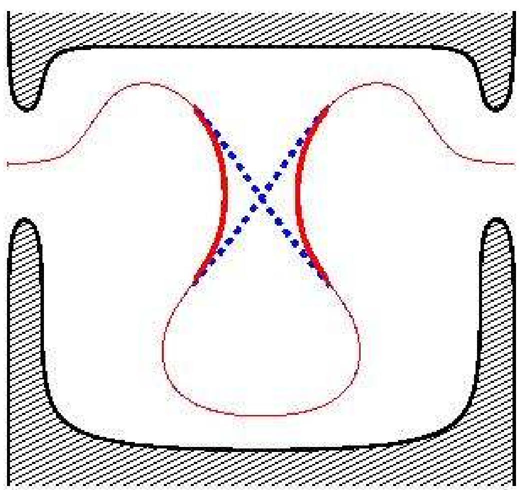
The classical limit of quantum transport

Saar Rahav

Dept. of Chemistry and Biochemistry

University of Maryland



With Piet Brouwer

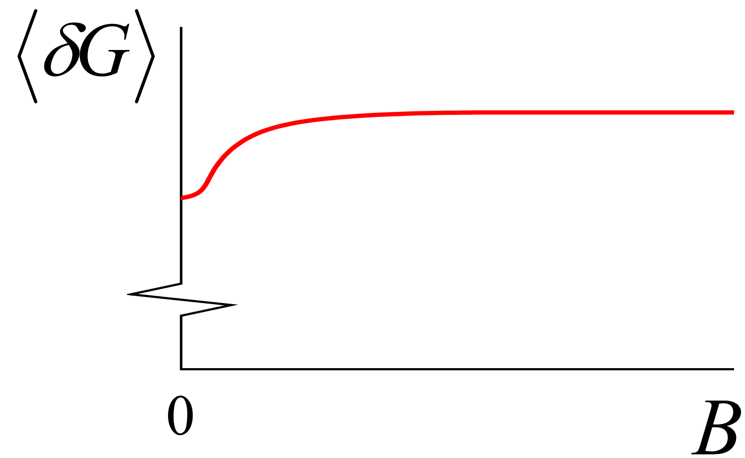
Cornell University

Support:NSF, Packard foundation

Interference effects in transport

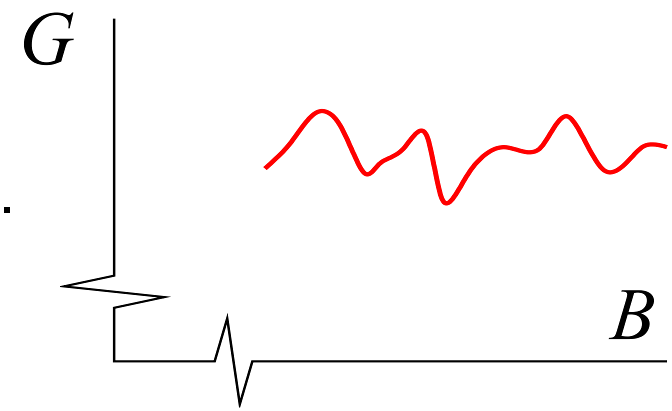
Weak Localization:

Small negative correction for the average conductance at $B = 0$



Conductance fluctuations:

Reproducible fluctuation of the conductance when a parameter is varied.



First studied for disordered samples.

Anderson, Abrahams, Ramakrishnan (1979)
Gorkov, Larkin, Khmelnitskii (1979)
Altshuler (1985)
Lee and Stone (1985)

Ballistic quantum dots

No impurities, smooth boundaries:

Chaotic classical dynamics

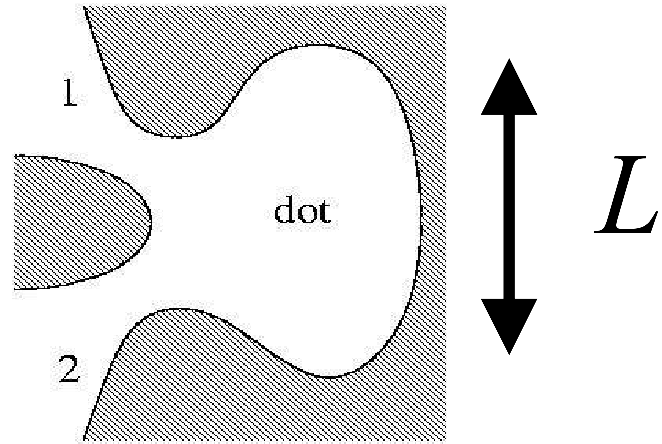
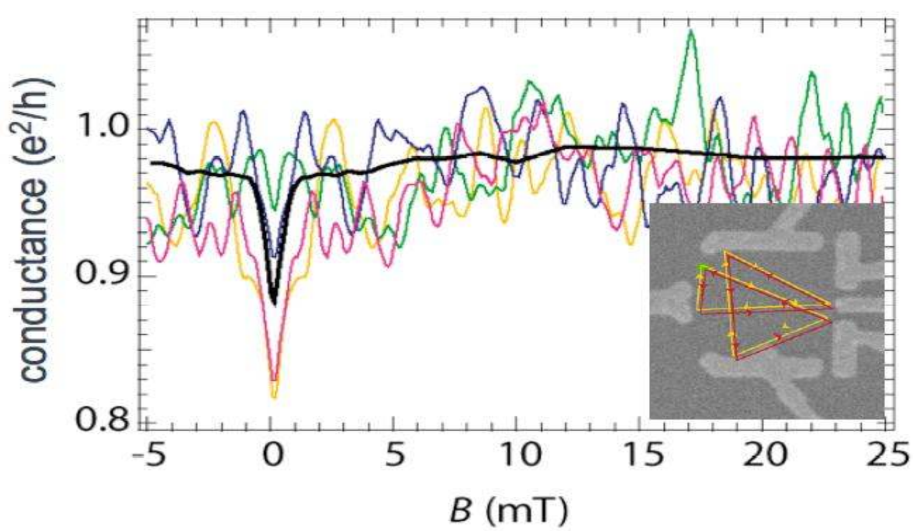


Exhibit interference effects:



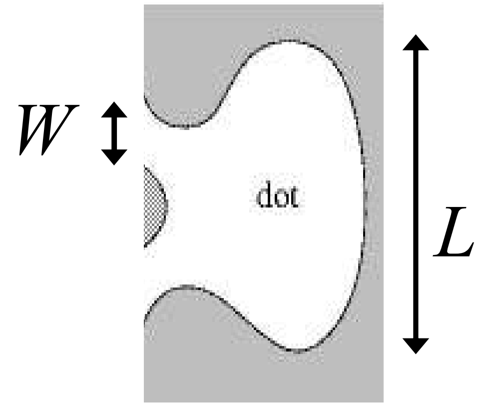
Marcus group

RMT, Universality

Random matrix theory $N = k_F W \gg 1$

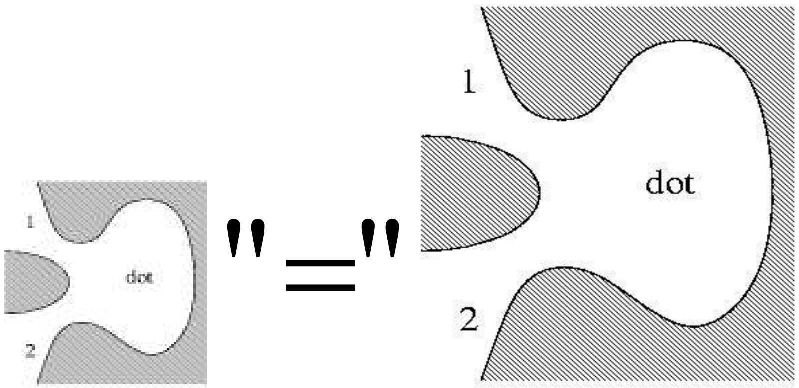
$$\delta G_{\mathrm{RMT}} = \frac{1}{4}\left(\frac{2e^2}{h}\right)$$

$$\mathrm{var}\, G_{\mathrm{RMT}} = \frac{1}{16}\left(\frac{2e^2}{h}\right)^2$$



Jalabert, Pichard, Beenakker (1994)
Baranger and Mello (1994)

RMT: δG and var G do not depend on dot size L



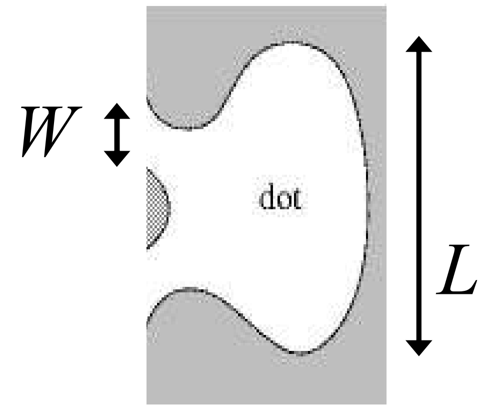
RMT, Universality ???

