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# DEPHASING IN QUANTUM CHAOTIC TRANSPORT

## A SEMICLASSICAL APPROACH

Dephasing  
&  
Quantum  
Transport

Cyril Petitjean<sup>1,2</sup>

Cyril Petitjean

Transport & Chaos

Dephasing

Microscopic Model

Phenomenological  
Model

Summary

<sup>1</sup>*Institut für Theoretische Physik, Universität Regensburg*  
<sup>2</sup>*Department of Theoretical Physics, University of Geneva*

**Banff, February 2008**

Collaborators :

- Philippe Jacquod (*University of Arizona*)
- Robert S. Whitney (*Institut Laue-Langevin*)



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Alexander von Humboldt  
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## 1 QUANTUM TRANSPORT AND CHAOS

## 2 QUANTUM TRANSPORT AND DEPHASING

- Microscopic Model
- Phenomenological Model



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# QUANTUM TRANSPORT AND CHAOS

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*"The conductance is  
proportional to the  
transmission probability  
through the sample"*

Quantum Mechanics :

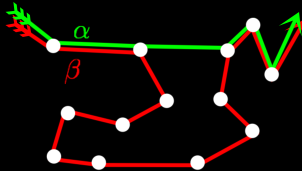
Probability is given by the square of an Amplitude.

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$$G \propto \sum_{\alpha} |A_{\alpha}|^2 + \sum_{\alpha \neq \beta} A_{\alpha} A_{\beta}^*$$

**Classical  
Conductance**

**Quantum  
Interference**

Anderson, Abrahams and Ramakrishnan (1979)

Gorkov, Larkin and Khmel'nitskii (1979)





# QUANTUM TRANSPORT AND CHAOS

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*"The conductance is proportional to the transmission probability through the sample"*

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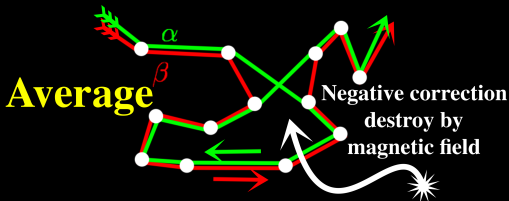
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$$\overline{G} \propto \sum_{\alpha} |A_{\alpha}|^2 + \sum_{\alpha \neq \beta} A_{\alpha} A_{\beta}^*$$

Weak localization

Anderson, Abrahams and Ramakrishnan (1979)

Gorkov, Larkin and Khmel'nitskii (1979)



# QUANTUM TRANSPORT AND CHAOS

## EXPERIMENTS : OPEN BALLISTIC QUANTUM DOT

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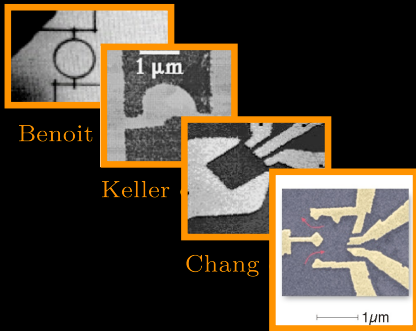
### BALLISTIC CAVITY :

- ✓ Clean sample, very few impurity
- ✓ Randomness / chaoticity  $\Rightarrow$  sample boundaries

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Benoit

Keller

Chang

Marcus et al.



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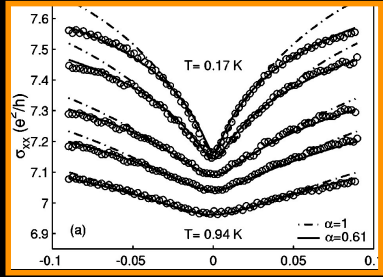
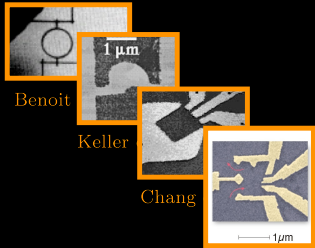
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### BALLISTIC CAVITY :

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Senz et al. (2000)



# WHY BALLISTIC TRANSPORT IS DIFFERENT ?

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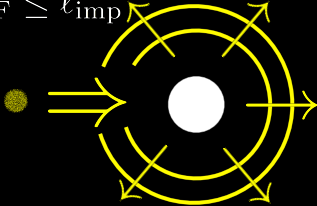
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## ■ Quantum Disorder

