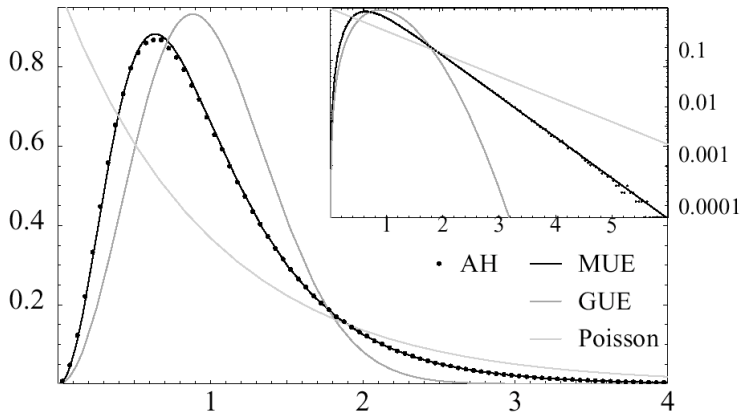
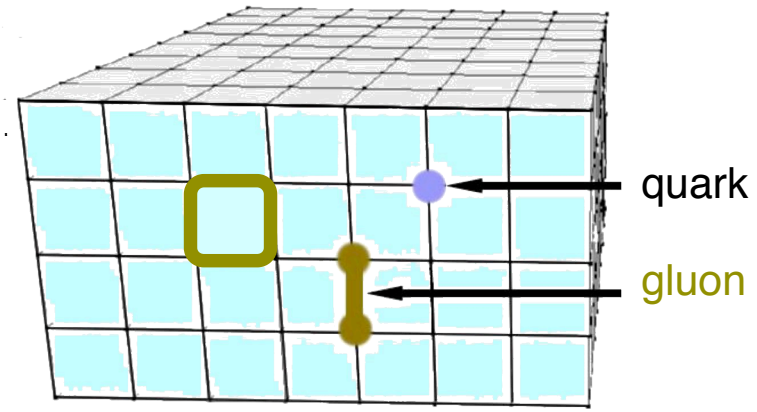
Critical Level Statistics and QCD Phase Transition

S.M. Nishigaki

Dept. Mat. Sci, Shimane Univ.



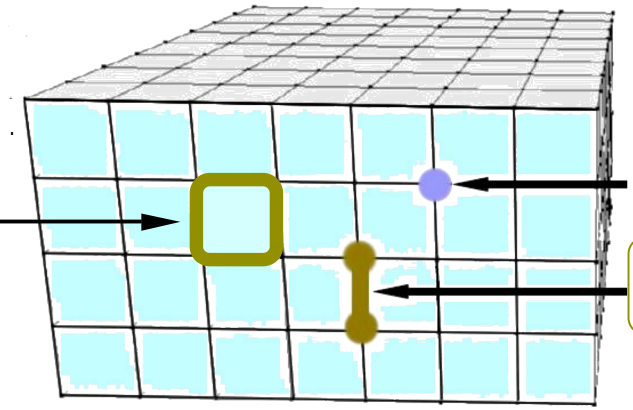


Wilson's Lattice Gauge Theory

= random Dirac op

quenched
Boltzmann weight

$$\beta \sum_{\square} \mathrm{tr}\, \underbrace{U.U.U.U.}_{\square}$$



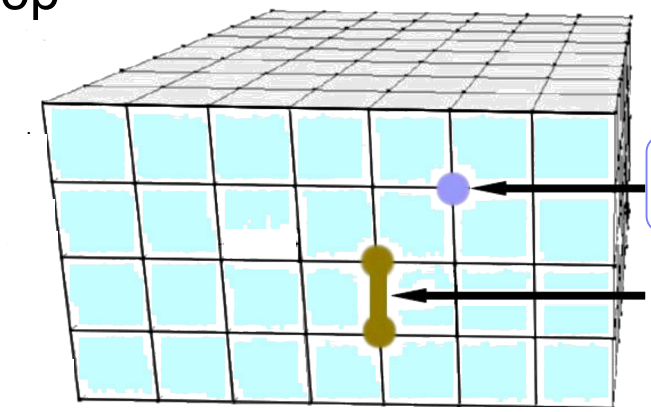
4-spinor & N-color d.o.f.

random $\mathrm{SU}(N)$ variable $U_{n,\mu}$

× Dirac γ_μ

Anderson's tight-binding H

= random Schrodinger op



i.i.d. random variable V_n

fixed const

OUTLINE

Part I : Critical Level Statistics & RMT

- CLS at localization transition — Shkhlovskii et al PRB'93
- deformed RMT — Muttalib et al PRL'93
- level spacing : CLS vs dRMT — SN PRE'98/99, Garcia^2-SN-Verbaarschot PRE'02

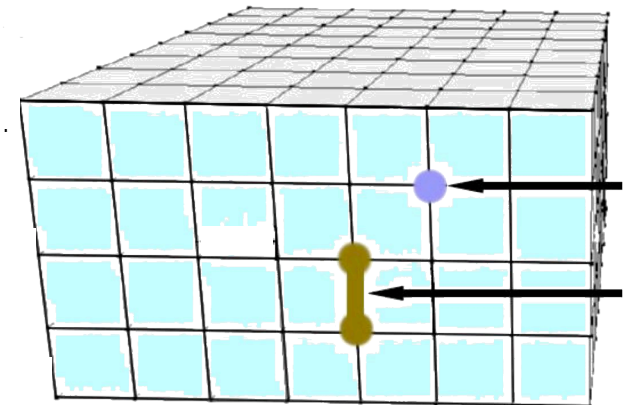
Part II : QCD transition & Dirac spectra

- chiral restoration by localization — Diakonov-Petrov NPB'86
- Dirac spectra at QCD transition — Garcia^2-Osborn NPA'06/PRD'07
- level spacing : LGT vs dRMT — Kato-SN *'08

