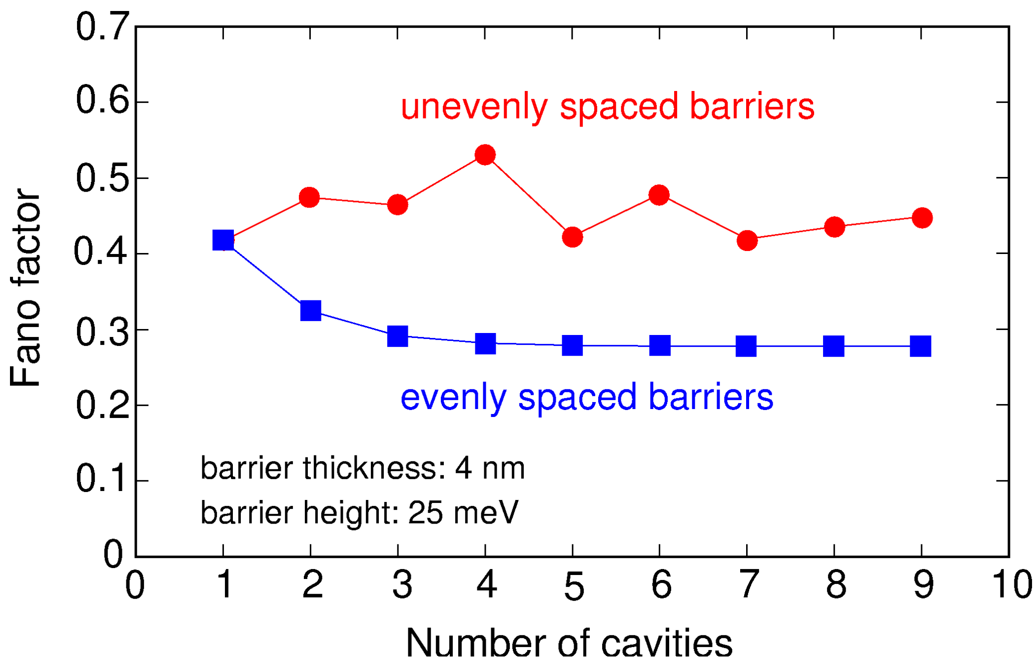
Cascaded barriers: resistance

- **The overall resistance is approximately equal to that of a single cavity if the barriers are evenly spaced, while it increases proportionally to the number of cavities if the barriers are unevenly spaced**