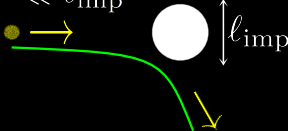
$$\lambda_F \leq l_{\text{imp}}$$



- ✓ Huygens diffraction
- ✓ Universal average properties

## ■ Quantum Chaos

$$\lambda_F \ll l_{\text{imp}}$$



- ✓ Classical trajectories resolved
- ✓ Features of Classical dynamic
- ✓ ( ergodicity + mixing )



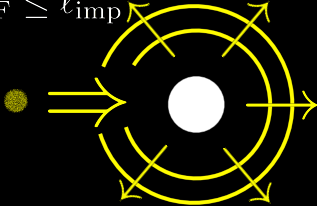
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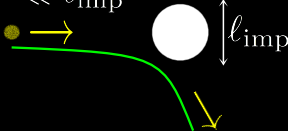
$$\lambda_F \leq l_{\text{imp}}$$



- ✓ Huygens diffraction
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## ■ Quantum Chaos

$$\lambda_F \ll l_{\text{imp}}$$



- !! Classical Chaotic Dynamics !!
- ✓ New "Ehrenfest " time scale
- ✓ Short time correction to the universality

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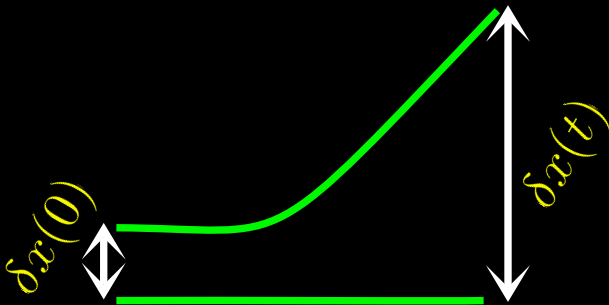
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# THE EHRENFEST TIME SCALE

LARKIN AND OVCHINIKOV, JETP 28, 1200(1969) & BERMAN AND ZASLAVSKY, PHYSICA A 91,450 (1978)

- Classical Chaos  
Local exponential divergence (Lyapunov exponent)
- Quantum Chaos

$$\delta \mathbf{x}(\mathbf{t}) = \delta \mathbf{x}(\mathbf{0}) \exp[\lambda \mathbf{t}]$$





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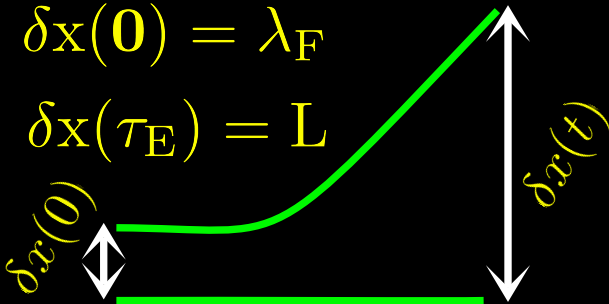
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$$\delta \mathbf{x}(\mathbf{t}) = \delta \mathbf{x}(\mathbf{0}) \exp[\lambda \mathbf{t}]$$

$$\delta \mathbf{x}(\mathbf{0}) = \lambda_F$$

$$\delta \mathbf{x}(\tau_E) = L$$





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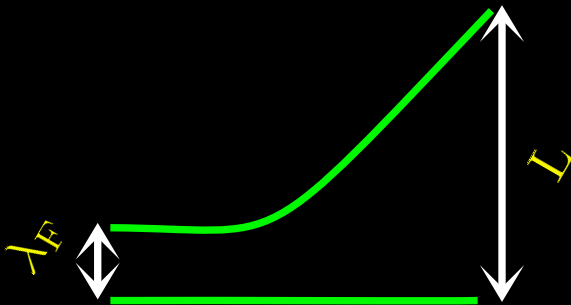
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$$\tau_E \simeq \lambda^{-1} \ln[\mathbf{L}/\lambda_F]$$







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# SCATTERING APPROACH AND SEMICLASSICAL FORMALISM

## Scattering matrix

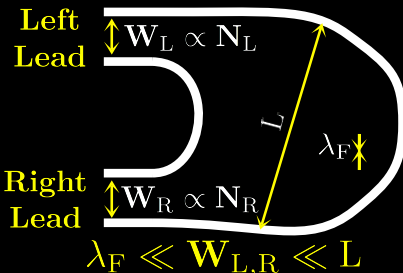
Büttiker, Phys. Rev. B **33**, 3020 (1986).

$$\hat{S} = \begin{pmatrix} \mathbf{r} & \mathbf{t} \\ \mathbf{t}' & \mathbf{r}' \end{pmatrix}$$