Part I

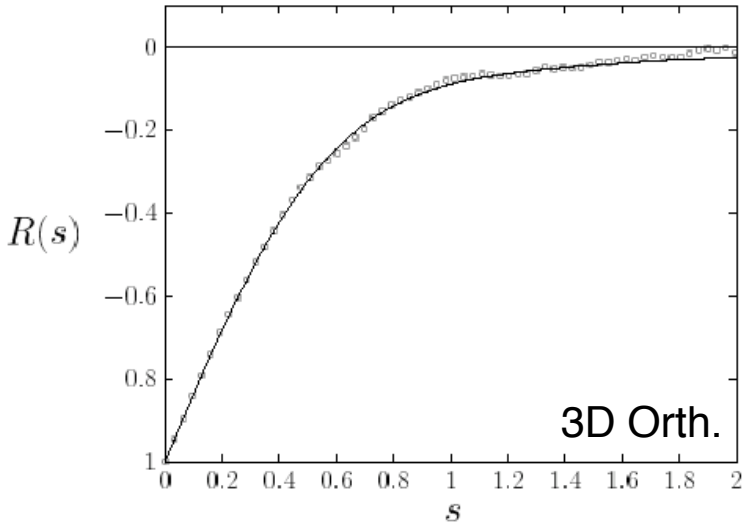
Critical Level Statistics & RMT

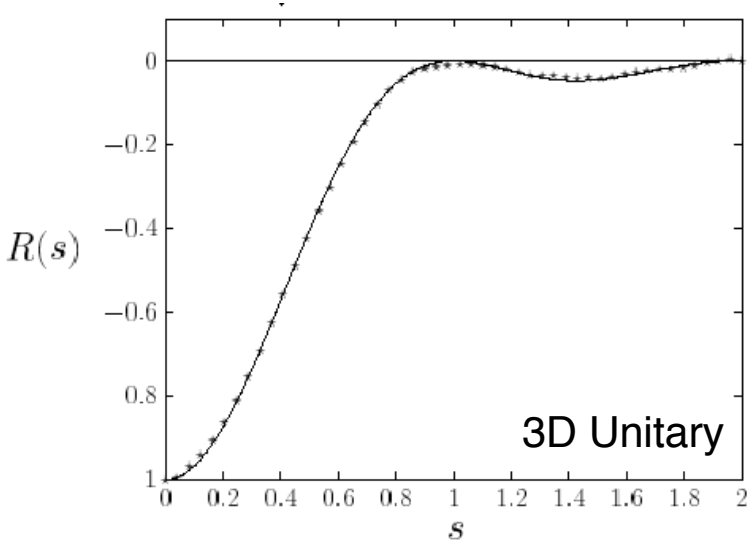
Anderson Hamiltonian



Two-level correlator

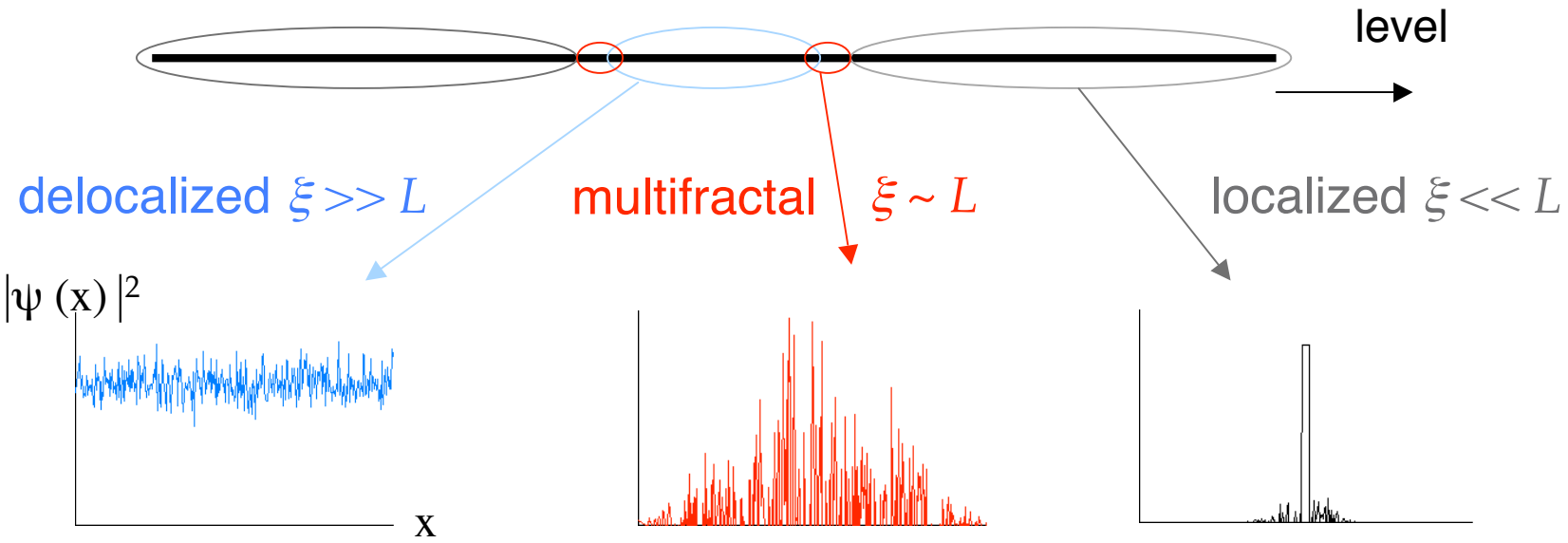
Braun-Montambaux 95





weak randomness : level statistics ⊂ RMT universality

Anderson Hamiltonian



level repulsion
→ Wigner

scale invariant
Critical Statistics

no repulsion
→ Poisson

Critical Level Statistics

sparse overlap $\sum_x \left\langle |\psi_i(x)|^2 |\psi_j(x)|^2 \right\rangle \propto |\varepsilon_i - \varepsilon_j|^{-(1-D_2)/d}$ Chalker 90

↘ distant levels becomes less repulsive

↓

		s small	s large
level spacings	$P(s)$	$\propto s^{\beta}$	$\propto \mathrm{e}^{-\kappa s}$
level # variance	$\Sigma^2(S)$	$\propto \log s$	$\propto \chi S$

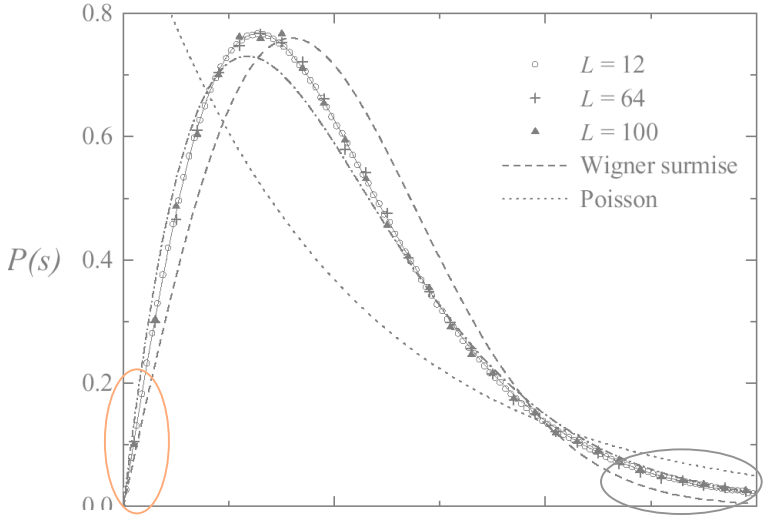
} Poisson-like

"Level Repulsion w/o Rigidity"