Semiclassical theory

$$\delta G = \delta G_{\mathrm{RMT}} \, (k_F L)^{-2/\lambda\tau_{\mathrm{D}}}$$

large small

Aleiner, Larkin (1996)



$$\lambda \sim \frac{v_F}{L} \quad \text{Lyapunov exponent}$$

$$\tau_{\mathrm{D}} \sim \frac{L^2}{W v_F} \quad \text{dwell time}$$

$$\frac{1}{\lambda\tau_{\mathrm{D}}} \sim \frac{W}{L}$$

RMT, Universality ???

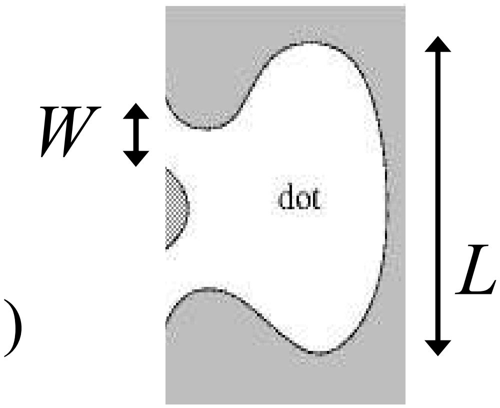
Semiclassical theory

$$\delta G = \delta G_{\mathrm{RMT}} \, (k_F L)^{-2/\lambda\tau_{\mathrm{D}}}$$

Aleiner, Larkin (1996)

$$\delta G = \delta G_{\mathrm{RMT}} \, (k_F L)^{-1/\lambda\tau_{\mathrm{D}}}$$

Adagideli (2003)



Numerical simulations

$$\mathrm{var}\, G = \mathrm{var}\, G_{\mathrm{RMT}}$$

$$\delta G = \delta G_{\mathrm{RMT}}$$

Tworzydlo, Tajic, Beenakker (2004)

Jacquod and Sukhorukov (2004)

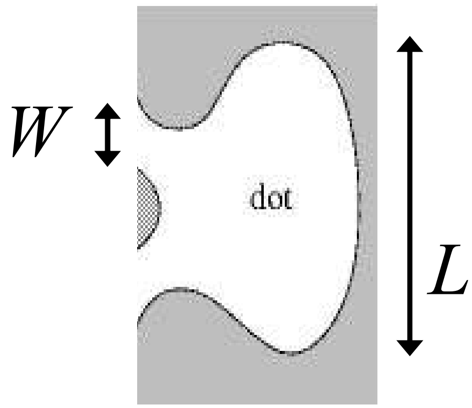
Theory and numerics disagree!

RMT, Universality ???

Semiclassical theory

$$\delta G = \delta G_{\mathrm{RMT}} \, (k_F L)^{-2/\lambda\tau_{\mathrm{D}}}$$

$$\delta G = \delta G_{\mathrm{RMT}} \, (k_F L)^{-1/\lambda\tau_{\mathrm{D}}}$$



Numerical simulations

$$\mathrm{var}\, G = \mathrm{var}\, G_{\mathrm{RMT}}$$

$$\delta G = \delta G_{\mathrm{RMT}}$$

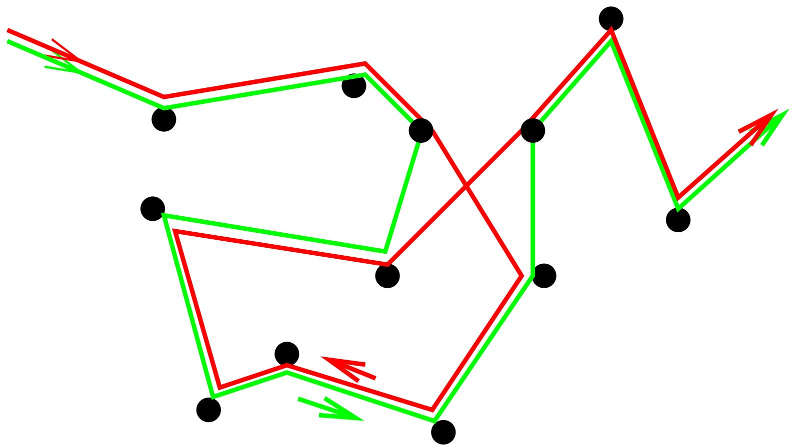
This talk: Classical correlations for

- Weak localization - δG
- Conductance fluctuation - $\mathrm{var}\, G$

Remark: Parametric correlations

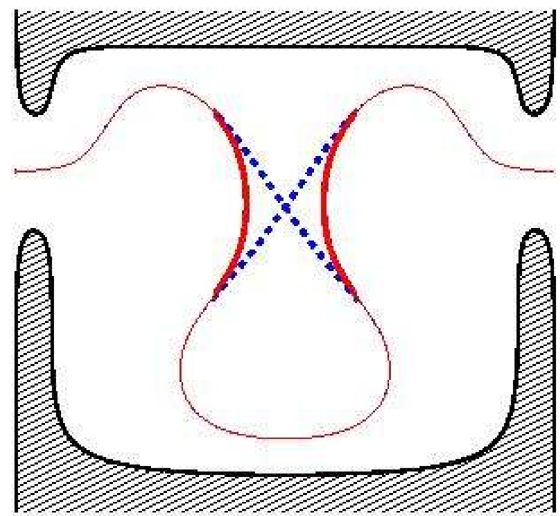
Semiclassical picture of weak localization

Disordered systems:



Anderson, Abrahams, Ramakrishnan (1979), Gorkov, Larkin, Khmelnitskii (1979)

Ballistic systems:

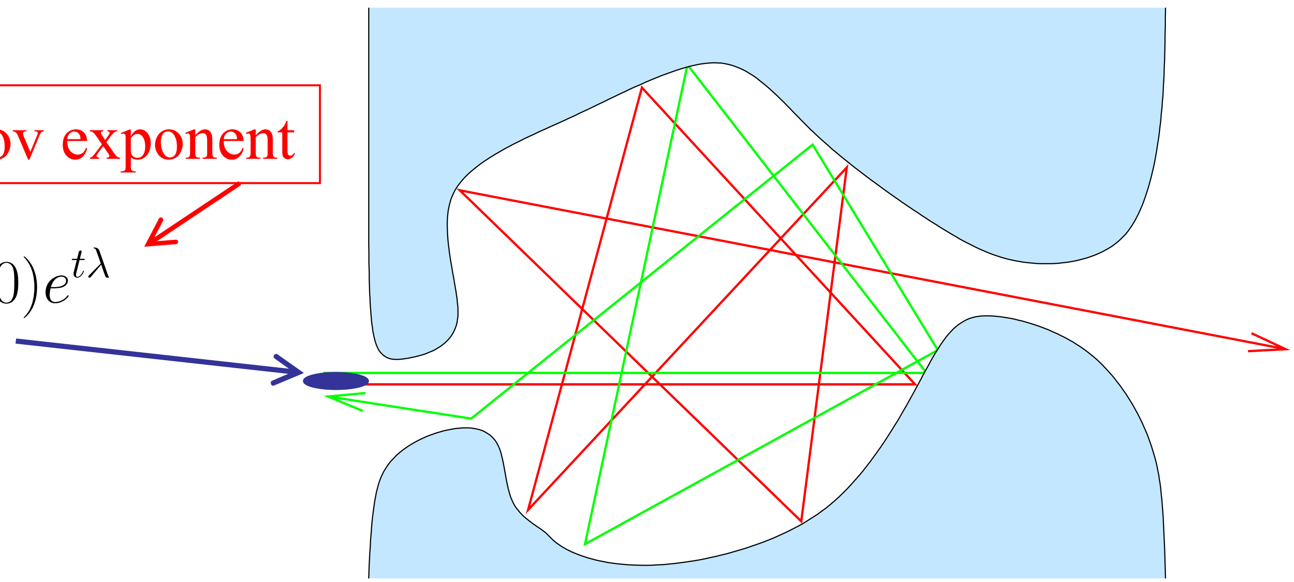


Aleiner and Larkin (1996), Richter and Sieber (2001).

A 'small region' responsible for phase difference

λ: Lyapunov exponent

$$\delta\mathbf{r}(t) = \delta\mathbf{r}(0)e^{t\lambda}$$



Quantum uncertainty in position or direction of incoming wavepacket is magnified by chaotic boundary scattering.