## Landauer- Büttiker Formula :

$$\mathbf{g} = \text{Tr}[\mathbf{t}^\dagger \mathbf{t}]$$

## Open Quantum dot





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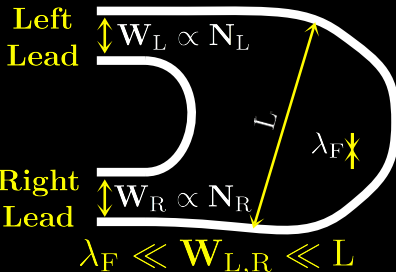
Semiclassic :  $\sum$  over classical paths from mode  $m$  to  $n$

$$t_{nm} = \sum_{\gamma} B_{nm} A_{\gamma}^{\frac{1}{2}} \exp[iS_{\gamma}/\hbar]$$

join

lead-modes

## Open Quantum dot





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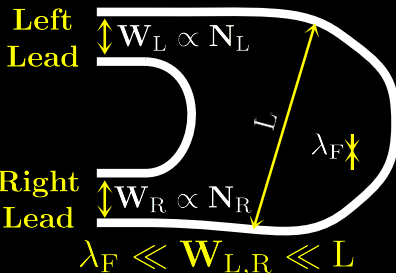
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join  
lead-modes

classical  
action

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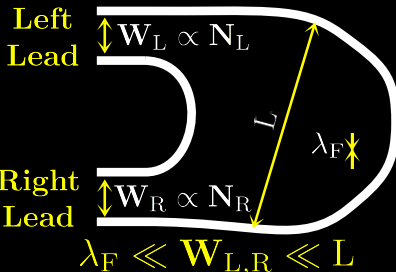
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join  
lead-modes

classical  
stability

classical  
action

## Open Quantum dot





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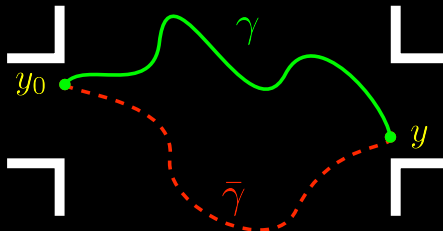
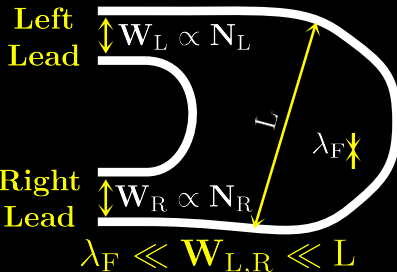
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Left lead

Right lead

## Open Quantum dot





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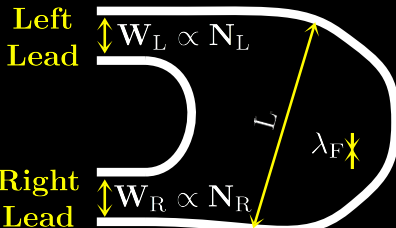
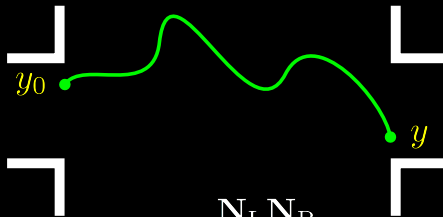
### Landauer- Büttiker Formula :

$$\mathbf{g} = \text{Tr}[\mathbf{t}^\dagger \mathbf{t}]$$

Left lead

$$\gamma = \bar{\gamma}$$

Right lead



$$\lambda_F \ll \mathbf{W}_{L,R} \ll L$$

$$\mathbf{g}_D = \frac{\mathbf{N}_L \mathbf{N}_R}{(\mathbf{N}_L + \mathbf{N}_R)}$$



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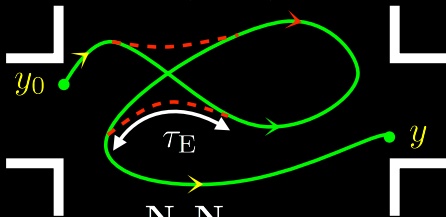
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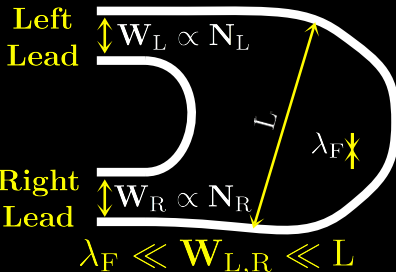
$$\gamma \approx \bar{\gamma}$$

Right lead



$$\delta g = \frac{N_L N_R}{(N_L + N_R)^2} \exp[-\tau_E / \tau_D]$$

Aleiner, Larkin, (1996)  
Sieber, Richter, (2001)  
Adagideli, (2003)  
Rahav, Brouwer, (2005)  
Jacquod, Whitney, (2006)