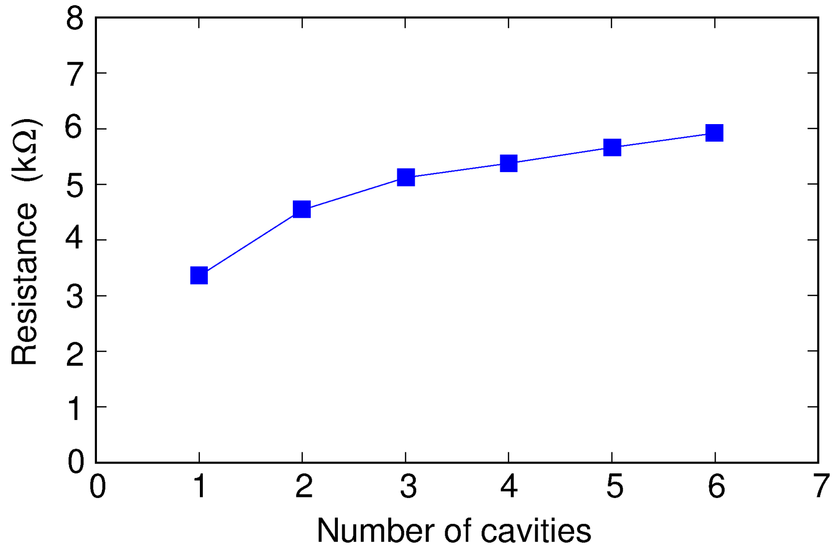
Cascaded barriers: Fano factor

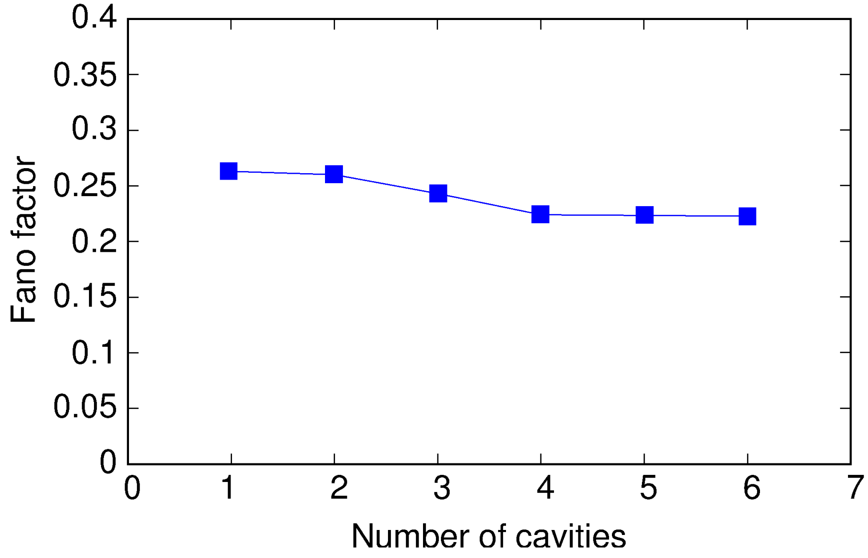
- For a given choice of barrier transparency the Fano factor decreases down to about 0.28 for evenly spaced barriers
- For unevenly spaced barriers it increases to a larger value
- Both results differ from the value 1/3 predicted by the semiclassical model



Series of evenly spaced constrictions

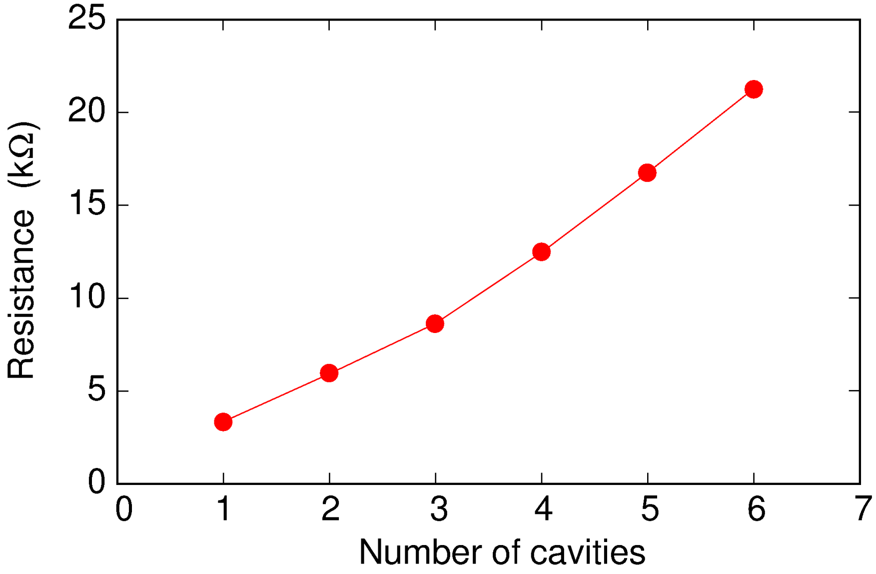
- On the basis of a semiclassical model, the resistance of a series of narrow constrictions should be equal to the sum of their resistances, while the Fano factor should approach 1/3, as the number of constrictions increases
- For evenly spaced constrictions the resistance doubles at most, with respect to that of a single cavity, while the Fano factor slightly decreases