Aleiner and Larkin (1996)

Ehrenfest time

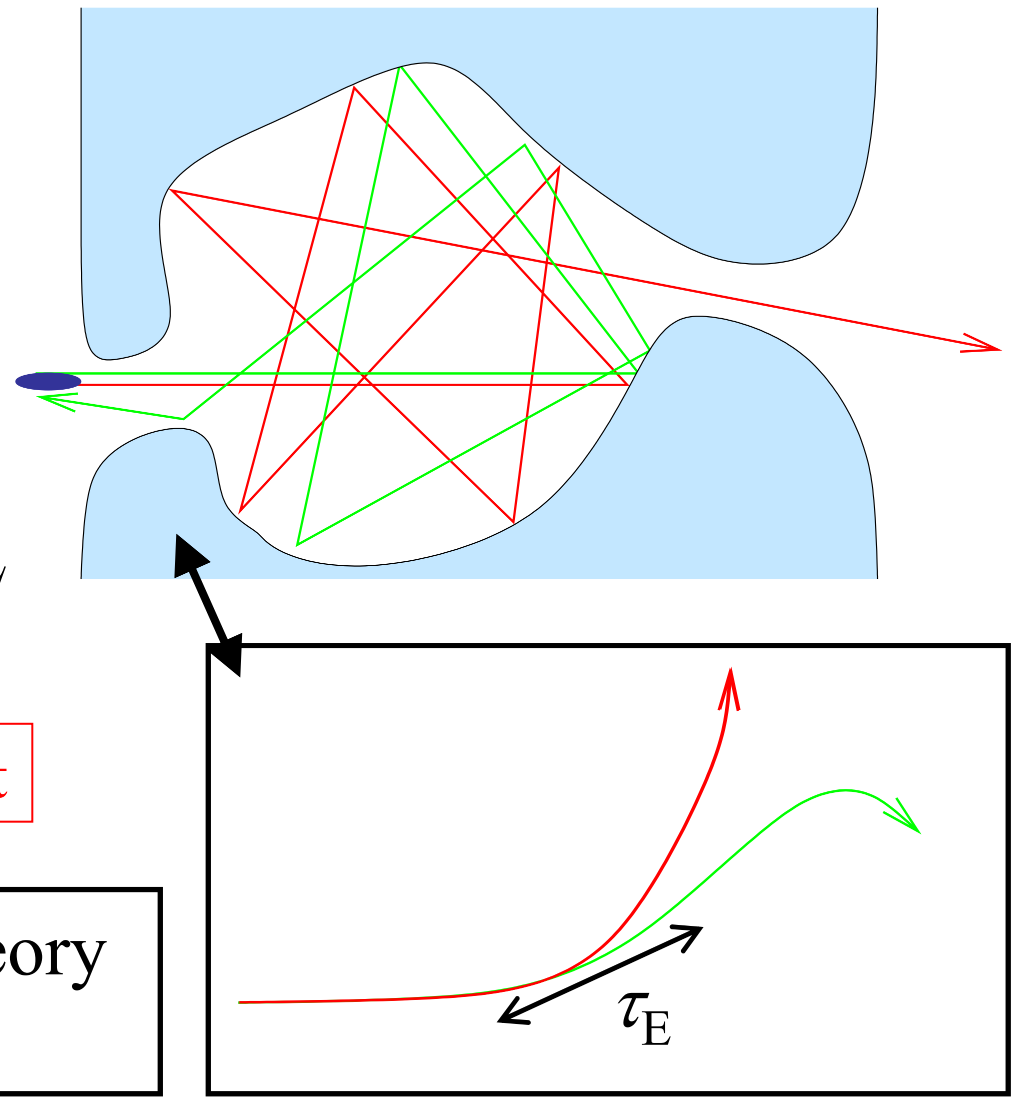
\`Ehrenfest time' τ_E:
Time until initial uncertainty $1/k_F$ has reached dot size L:

$$L = \frac{1}{k_F} e^{\lambda \tau_E}$$

$$\tau_E = \frac{1}{\lambda} \ln k_F L$$

λ: Lyapunov exponent

Random Matrix Theory valid if $\tau_E << \tau_D$



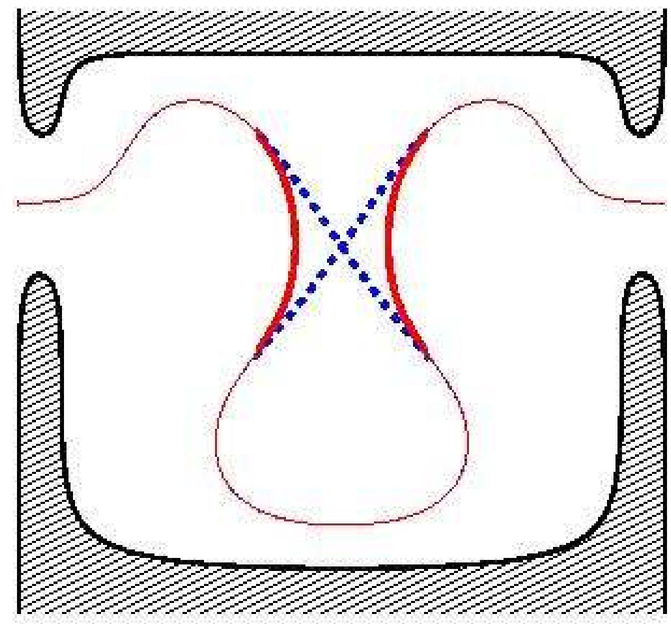
Theory for $\tau_E << \tau_D$

$$G = \frac{2e^2}{h} \sum_{m,n} |t_{mn}|^2$$

Landauer formula

$$|t_{mn}|^2 \sim \sum_{\alpha,\beta} A_\alpha A_\beta e^{i(\mathcal{S}_\alpha - \mathcal{S}_\beta)/\hbar},$$

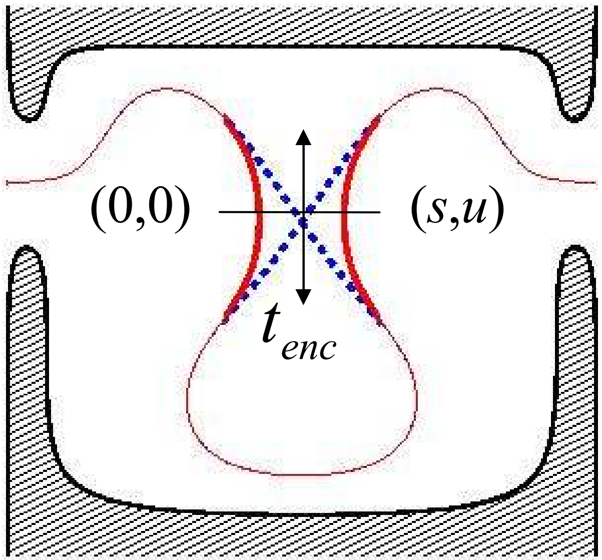
Jalabert, Baranger, Stone (1990)

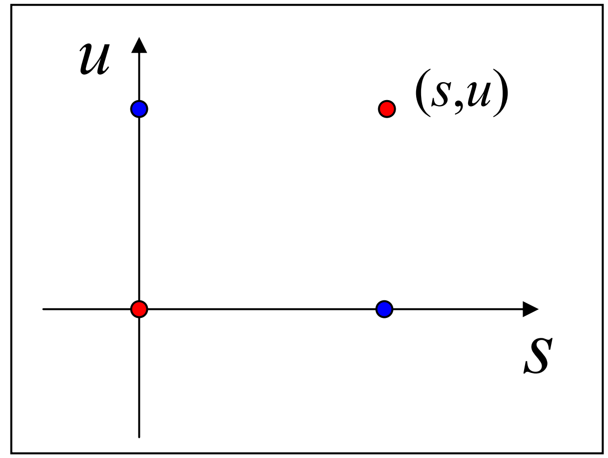


- S_α, S_β: classical action
- angles of α, β consistent with transverse momentum in lead,

$$p_\perp(m) = \pm\pi\hbar m / W_j, \quad m = 1, \ldots, N_j,$$

- A_α, A_β: stability amplitudes





s, u: distances along stable, unstable phase space directions

encounter region: $|s|,|u| < c$

c: classical cut-off scale

$$t_{\text{enc}} = \frac{1}{\lambda}\left(\ln\frac{c}{|s|} + \ln\frac{c}{|u|}\right)$$

Action difference S_α-$S_\beta = su$

$s \propto e^{-\lambda t}$
$u \propto e^{\lambda t}$

Richter and Sieber (2003)
Spehner (2003)
Turek and Richter (2003)
Müller *et al*. (2004)
Heusler *et al*. (2006)