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# QUANTUM TRANSPORT AND DEPHASING

## WHAT IS LEFT OUT IN THE STANDARD SCATTERING APPROACH ?

- ✓ All dissipative processes occur in the leads
- ✓ Apart from this lead connection System is isolated

Effects due to the coupling with an external environment  
cannot be described

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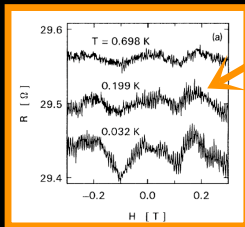
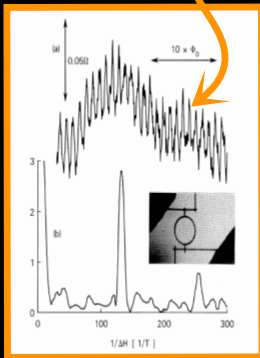
- ✓ All dissipative processes occur in the leads
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Effects due to the coupling with an external environment cannot be described

## Magnetoresistance

Amplitude of oscillations decreases with increasing Temperature

~ Decoherence



Webb, Washburn, Umbach, and Laibowitz, Phys. Rev. Lett. 54, 2696 (1985)



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Treatment of Decoherence  $\neq$  Ways :

**MICROSCOPIC MODEL**  $\Rightarrow$  New scattering formalism

**PHENOMENOLOGICAL MODEL**  $\Rightarrow$  Dephasing Model



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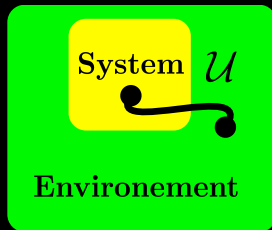
Summary

# STANDARD TREATMENT OF DECOHERENCE

JOOS, ZEH, KIEFER, GIULINI, KUPSCH, AND STAMATESCU, SPRINGER, (2003).

ZUREK, REV. MOD. PHYS. **75**, 715 (2003).

*"There is no fully  
isolated quantum system.  
When we measure one, we implicitly  
intergrate out its environment"*



## Reduced Density Matrix

$$\eta_{\text{Sys}} = \text{Tr}_{\text{Env}}[\eta_{\text{Tot}}]$$

Application to :

- ✓ Coupled chaotic system
  - Jacquod, Phys. Rev. Lett. **92**, 150403 (2004)
  - C.P., Jacquod, Phys. Rev. Lett. **97**, 194103 (2006)
- ✓ Fidelity ( Boltzmann echo)
  - C.P., Jacquod, Phys. Rev. Lett. **97**, 124103 (2006)
- ✓ Quantum Transport, ... Now
  - C.P., Jacquod, Whitney, JETP Lett. **86**, 736 (2007)
  - Whitney, Jacquod, C.P., Phys. Rev. **B 77**, 045315 (2008)



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# SCATTERING FORMALISM IN PRESENCE OF AN ENVIRONMENT

C.P., JACQUOD, WHITNEY, JETP LETT. 86, 736 (2007)

WHITNEY, JACQUOD, C.P., PHYS. REV. B 77, 045315 (2008)

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## Coupled Dot Model

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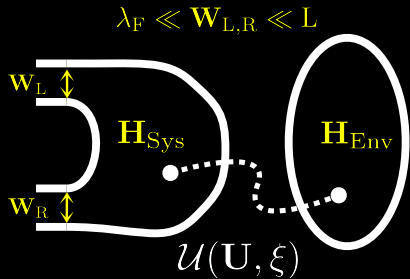
Phenomenological  
Model

Summary

$$\mathcal{H} = \mathbf{H}_{\text{Sys}} + \mathbf{H}_{\text{Env}} + \mathcal{U}(\mathbf{U}, \xi)$$

$$\mathbf{U} = \text{Strength}$$

$$\xi = \text{Correlation length}$$





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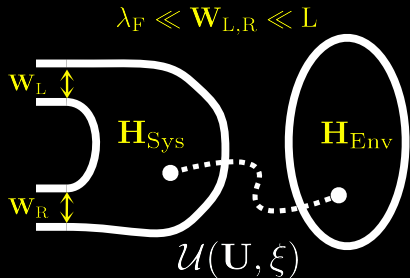
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## Coupled Dot Model

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$\Rightarrow$  New Scattering matrix :

- 1** Include external non- current carrying degrees of freedom
- 2** in a Time-resolved manner .