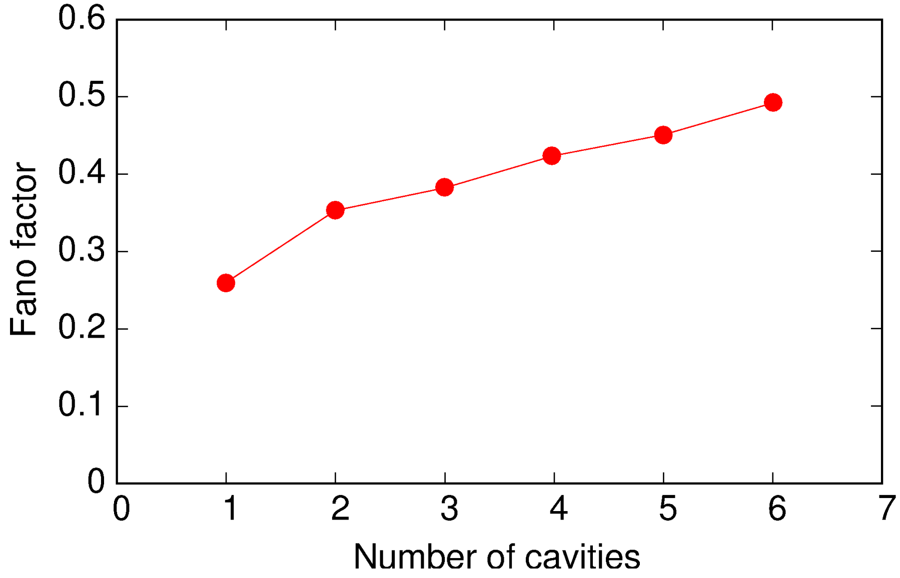




Series of unevenly spaced constrictions

- **For unevenly spaced constrictions the resistance increases linearly or more than linearly with the number of cavities, while the Fano factor increases towards a value significantly larger than 1/3**



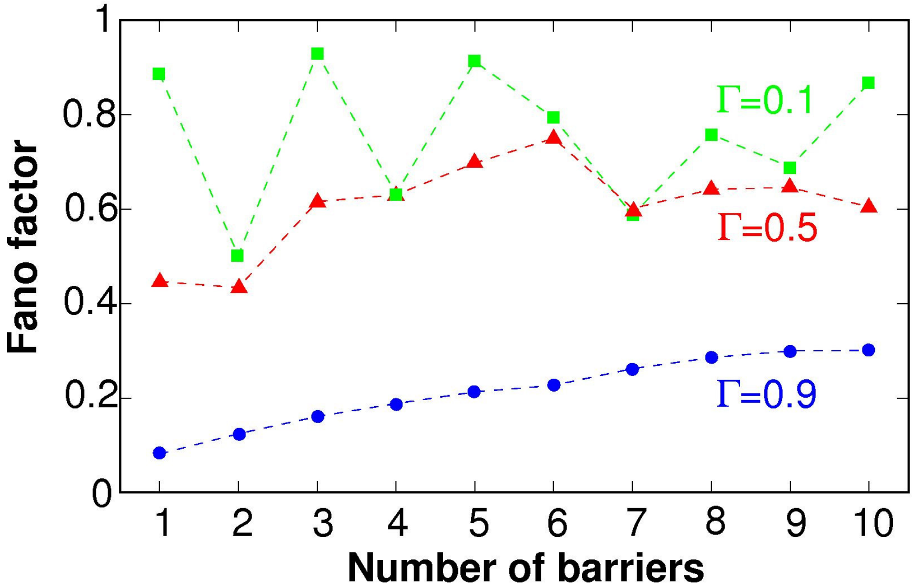


Semiclassical vs. quantum

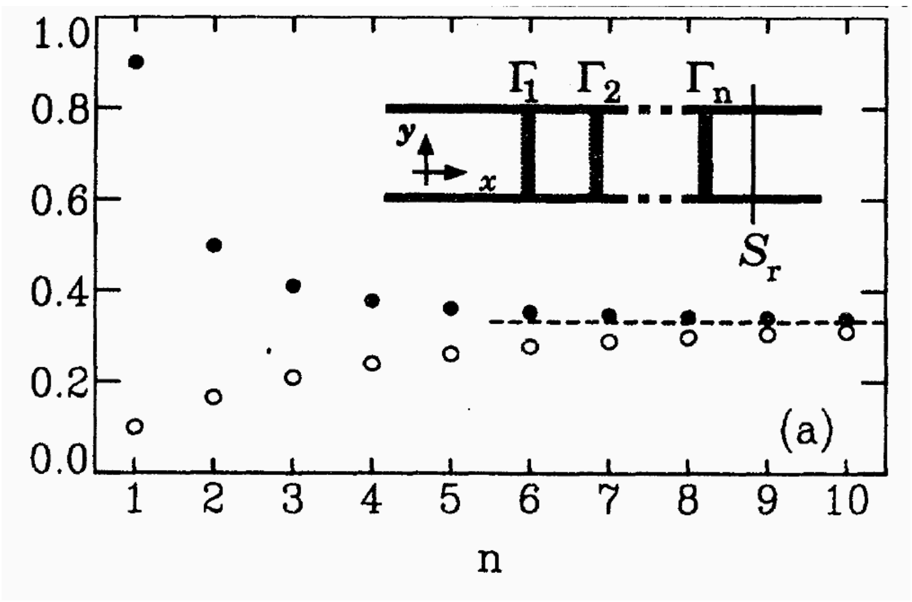
- The main issue is now about the source of the discrepancy between the two approaches
- We focus on the simpler problem, that of cascaded barriers, since it allows an extremely simple treatment, as a result of the absence of mode mixing
- Indeed the mode overlaps at each interface are identity matrices, so that we just need to study a collection of one-dimensional problems!
- Let us first compare the quantum results for unevenly spaced barriers with the well known semiclassical results, for three choices of barrier transparency