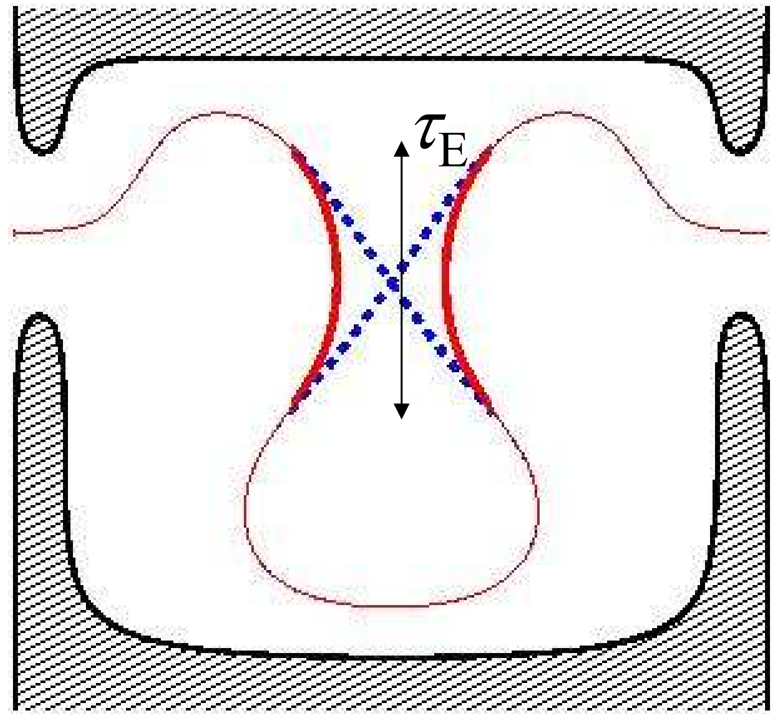
Theory for $\tau_E \sim \tau_D$

Encounters have 'size':

$$\mathcal{S}_\alpha - \mathcal{S}_\beta \simeq \hbar$$

⇓

$$t_{\rm enc} \simeq \frac{1}{\lambda} \ln \frac{c^2}{\hbar} = \tau_{\rm E}$$

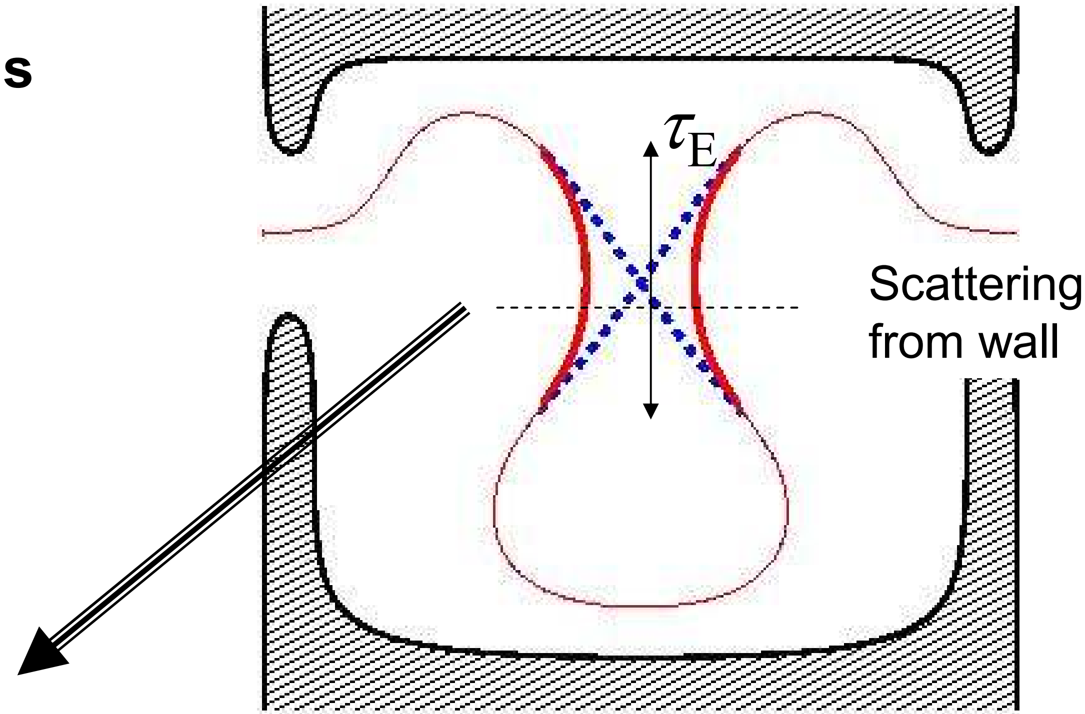


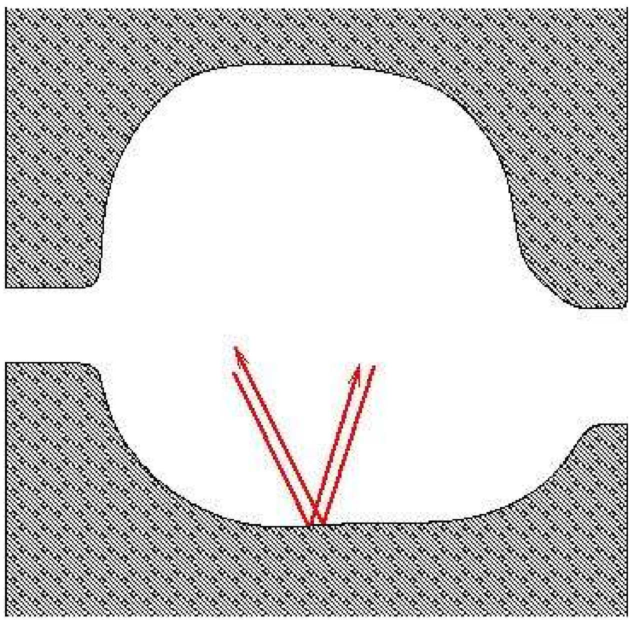
Uncorrelated escape: $P(t) = e^{-t/\tau_{\rm D}}$

$$P_{stay} = ??$$

Theory for $\tau_E \sim \tau_D$

Correlated motion means correlated escape !

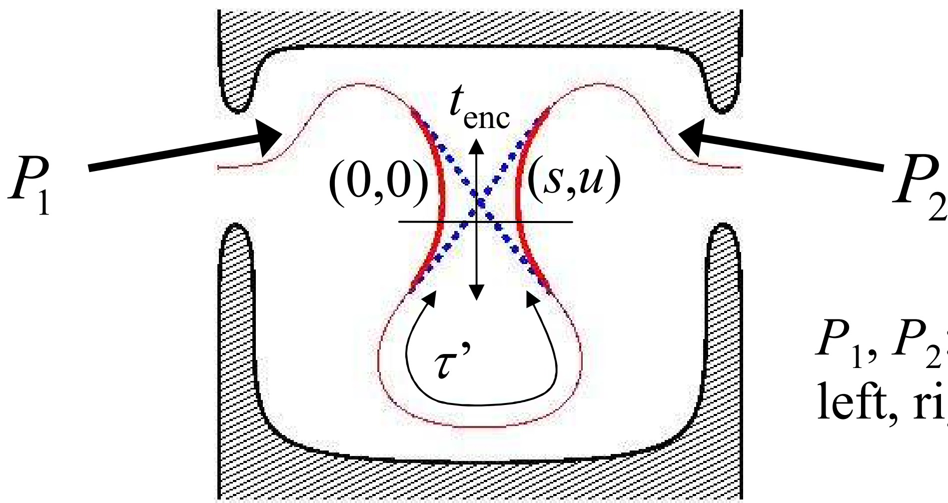




$$P_{stay} = e^{-t_{\rm enc}/\tau_{\rm D}} \simeq e^{-\tau_{\rm E}/\tau_{\rm D}}$$

Not $e^{-2\tau_{\rm E}/\tau_{\rm D}}$

Theory for $\tau_E \sim \tau_D$

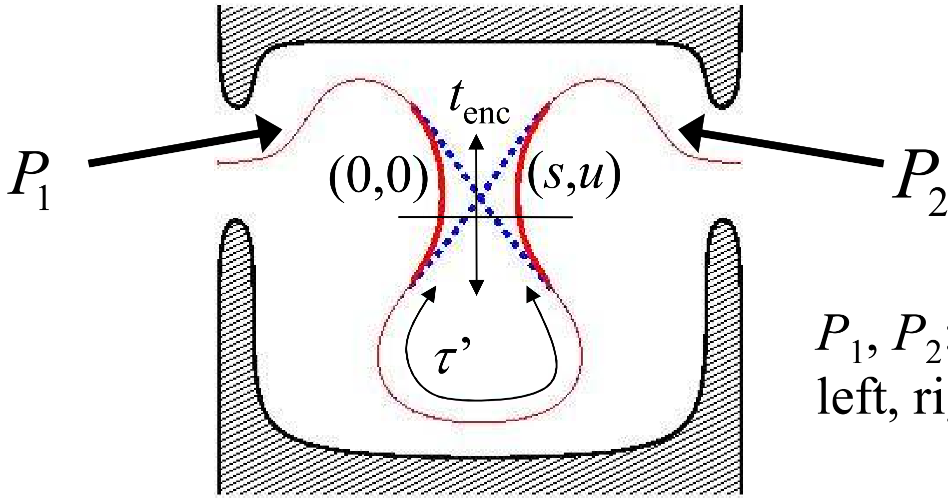


P_1, P_2: probabilities to enter, exit through left, right contacts

$$\delta G = -\frac{2e^2}{h} P_1 P_2 \int d\tau' \int_{-c}^{c} ds du \frac{e^{isu/\hbar - (t_{\text{enc}}+\tau')/\tau_{\text{D}}}}{2\pi\hbar t_{\text{enc}}}$$
$$= \frac{2e^2}{h} P_1 P_2 e^{-\tau_{\text{E}}/\tau_{\text{D}}}$$

SR and Brouwer (2005), Brouwer and SR (2006), Jacquod and Whitney (2006).