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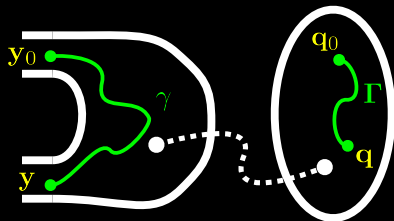
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$$S_{nm}(\mathbf{q}_0, \mathbf{q}) = \sum_{\gamma, \Gamma} \mathcal{B}_{nm} \mathbf{A}_{\Gamma}^{\frac{1}{2}} \mathbf{A}_{\gamma}^{\frac{1}{2}} \exp \left[ i \frac{\mathbf{S}_{\gamma} + \mathbf{S}_{\Gamma}}{\hbar} + i \frac{\mathcal{S}_{\gamma, \Gamma}}{\hbar} \right]$$

Classical Paths  
determined by  $H_{\text{Sys}}, H_{\text{Env}}$



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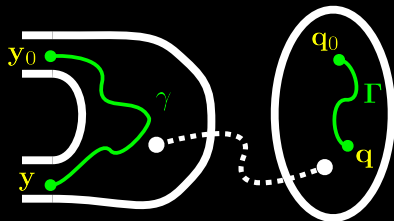
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$$\mathbf{U} = \text{Strength}$$

$$\xi = \text{Correlation length}$$



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$$S_{nm}(\mathbf{q}_0, \mathbf{q}) = \sum_{\gamma, \Gamma} B_{nm} A_{\Gamma}^{\frac{1}{2}} A_{\gamma}^{\frac{1}{2}} \exp \left[ i \frac{\mathbf{S}_{\gamma} + \mathbf{S}_{\Gamma}}{\hbar} + i \frac{\mathbf{S}_{\gamma, \Gamma}}{\hbar} \right]$$



Classical Paths

determined by  $H_{\text{Sys}}, H_{\text{Env}}$



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# SCATTERING FORMALISM IN PRESENCE OF AN ENVIRONMENT

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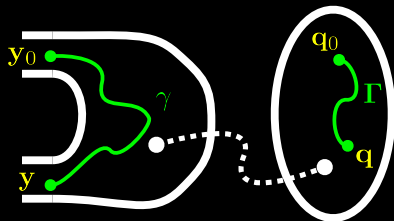
WHITNEY, JACQUOD, C.P., PHYS. REV. B 77, 045315 (2008)

## Coupled Dot Model

$$\mathcal{H} = \mathbf{H}_{\text{Sys}} + \mathbf{H}_{\text{Env}} + \mathcal{U}(\mathbf{U}, \xi)$$

$$\mathbf{U} = \text{Strength}$$

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Summary

$$S_{nm}(\mathbf{q}_0, \mathbf{q}) = \sum_{\gamma, \Gamma} \mathcal{B}_{nm} \mathbf{A}_{\Gamma}^{\frac{1}{2}} \mathbf{A}_{\gamma}^{\frac{1}{2}} \exp \left[ i \frac{\mathbf{S}_{\gamma} + \mathbf{S}_{\Gamma}}{\hbar} + i \frac{\mathcal{S}_{\gamma, \Gamma}}{\hbar} \right]$$

join  
lead-modes

classical  
stability

Classical Paths

determined by  $H_{\text{Sys}}, H_{\text{Env}}$





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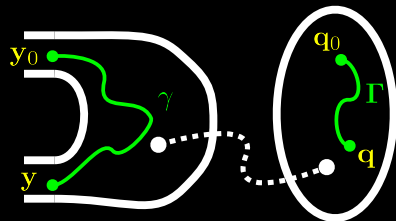
WHITNEY, JACQUOD, C.P., PHYS. REV. B 77, 045315 (2008)

## Coupled Dot Model

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Summary

$$S_{nm}(q_0, q) = \sum_{\gamma, \Gamma} B_{nm} A_{\Gamma}^{\frac{1}{2}} A_{\gamma}^{\frac{1}{2}} \exp \left[ i \frac{\mathbf{S}_{\gamma} + \mathbf{S}_{\Gamma}}{\hbar} + i \frac{\mathbf{S}_{\gamma, \Gamma}}{\hbar} \right]$$

Classical Paths determined by  $H_{\text{Sys}}, H_{\text{Env}}$  (via join lead-modes)

classical stability (via  $A_{\Gamma}^{\frac{1}{2}} A_{\gamma}^{\frac{1}{2}}$ )

One-particle actions (via  $i \frac{\mathbf{S}_{\gamma} + \mathbf{S}_{\Gamma}}{\hbar}$ )

Two-particle action (via  $i \frac{\mathbf{S}_{\gamma, \Gamma}}{\hbar}$ )