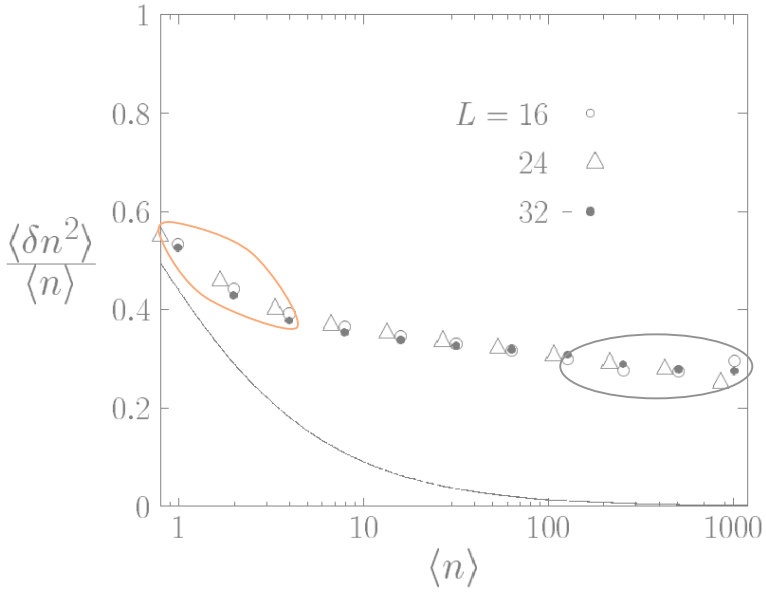
Critical Level Statistics

AH (3D Orth.) Zharekeshev-Kramer 97

level spacings



number variance



- indep of scale
- indep of randomness type
- dep on dimensionality, b.c.

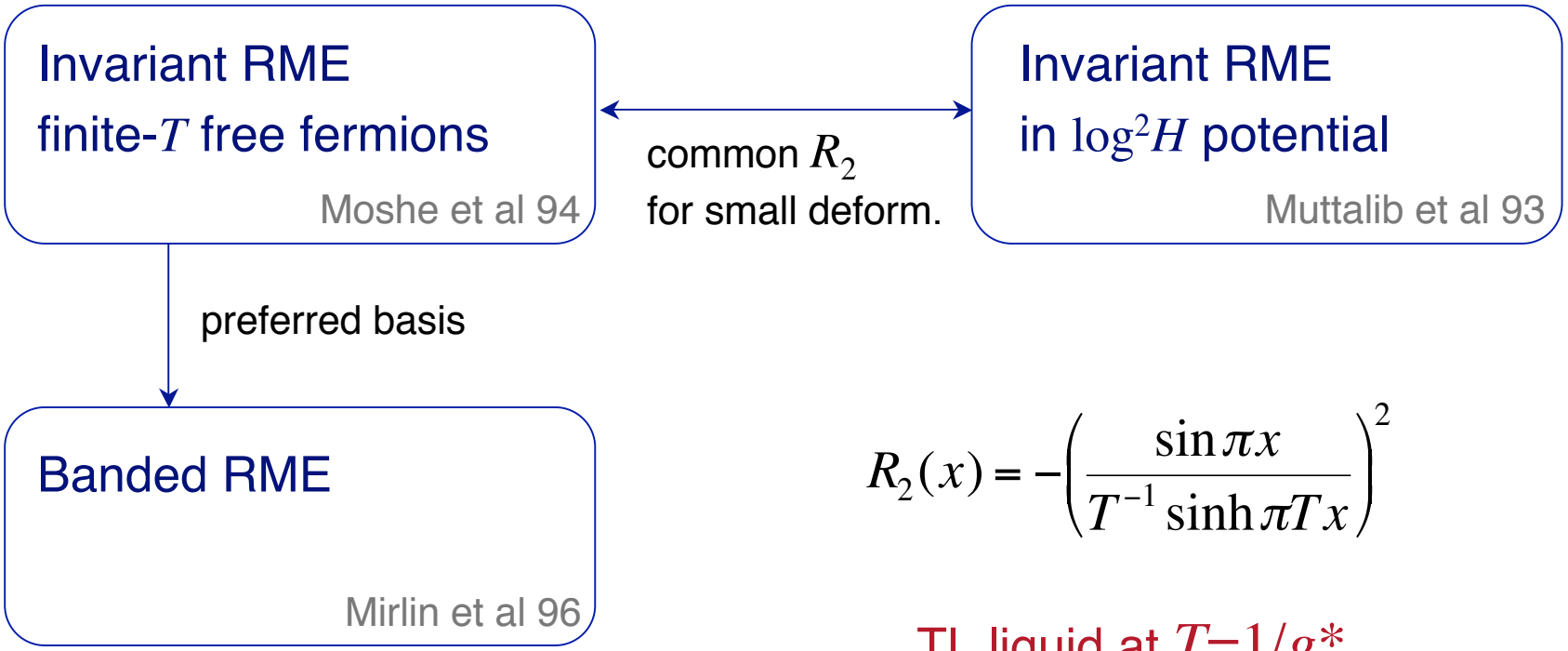
IR fixed pt

quasi-universality

conductance at fixed pt g^*

Deformed RMT

phenomenological model for CLS



$$R_2(x) = -\left(\frac{\sin \pi x}{T^{-1} \sinh \pi T x}\right)^2$$

TL liquid at $T=1/g^*$

Deformed RMT

before unfolding, inv RME always gives kernel $K(\lambda,\lambda') = \dfrac{\sin\pi\left(\int^{\lambda}\rho - \int^{\lambda'}\rho\right)}{\lambda-\lambda'}$

unfolding $\lambda \to x(\lambda) = \int^{\lambda}\rho_{\rm av}(\lambda)d\lambda$

for $V(H)\sim\dfrac{1}{2a}(\log H)^2$ and T small, $\rho_{\rm av}(\lambda) = \int_{\lambda-\Delta/2}^{\lambda+\Delta/2}\rho(\lambda')d\lambda' \sim \dfrac{1}{2a\lambda}$