Theory for $\tau_E \sim \tau_D$



P_1, P_2: probabilities to enter, exit through left, right contacts

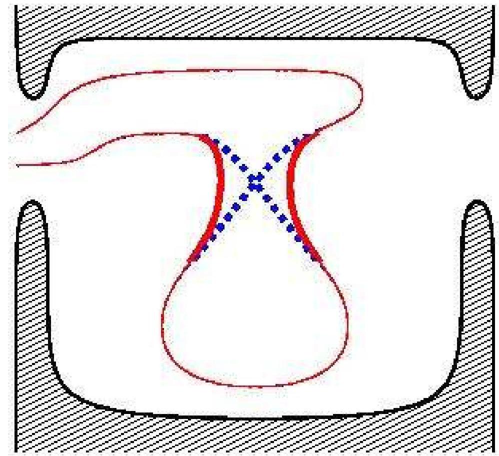
$$\begin{aligned} \delta G &= \delta G_{\rm RMT} e^{-\tau_{\rm E}/\tau_{\rm D}} \\ &= \delta G_{\rm RMT} \left(k_{\rm F} L\right)^{-1/\lambda\tau_{\rm D}} \end{aligned}$$

SR and Brouwer (2005), Brouwer and SR (2006), Jacquod and Whitney (2006).

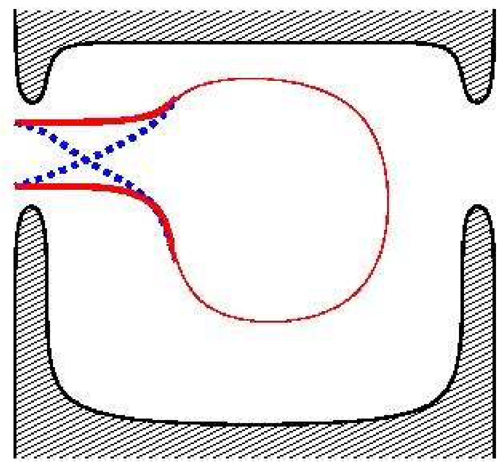
Unitarity

δG can also be calculated through the dot's reflection

$$\begin{aligned}\delta G &= -2\frac{e^2}{h}P_1^2 e^{-\tau_E/\tau_D} + 2\frac{e^2}{h}P_1 e^{-\tau_E/\tau_D} \\ &= 2\frac{e^2}{h}P_1\,(1-P_1)\,e^{-\tau_E/\tau_D} \\ &= \delta G_{RMT} e^{-\tau_E/\tau_D}\end{aligned}$$



Weak localization



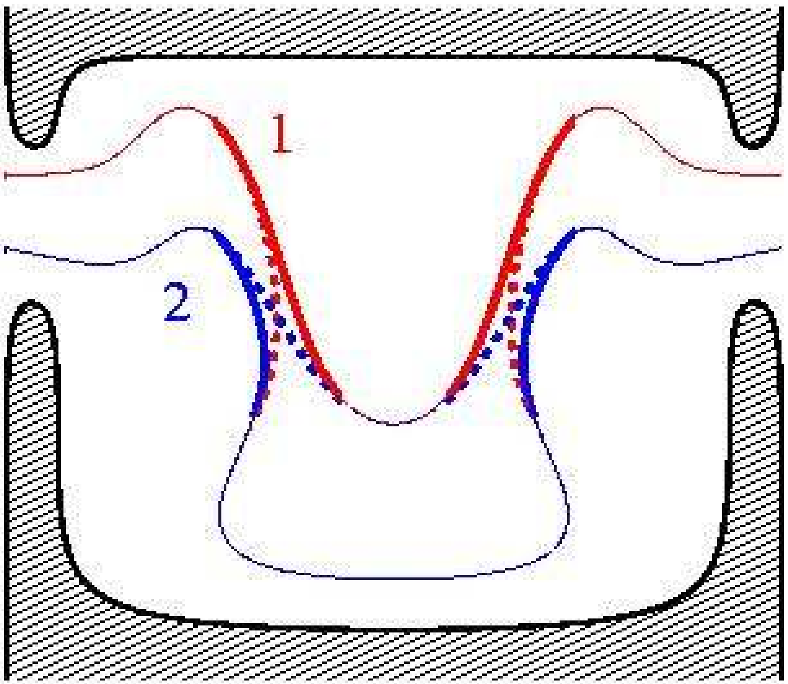
Coherent backscattering

P_1, P_2=1-P_1: probabilities to enter, exit through left, right contacts

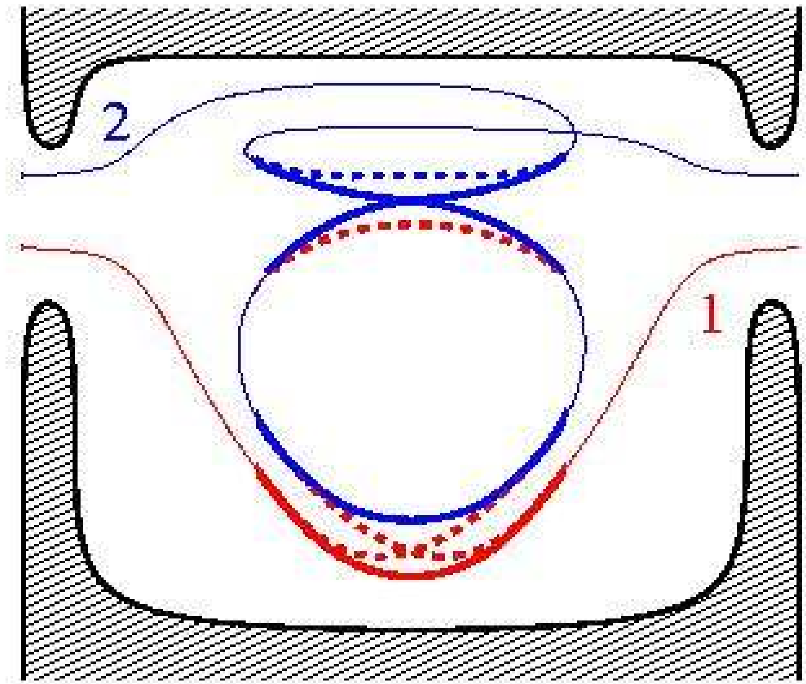
Conductance fluctuations

Two classes of trajectories:

"Diffusion constant"