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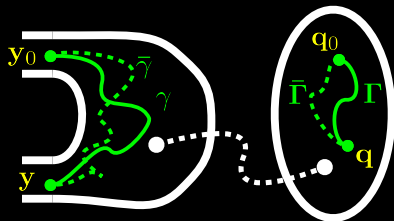
Summary

## Coupled Dot Model

$$\mathcal{H} = \mathbf{H}_{\text{Sys}} + \mathbf{H}_{\text{Env}} + \mathcal{U}(\mathbf{U}, \xi)$$

$$\mathbf{U} = \text{Strength}$$

$$\xi = \text{Correlation length}$$



## Conductance

$$\mathbf{g} = \sum_{\mathbf{nm}} \text{Tr}_{\text{Env}} \left[ \mathbf{S}_{\mathbf{mn}} (\mathbb{I}_{\mathbf{n}} \otimes \eta_{\text{Env}}) \mathbf{S}_{\mathbf{nm}}^* \right]$$

$$\mathbf{g} \propto \int_{\text{time}} \int_{\text{Env}} \int_{\text{Lead}} d \cdots \sum_{\text{paths}} \mathbf{A}_{\gamma \bar{\gamma} \Gamma \bar{\Gamma}}^{\frac{1}{2}} \exp \left[ i \frac{\Phi_{\text{Sys}} + \Phi_{\text{Env}}}{\hbar} + i \frac{\Phi_{\mathbf{U}}}{\hbar} \right]$$



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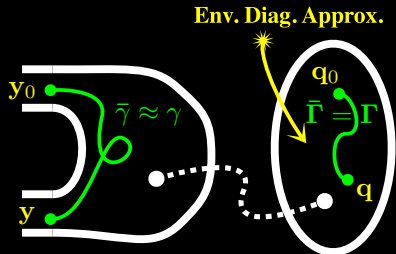
Summary

## Coupled Dot Model

$$\mathcal{H} = \mathbf{H}_{\text{Sys}} + \mathbf{H}_{\text{Env}} + \mathcal{U}(\mathbf{U}, \xi)$$

$$\mathbf{U} = \text{Strength}$$

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## Conductance

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$$\mathbf{g} \propto \int_{\text{time}} \int_{\text{Lead}} d\cdots \sum_{\text{paths}} \mathbf{A}_{\gamma\Gamma} \exp \left[ i \frac{\Delta\Phi_{\text{Sys}}}{\hbar} + i \frac{\Delta\Phi_{\mathcal{U}}}{\hbar} \right]$$



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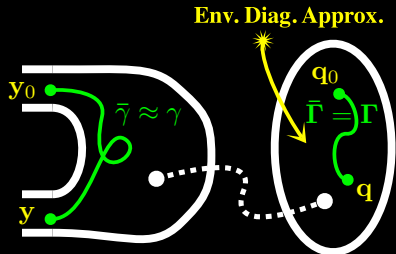
WHITNEY, JACQUOD, C.P., PHYS. REV. B 77, 045315 (200)

## Coupled Dot Model

$$\mathcal{H} = \mathbf{H}_{\text{Sys}} + \mathbf{H}_{\text{Env}} + \mathcal{U}(\mathbf{U}, \xi)$$

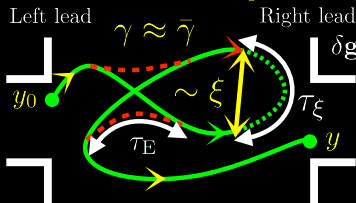
$$\mathbf{U} = \text{Strength}$$

$$\xi = \text{Correlation length}$$



## Conductance

$$g \propto \int_{\text{time}} \int_{\text{Lead}} d \cdots \sum_{\text{paths}} \mathbf{A}_{\gamma \Gamma} \exp \left[ i \frac{\Delta \Phi_{\text{Sys}}}{\hbar} + i \frac{\Delta \Phi_{\mathcal{U}}}{\hbar} \right]$$



$$\delta g = \frac{\mathbf{N}_L \mathbf{N}_R \exp[-\tau_E/\tau_D]}{(\mathbf{N}_L + \mathbf{N}_R)^2} \frac{\exp[-\tau_\xi/\tau_\phi]}{1 + \tau_D/\tau_\phi}$$

$$\tau_\xi = \lambda^{-1} \ln \left[ \left( \mathbf{L}/\xi \right)^2 \right] \neq \tau_E$$