unusual unfolding $\lambda \to x = \dfrac{1}{2a}\log\lambda$

$\Downarrow$

$$K(x,x') = \frac{\sin\pi(x-x')}{(\pi/a)\sinh a(x-x')}$$

Deformed RMT

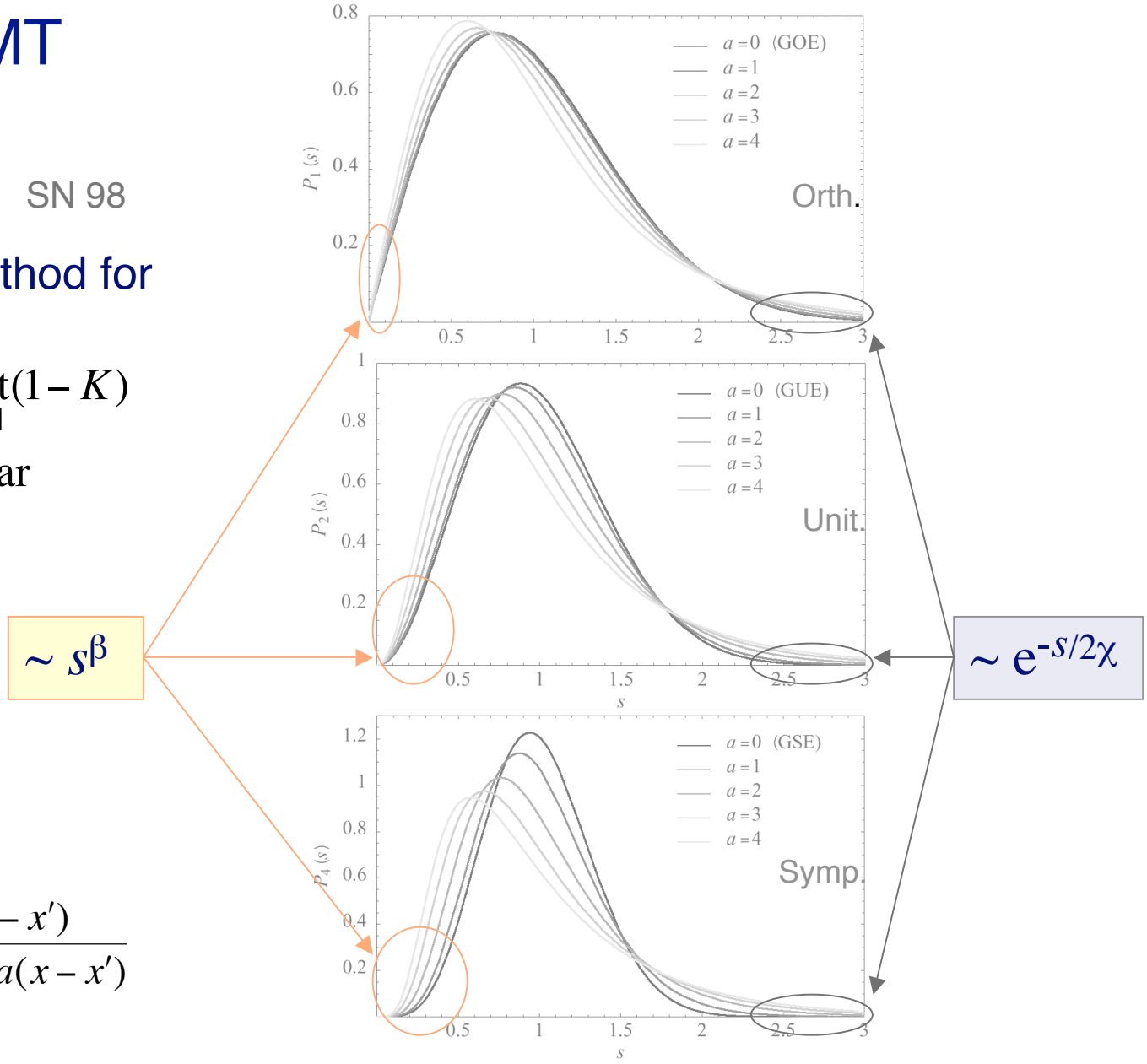
level spacings : SN 98

Tracy-Widom method for

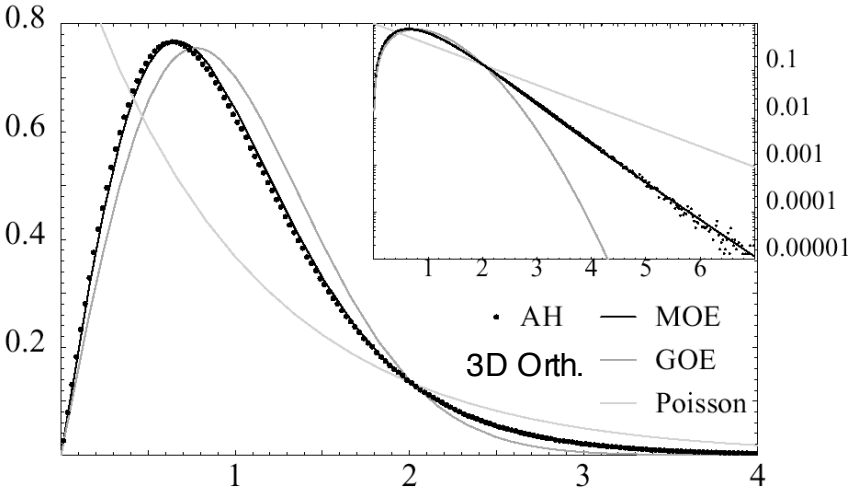
$$P_{\beta=2}(s) = \frac{d^2}{ds^2}\underset{[0,s]}{\mathrm{Det}}(1-K)$$