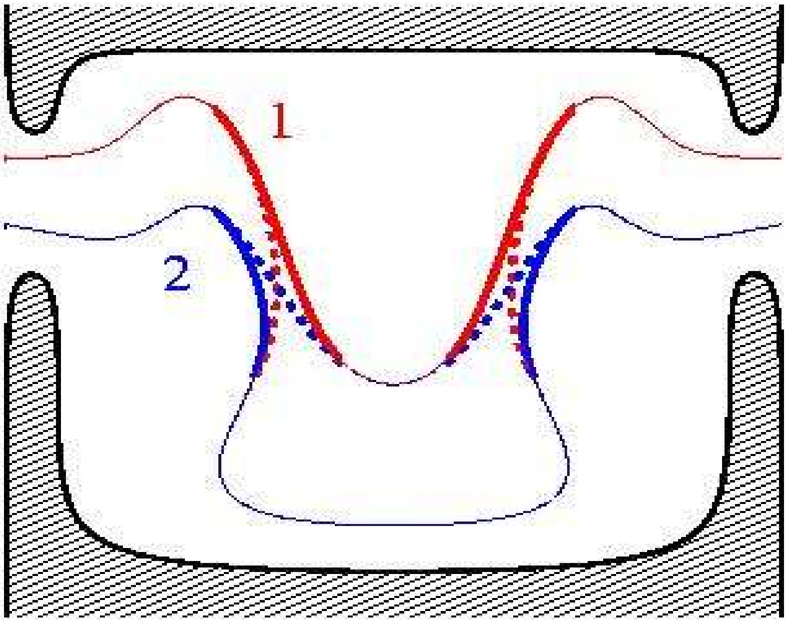


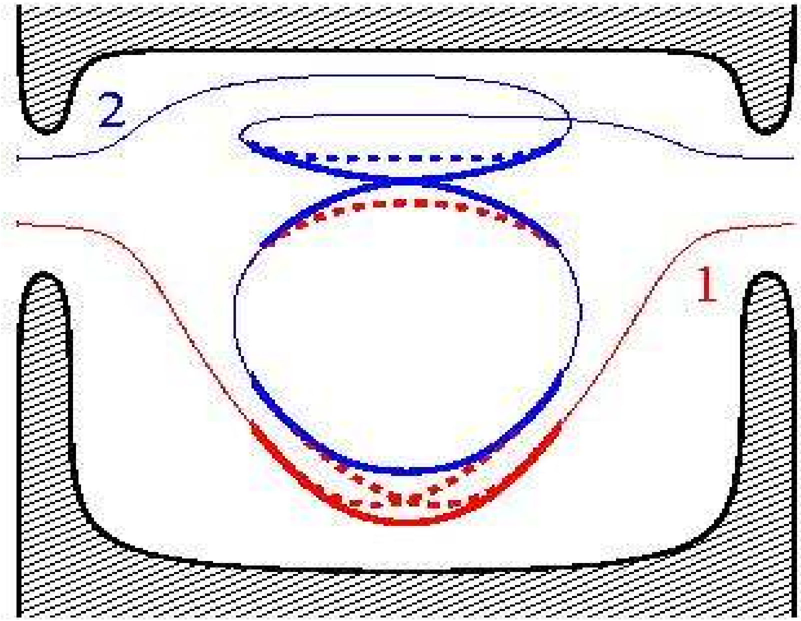
"Density of states"



RMT: 1 : 0

Conductance fluctuations





Semiclassical theory:

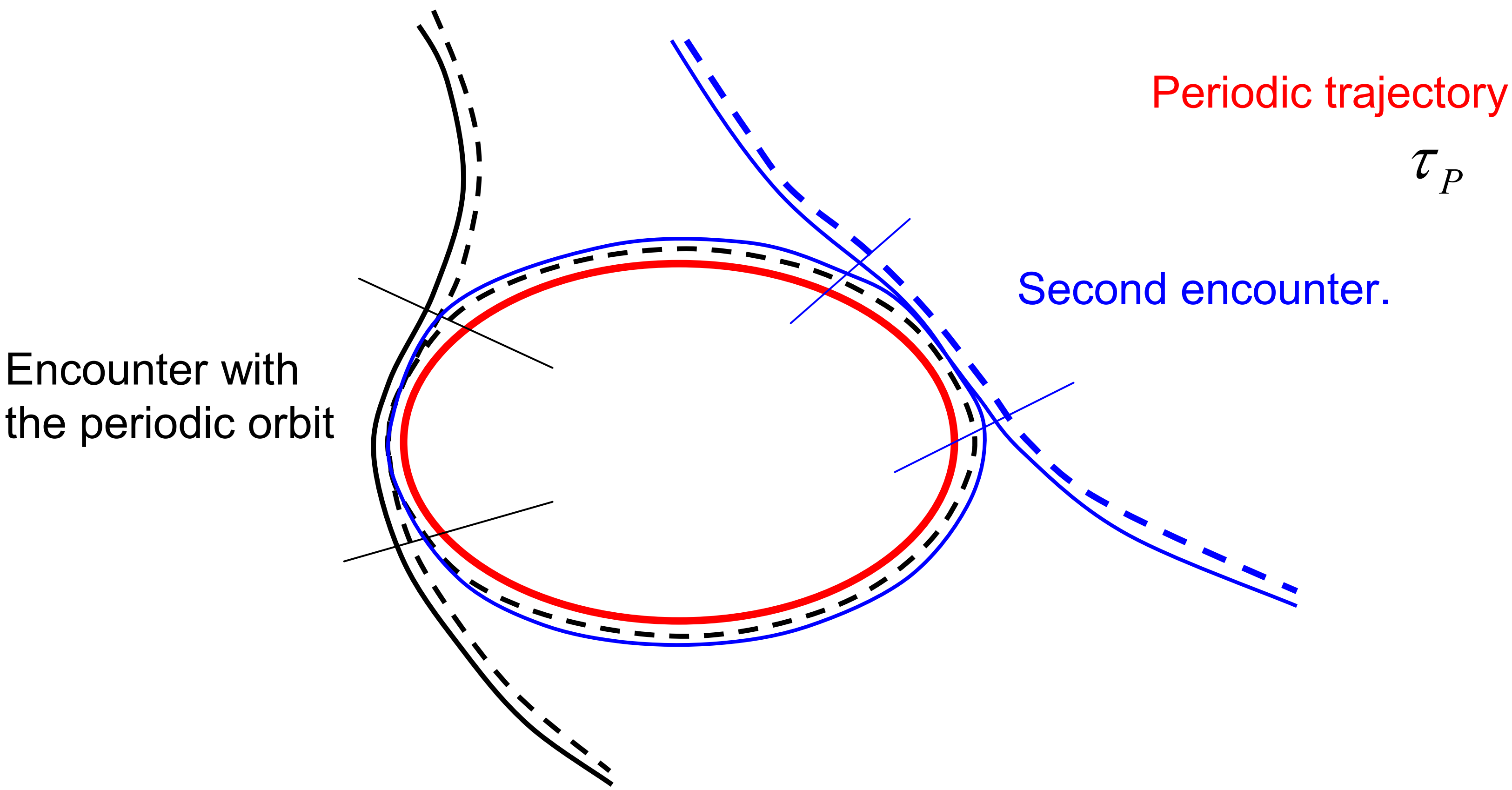
$$\mathrm{var}\, G_{\mathrm{RMT}} \times e^{-2\tau_{\mathrm{E}}/\tau_{\mathrm{D}}} \quad + \quad \mathrm{var}\, G_{\mathrm{RMT}} \times (1 - e^{-2\tau_{\mathrm{E}}/\tau_{\mathrm{D}}})$$

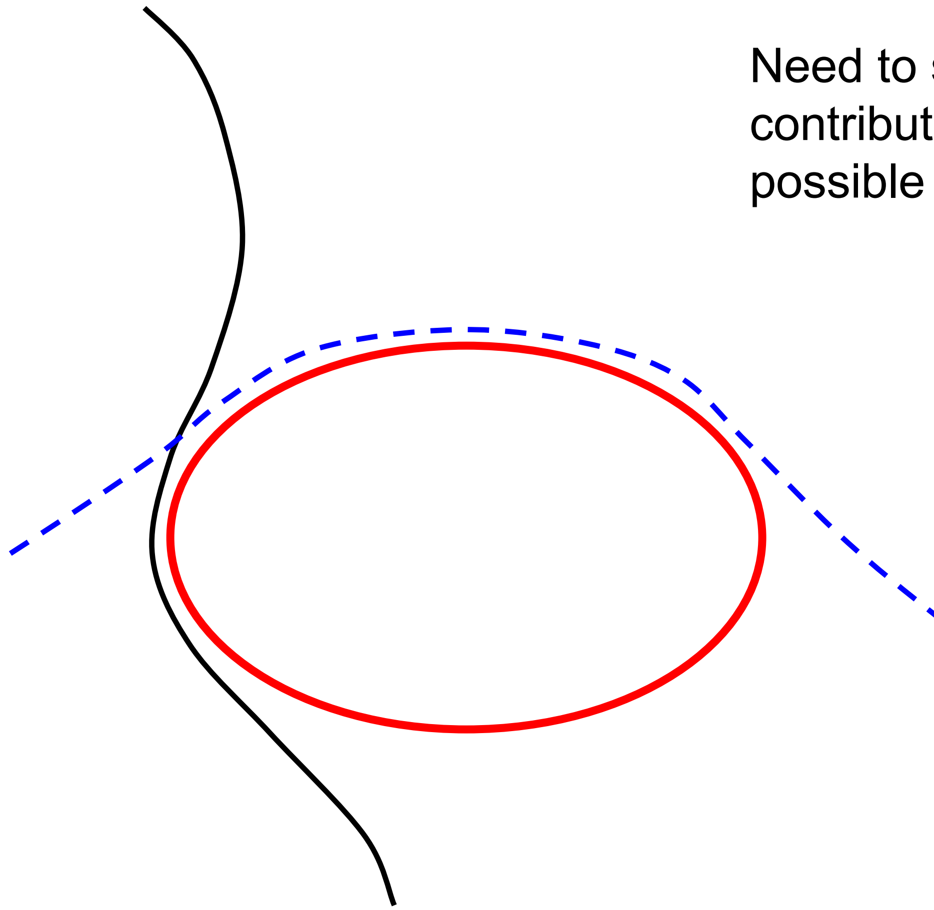
$$\mathrm{var}\, G = \mathrm{var}\, G_{\mathrm{RMT}}$$

Brouwer and SR (2006). Brouwer (2007).