Fano factor vs. number of barriers

Quantum calculation: no clear asymptotic behavior

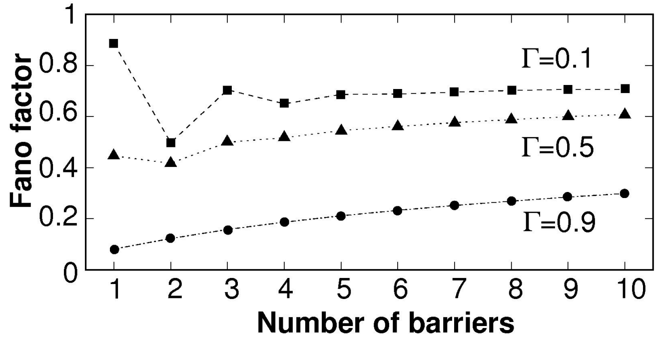


Semiclassical calculation: a clear asymptotic value of 1/3

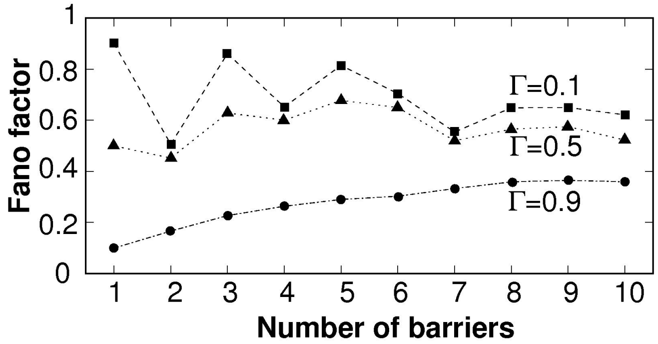


de Jong and Beenakker, PRB 51, 16867 (1995)

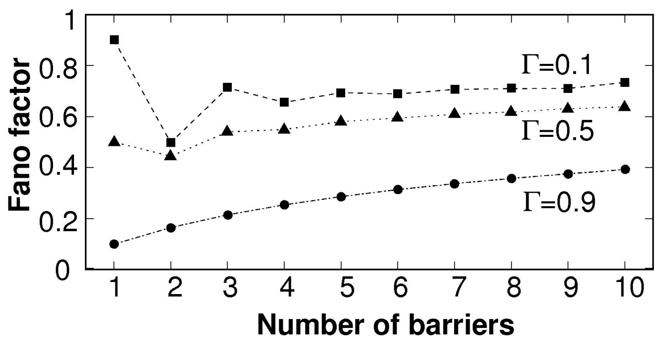
Searching for the origin of the discrepancy