$P_{\beta=1,4}(s)$ similar

$$K(x,x') = \frac{\sin\pi(x-x')}{(\pi/a)\sinh a(x-x')}$$

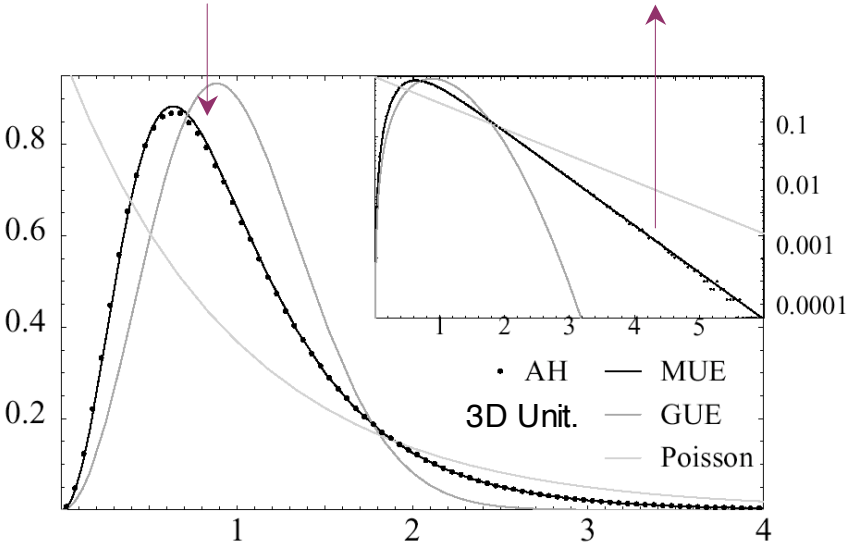


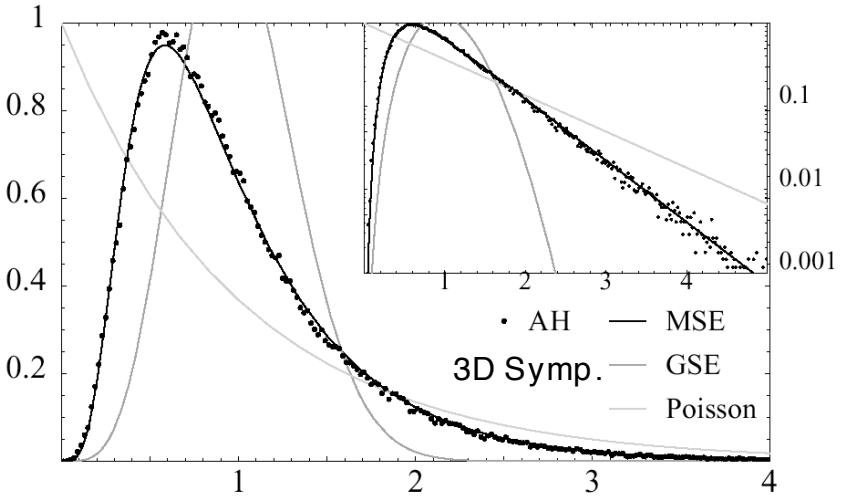
Level spacings : CLS vs dRMT



SN 99

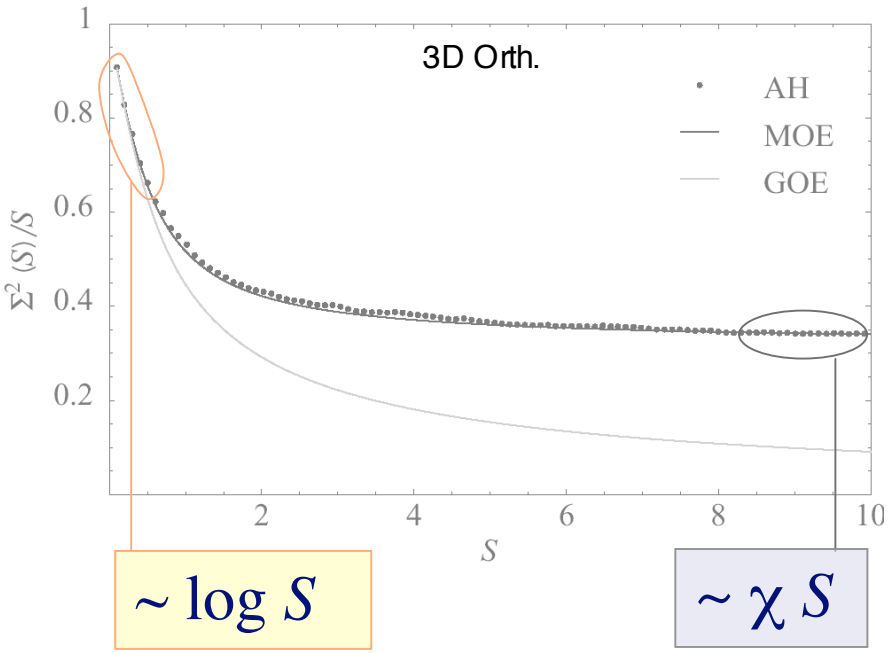
choose a=3.55 from tail fit s>>1





CLS vs dRMT

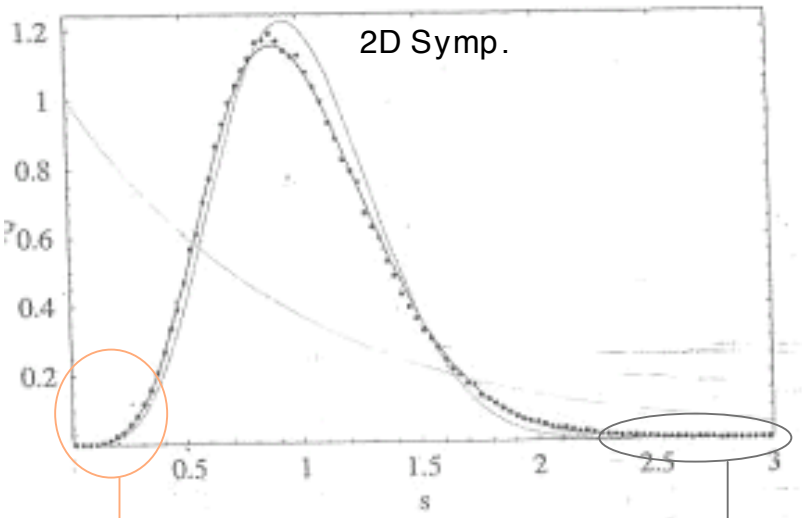
number variance



$\sim \log S$

$\sim \chi S$

level spacings : 2D AH



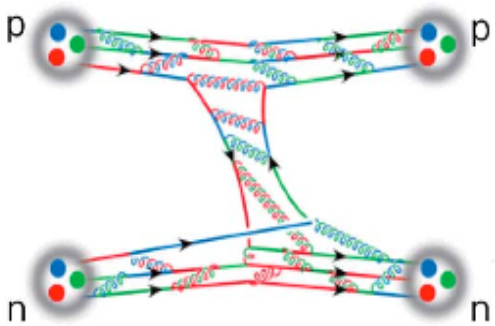
$\sim s^4$

$\sim e^{-s/2\chi}$

Part II

QCD transition & Dirac spectra

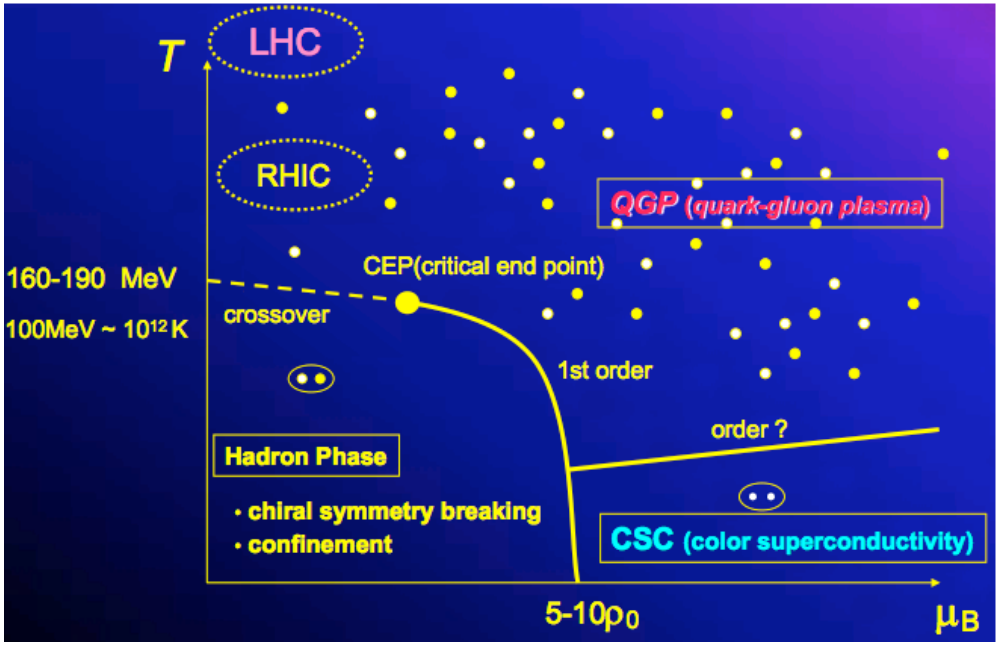
QCD transition



$$S_{\mathrm{QCD}} = \int_0^{1/T} d\tau \int d^3\vec{x} \;\; \beta \,\mathrm{tr}\, F_{\mu\nu}^2 + \bar{q}\overbrace{(i\partial_\mu + A_\mu)\gamma^\mu}^{i\not{D}} q + m\bar{q}q + \mu_{\mathrm{B}} q^{+} q$$

hadron phase

- chiral symm breaking $\langle \bar{q}q \rangle \neq 0$
- color confinement $V_{q\text{-}\bar{q}}(r) \propto r$