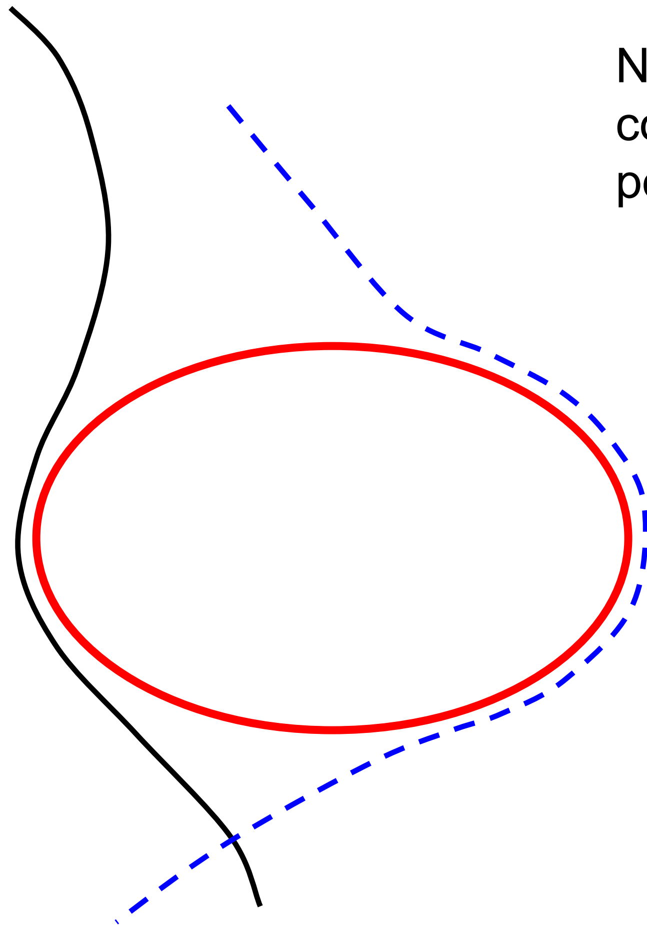
Conductance fluctuations independent of Ehrenfest time !

The ‘density-of-states’ configurations:

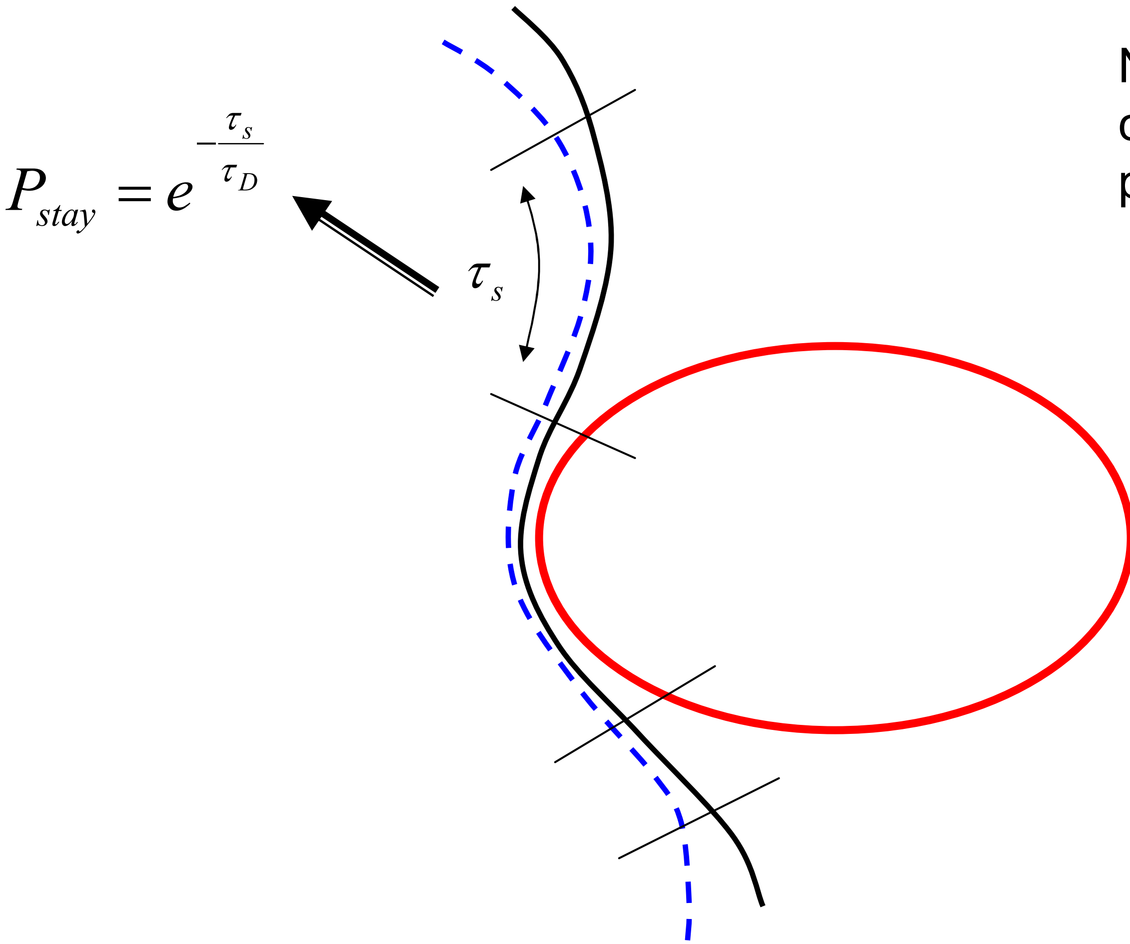




Need to sum contributions from all possible encounters



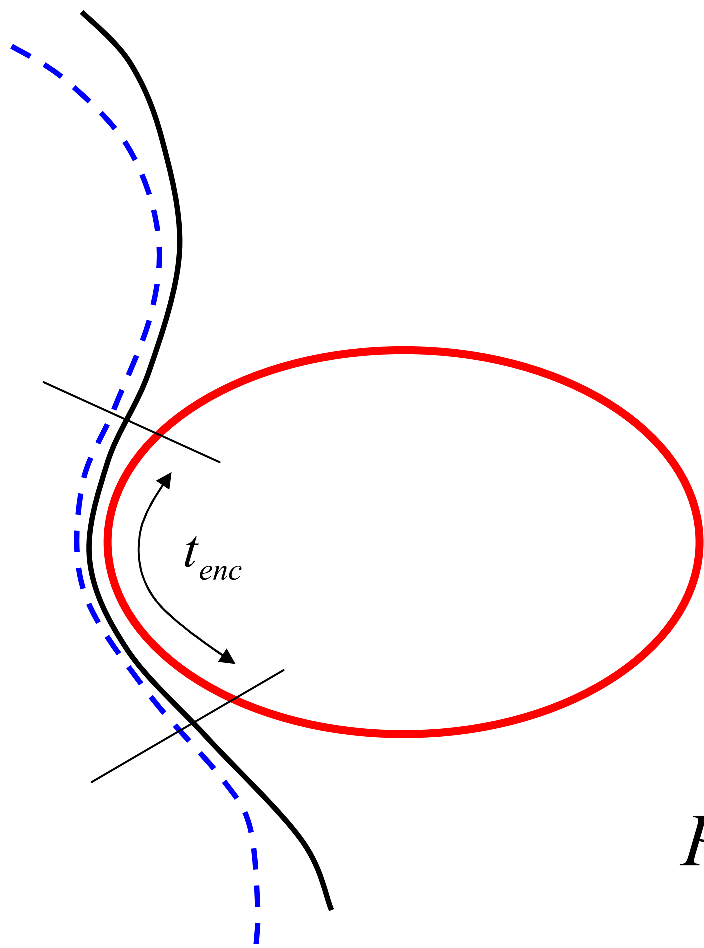
Need to sum contributions from all possible encounters