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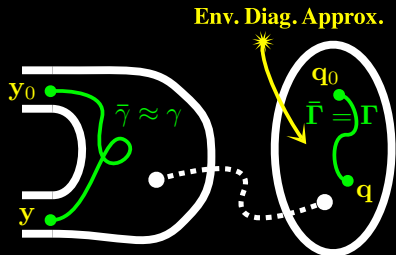
WHITNEY, JACQUOD, C.P., PHYS. REV. B 77, 045315 (2008)

## Coupled Dot Model

$$\mathcal{H} = \mathbf{H}_{\text{Sys}} + \mathbf{H}_{\text{Env}} + \mathcal{U}(\mathbf{U}, \xi)$$

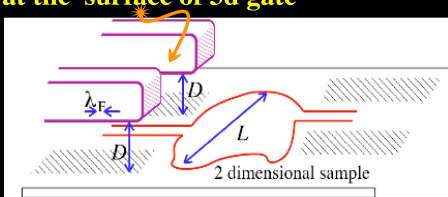
$$\mathbf{U} = \text{Strength}$$

$$\xi = \text{Correlation length}$$



## Diagonal Approximation for Environment $\simeq$ Noise

Noise generated by  
the fluctuations  
at the surface of 3d gate



$$\Delta\Phi_{\mathcal{U}} \Rightarrow \Delta\Phi_{\text{Noise}}$$

$$\delta g = \frac{N_L N_R \exp[-\tau_E/\tau_D]}{(N_L + N_R)^2} \frac{\exp[-\tau_\xi/\tau_\Phi]}{1 + \tau_\xi/\tau_\Phi}$$

Natural cutoff provide by  $D$

$$\xi \propto \sqrt{D}$$



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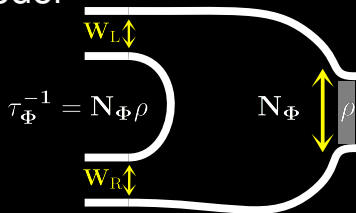
Summary

## Voltage<sup>1</sup>-Dephasing<sup>2</sup> Model

- Add Fictitious probe
- Net current is Null

1. Büttiker, Phys. Rev. B **33**, 3020 (1986)
2. De Jong, Beenakker, Physica A **230**, 219 (1996)

$$\lambda_F \ll W_{L,R} \ll L$$





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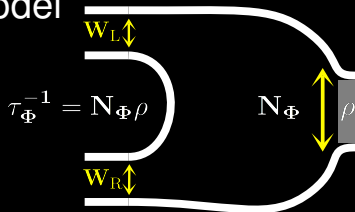
WHITNEY, JACQUOD, C.P., PHYS. REV. B **77**, 045315 (2008)

## Voltage<sup>1</sup>-Dephasing<sup>2</sup> Model

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1. Büttiker, Phys. Rev. B **33**, 3020 (1986)
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$$\lambda_F \ll W_{L,R} \ll L$$



Two terminal conductance : Büttiker, Phys. Rev. B **33**, 3020 (1986)

$$\mathbf{g} = \mathbf{T}_{RL} + \frac{\mathbf{T}_{R\Phi} \mathbf{T}_{\Phi L}}{\mathbf{T}_{R\Phi} + \mathbf{T}_{\Phi L}}$$

$$\text{with, } \mathbf{T}_{ij} = \mathbf{T}_{ij}^D + \delta \mathbf{T}_{ij}$$



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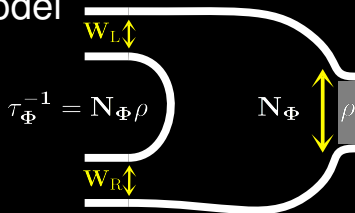
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- Add Fictitious probe
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1. Büttiker, Phys. Rev. B **33**, 3020 (1986)
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$$\lambda_F \ll W_{L,R} \ll L$$



Tunnel barriere semiclassical : Whitney, Phys. Rev. B **75**, 235404 (2007)

$$\delta \mathbf{T}_{ij} = \frac{N_i N_j}{N_T^2} \exp[-\tau_E / \tau_D^*]$$

$$\tau_D^* = \tau_D (1 + \tau_D / \tau_\phi)^{-1}$$





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## Voltage<sup>1</sup>-Dephasing<sup>2</sup> Model

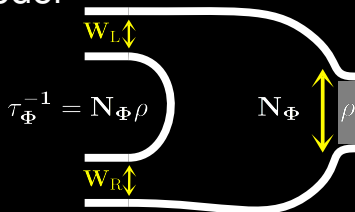
- Add Fictitious probe

- Net current is Null

1. Büttiker, Phys. Rev. B **33**, 3020 (1986)

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$$\lambda_F \ll W_{L,R} \ll L$$



$$\delta g = \frac{N_L N_R \exp[-\tau_E/\tau_D]}{(N_L + N_R)^2} \frac{\exp[-2\tau_E/\tau_\phi]}{1 + \tau_D/\tau_\phi}$$