Diakonov-Petkov scenario

localization of fermionic W.F. ⇒ chiral symm restoration

chiral quasi-zero mode Ψ on topological b.g.

$$\langle \bar{q} q \rangle = -\lim_{\lambda \to 0} \left(\lim_{V \to \infty} \frac{\pi \rho(\lambda)}{V} \right)$$: χSB needs energy band around origin

- low T: Ψ on instanton b.g.

  extended $\langle \Psi_I | \not{D} | \Psi'_{AI} \rangle \sim \frac{1}{r^3}$ ⇒ level repulsion ⇒ band around $\lambda = 0$

- high T : Ψ on periodic instanton b.g.

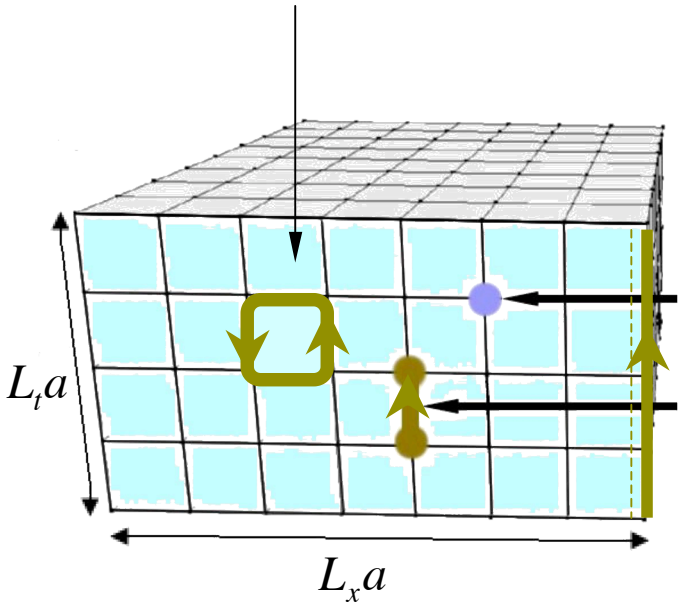
  localized in 3D $\langle \Psi_I | \not{D} | \Psi'_{AI} \rangle \sim e^{-\pi T r}$ ⇒ no level repulsion ⇒ collapse to $\lambda = 0$

  no band

Lattice Gauge Theory

quenched
Boltzmann weight

$$\beta \sum_{\square} \mathrm{tr}\, \underbrace{U_\bullet U_\bullet U_\bullet U_\bullet}_{\square}$$



spinor & N-color

SU(N) variable $U_{n,\mu}$

× Dirac γ_μ

- how to change T

$L_{x,y,z} >> L_t$ fixed

$\beta \nearrow \qquad a \propto \mathrm{e}^{-\beta/b_0} \searrow \qquad T = \dfrac{1}{L_t a} \nearrow$

- what to measure

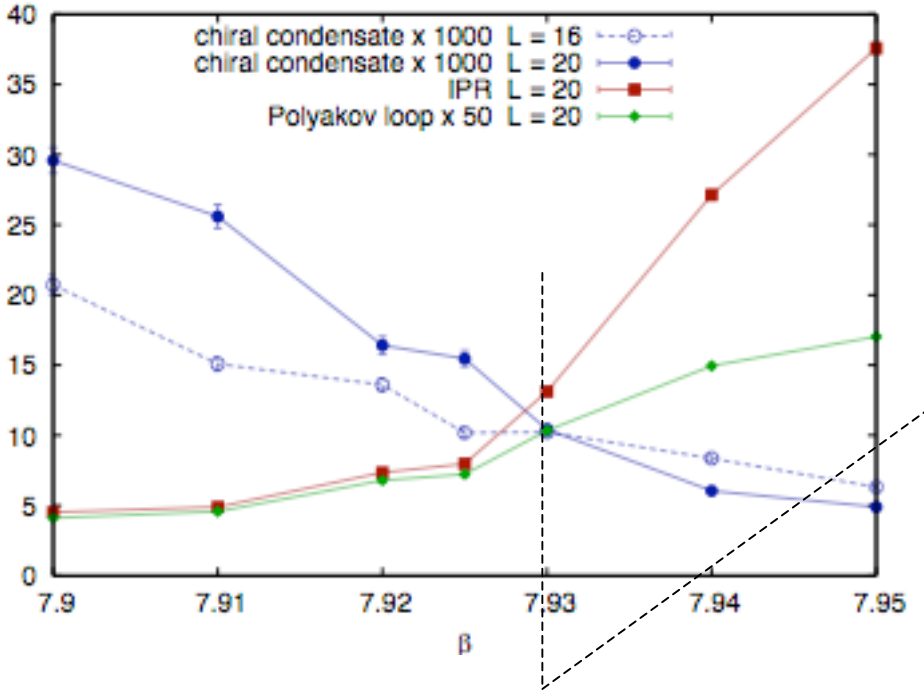
chiral SB $\quad \langle \bar{q} q \rangle = -\pi \rho_D(0)$

confinement $\quad \mathrm{e}^{-F_\mathrm{q}/T} \propto \left\langle \mathrm{tr}\, U_{(\vec{0},0),0} \cdots U_{(\vec{0},L_t),0} \right\rangle$

Dirac spectra at QCD transition

SU(3) quenched LGT
on $16^3 \sim 20^3 \times 4$, KS Dirac op.

Garcia^2-Osborn 07



- chiral symm restoration
- deconfinement transition
- localization

simultaneous!