Including a random phase to simulate dephasing

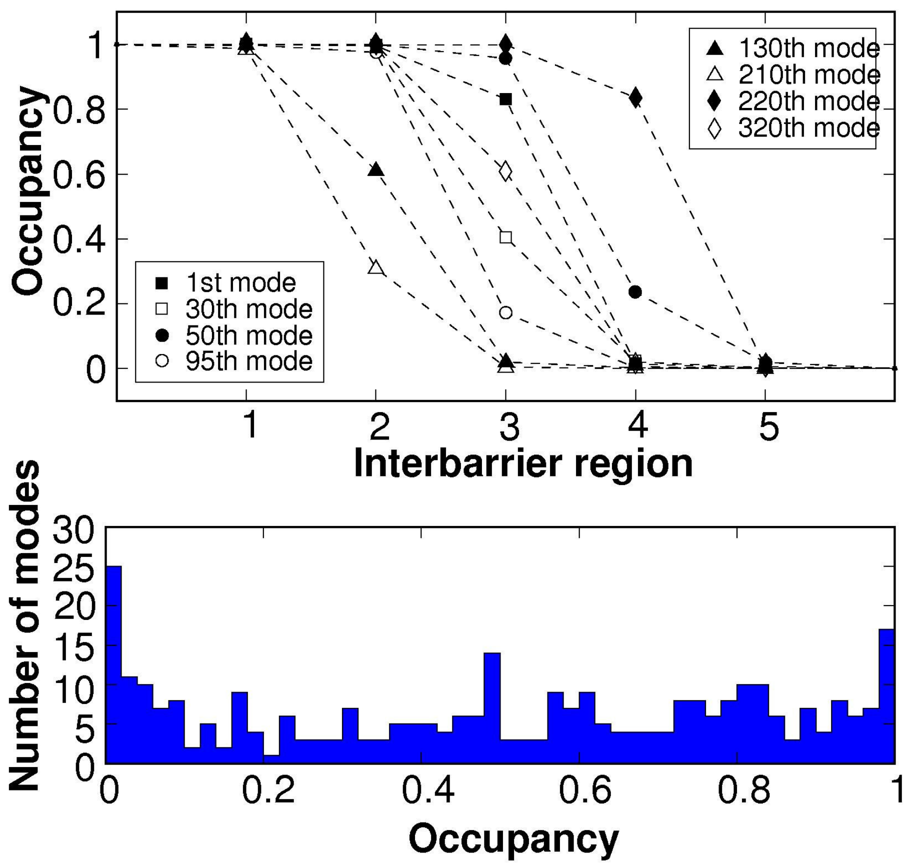


Considering barriers with a transparency independent of the wave vector



Including both barriers with a transparency independent of the wave vector and dephasing

Occupancy in the interbarrier regions



Since we have either left-going or right-going modes, the occupancy for each of them can be computed as:

$$f_{n_\Omega} = \frac{\int_\Omega |\psi_{n_L}|^2 \, dx\, dy}{\int_\Omega |\psi_{n_L}|^2 \, dx\, dy + \int_\Omega |\psi_{n_R}|^2 \, dx\, dy}$$

In the central region the occupancy for the different modes is scattered almost uniformly over the whole range from 0 to 1

Analytical expressions

For the case of barriers with a transparency independent of the wave vector and dephasing, we can work out an analytical expression for the Fano factor, at least up to three barriers

$$\gamma = \frac{\frac{1}{(2\pi)^n}\int_0^{2\pi} d\theta_1 \int_0^{2\pi} d\theta_2 \ldots \int_0^{2\pi} d\theta_n T(\theta_1,\theta_2,\ldots,\theta_n)[1-T(\theta_1,\theta_2,\ldots,\theta_n)]}{\frac{1}{(2\pi)^n}\int_0^{2\pi} d\theta_1 \int_0^{2\pi} d\theta_2 \ldots \int_0^{2\pi} d\theta_n T(\theta_1,\theta_2,\ldots,\theta_n)}$$

$$\gamma_{2B} = \frac{2(1-\Gamma)}{(2-\Gamma)^2}$$

Same as the semiclassical result!

$$\gamma_{3B} = \frac{3(4-8\Gamma+5\Gamma^2-\Gamma^3)}{16-24\Gamma+9\Gamma^2}$$

Different from the semiclassical result!

Conclusions

- Shot noise behavior of chaotic cavities is the result of diffraction of the openings rather than of their classically chaotic shape
- Also the dependence of the Fano factor on magnetic field is dominated by the characteristics of the constrictions
- The Fano factor for cascaded barriers is not properly described by otherwise very successful semiclassical models
- We have scrutinized this structure, a series of barriers, to understand the origin of the quantum-semiclassical discrepancy
- The origin of such a discrepancy is traced back to the impossibility of defining an occupancy depending only on particle energy in the interbarrier regions