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# DEPHASING MODEL VERSUS MICROSCOPIC MODEL

C.P., JACQUOD, WHITNEY, JETP LETT. 86, 736 (2007)

WHITNEY, JACQUOD, C.P., PHYS. REV. B 77, 045315 (2008)

$$\delta g = \frac{\mathbf{N}_L \mathbf{N}_R \exp[-\tau_E / \tau_D]}{(\mathbf{N}_L + \mathbf{N}_R)^2} \frac{\exp[-\tau^* / \tau_\phi]}{1 + \tau_D / \tau_\phi}$$

## ■ Microscopic Model

✓ Dephasing is model-dependent

$$\tau^* \propto \lambda^{-1} \ln [L / \xi]$$

## ■ Dephasing Model

✓ No independent parameter  $\xi$

$$\tau^* \propto \lambda^{-1} \ln [L / \lambda_F]$$

✓  $\xi$  is replaced by  $\lambda_F$



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# DEPHASING MODEL VERSUS MICROSCOPIC MODEL

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$$\delta g = \frac{\mathbf{N}_L \mathbf{N}_R \exp[-\tau_E / \tau_D]}{(\mathbf{N}_L + \mathbf{N}_R)^2} \frac{\exp[-\tau^* / \tau_\phi]}{1 + \tau_D / \tau_\phi}$$

## ■ Microscopic Model

✓ Dephasing is model-dependent

$$\tau^* \propto \lambda^{-1} \ln [L / \xi]$$

## ■ Dephasing Model

✓ No independent parameter  $\xi$

$$\tau^* \propto \lambda^{-1} \ln [L / \lambda_F]$$

✓  $\xi$  is replaced by  $\lambda_F$

Dephasing Model cannot mimic a system-environment model with  $\xi \sim L$



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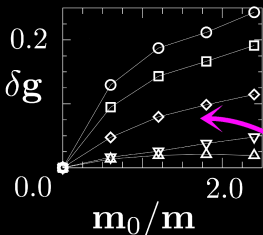
Dephasing

Microscopic Model

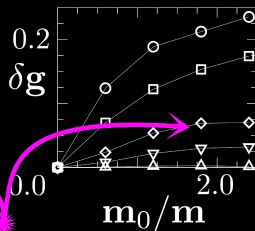
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## Coupled dot



## Dephasing lead



Different  
Behavior

$\infty$   
 $\tau_\phi/\tau_D$   
0



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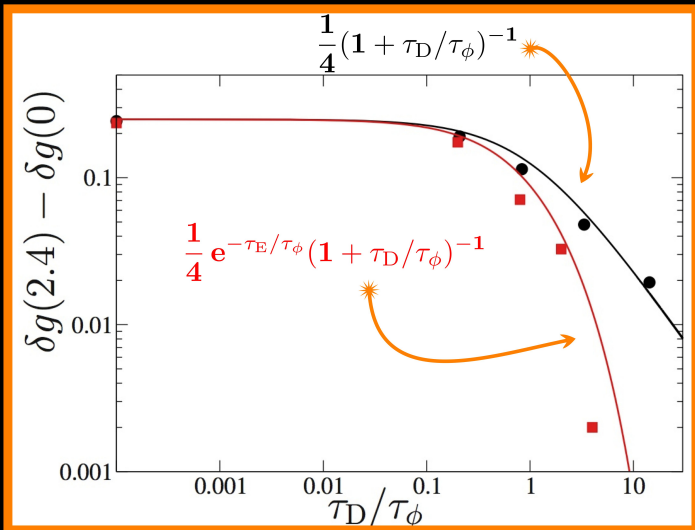
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Summary

Dephasing First trajectory based treatment of an  
✓ environment ("New scattering formalism")  
✓ of the dephasing model.

Dephasing is model-dependent  $\tau^* = \{\tau_\xi, \tau_E\}$

For more see (Shot noise, UCF, ...):

Whitney, Jacquod, C.P.,  
Phys. Rev. **B 77**, 045315 (2008)

See also

Altland, Brouwer, Tian,  
Phys. Rev. Lett. **99**, 036804 (2007)



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## ACKNOWLEDGMENTS

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Dephasing  
&  
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Transport

Cyril Petitjean

- Piet Brouwer
- Markus Büttiker
- Philippe Jacquod
- Mikhail Polianski
- Robert S. Whitney

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# Thank for your attention