Dirac spectra at QCD transition

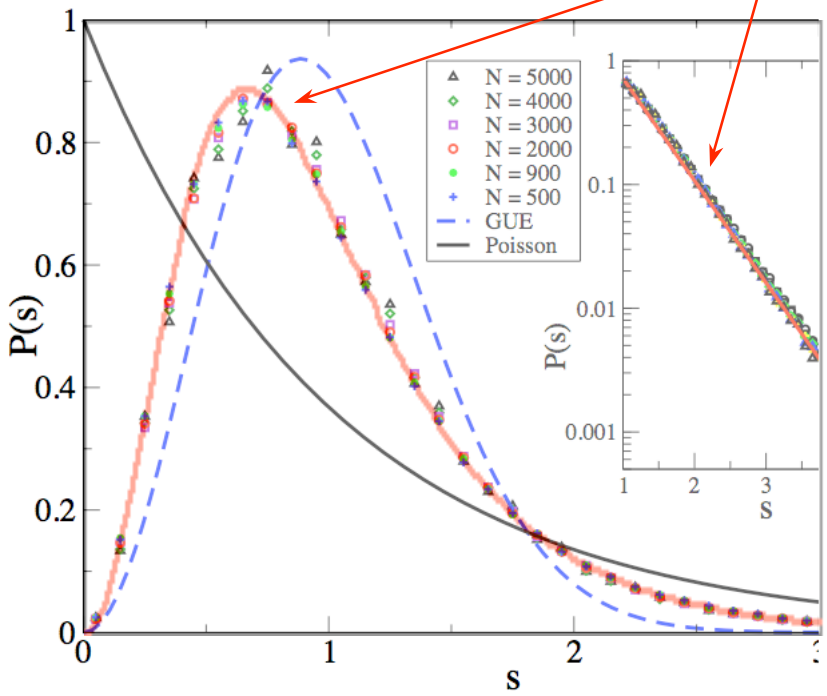
Attn: ILM is a priori semiclassically biased not the real QCD

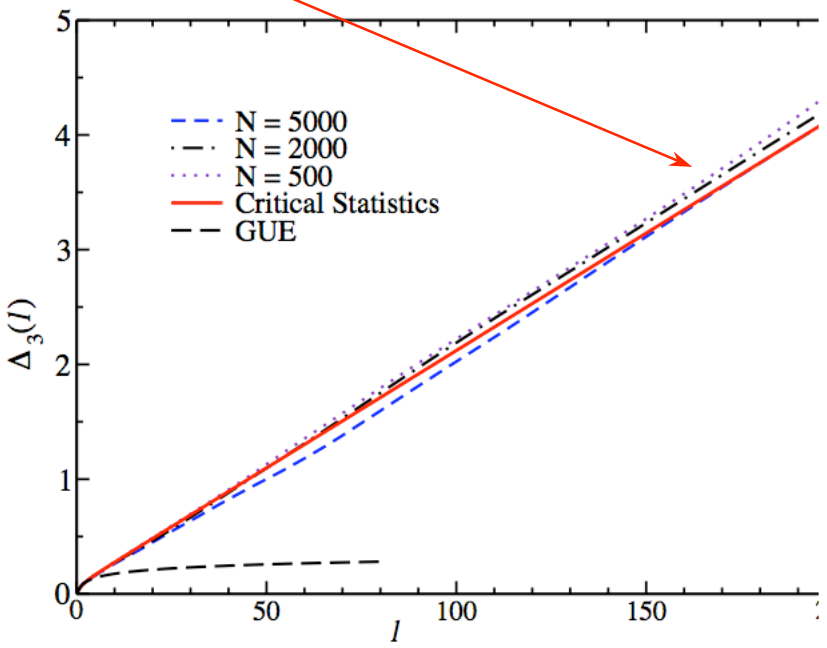
quenched ILM
at $T=\Lambda_{QCD}$, KS Dirac op.

Garcia^2-Osborn 06

unitary dRMT
a=3.2 SN 99

scale-inv critical statistics

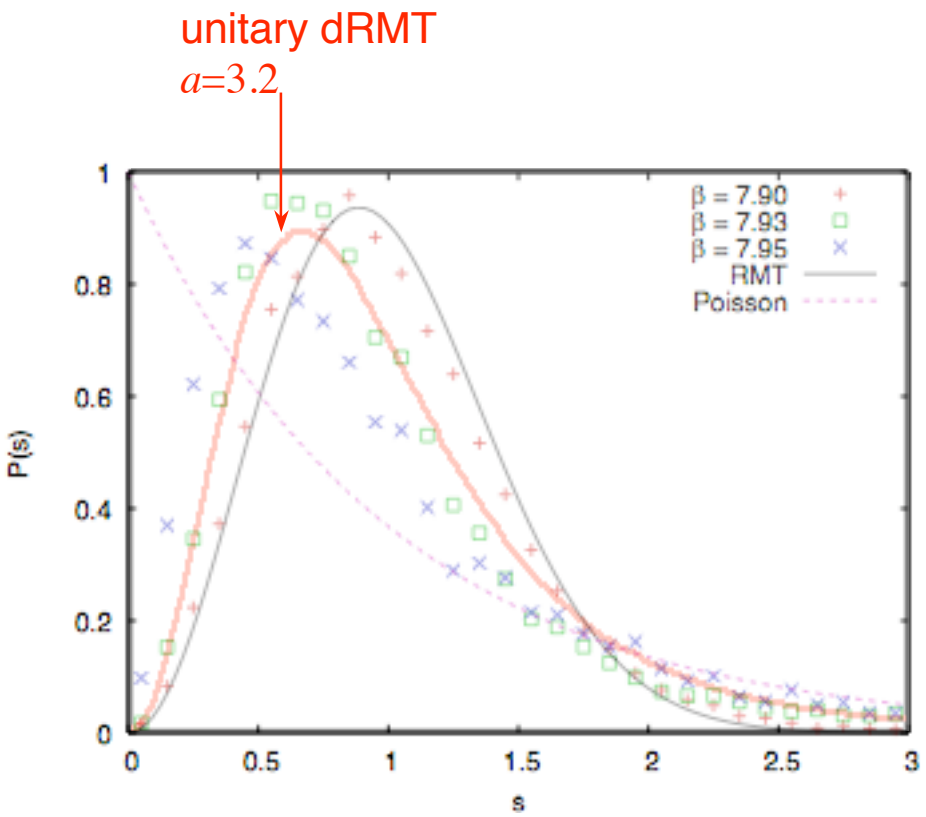




Dirac spectra at QCD transition

SU(3) quenched LGT
on $20^3 \times 4$, at $\beta = 7.93$, KS Dirac op.

Garcia^2-Osborn 07



Dirac spectra at QCD transition

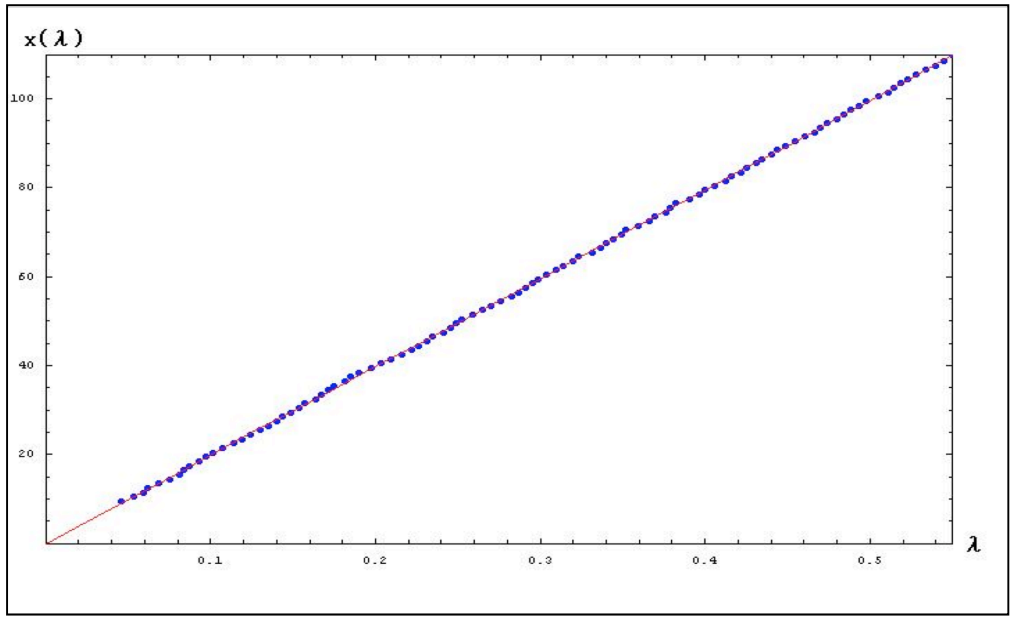
quenched SU(2) LGT
on $(7\times9\times11)\times4$, KS Dirac op.*

Kato-SN 08

2nd order
deconfinement

* extensive study $(T=0)$
by Guhr et al 99

cumulative
EV distribution
$x(\lambda) = \#(\lambda_i \le \lambda)$



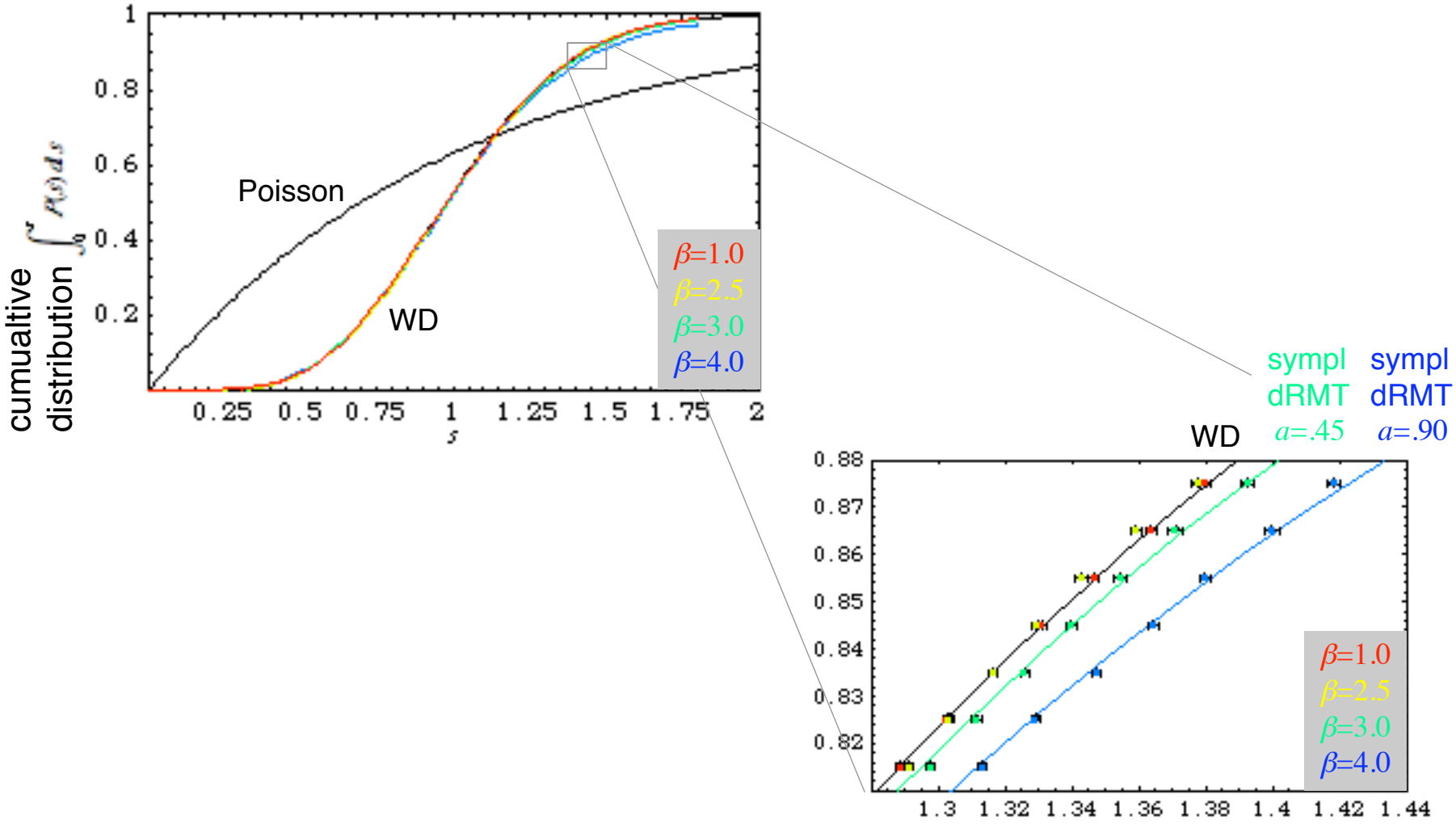
$$x(\lambda) = x_{\rm av}(\lambda) + x_{\rm osc}(\lambda) \longrightarrow s_i = \frac{\lambda_{i+1}-\lambda_i}{\Delta(\lambda_i)} \Rightarrow \frac{\lambda_{i+1}-\lambda_i}{x'_{\rm av}(\lambda_i)^{-1}}$$

polynomial fit for each config.

spectral unfolding

Dirac spectra at QCD transition

SU(2) quenched LGT on $(7\times9\times11)\times4$ Kato-SN 08



Summary

- Diakonov-Petkov scenario:

Localization of Fermionic WF ⇒ QCD Phase Transition

confirmed via Dirac spectra

- Muttalib conjecture:

Critical Level Statistics at Mobility Edge $\overset{\text{phenomenologically}}{\underset{\text{modelled by}}{\sim}}$ Deformed RMT

works both in AH, QCD

thanks: Damgaard, Verbaarschot, Garcia^2, Nagao, Kato - collaborator
Zharekeshev, Schweizer, Kawarabayashi, Evangelou, Ohtsuki - AH data
JSPS - grant