Need to sum contributions from all possible encounters

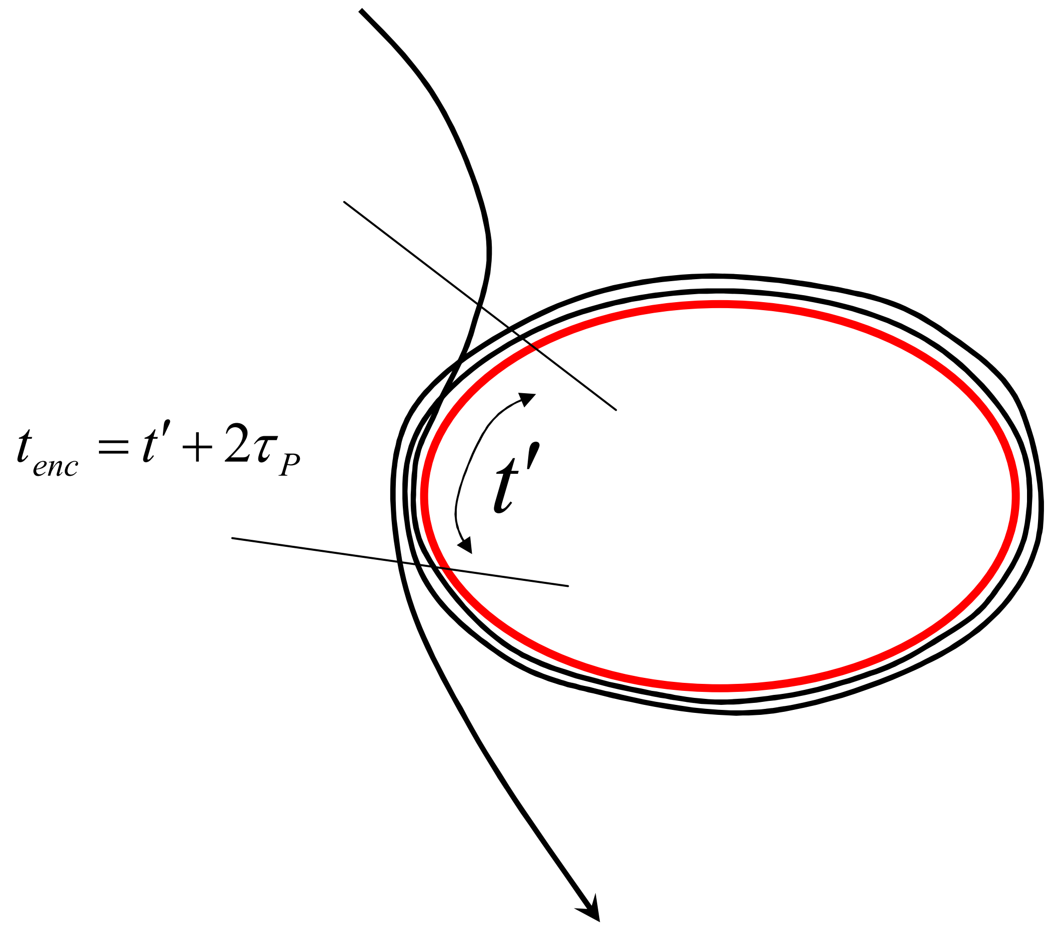
Systematic, non-oscillatory contribution from trajectories correlated away from the periodic orbit

Survival probability $t_{enc} < \tau_P$



$$P = \exp\left[-\frac{t_{enc}}{\tau_D}\right]$$

Survival probability: $t_{enc} > \tau_P$



$$P = \exp\left[-\frac{\tau_P}{\tau_D}\right]$$

Can encircle many times without affecting the survival probability!

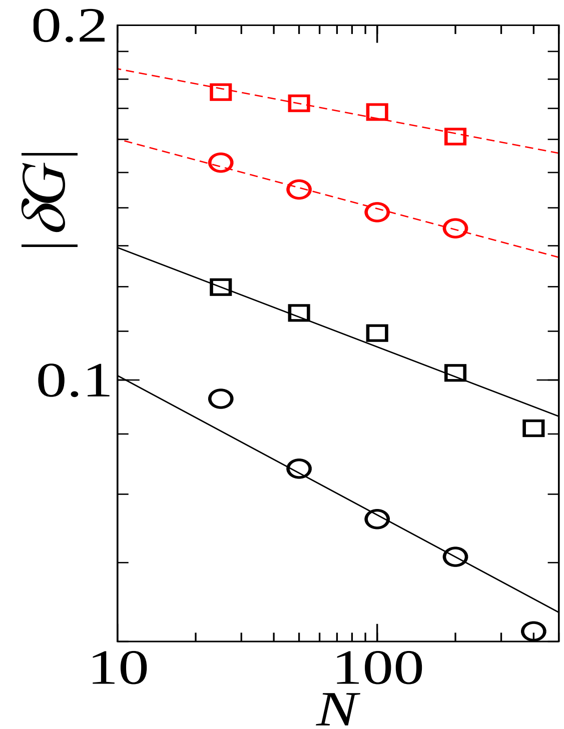
Numerical results

$$\delta G = \delta G_{\mathrm{RMT}} e^{-\tau_{\mathrm{E}}/\tau_{\mathrm{D}}}$$
$$= \delta G_{\mathrm{RMT}} \left(k_{\mathrm{F}} L\right)^{-1/\lambda\tau_{\mathrm{D}}}$$

$$\ln|\delta G| = -\frac{1}{\lambda \tau_D} \ln N + \mathrm{const}$$

$\lambda, \tau_{\mathrm{D}}$ are known

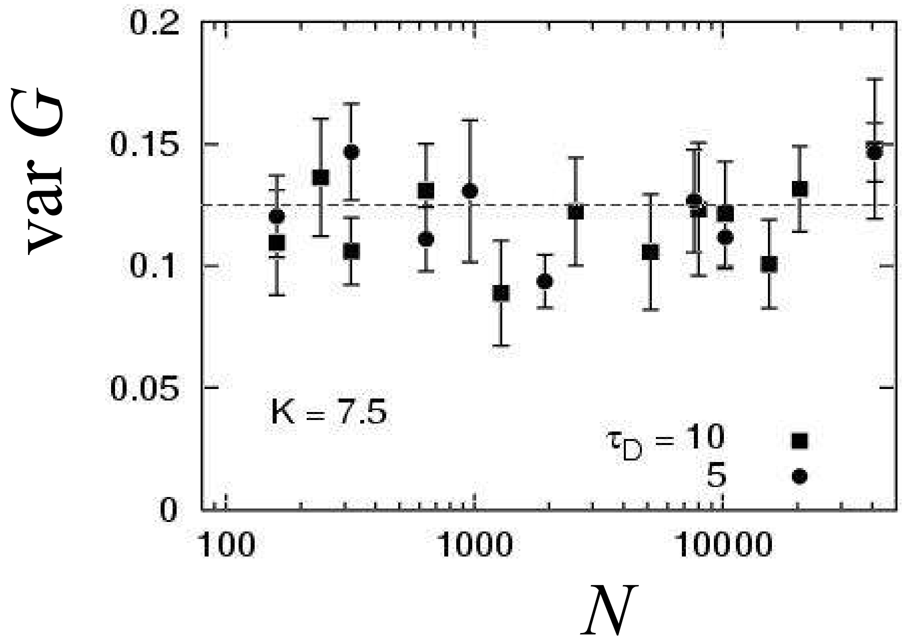
Numerical results consistent with theory !



SR and Brouwer (2005)

Numerical results

$$\mathrm{var}\, G = \mathrm{var}\, G_{\mathrm{RMT}}$$



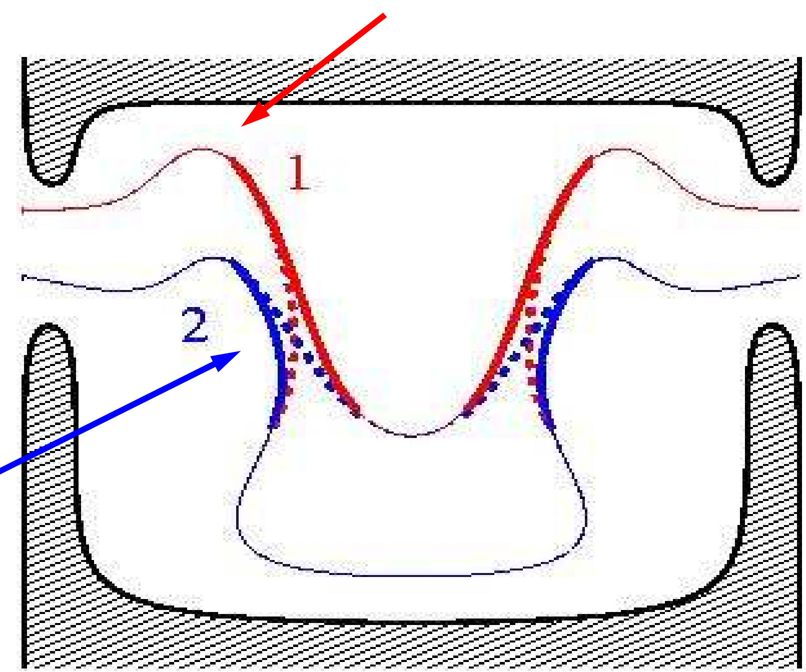
Tworzydlo *et al.* (2004)
Jacquod and Sukhorukov (2004)

Parametric correlations

Is there a deep reason for τ_E independence of Var(G)?

Study:

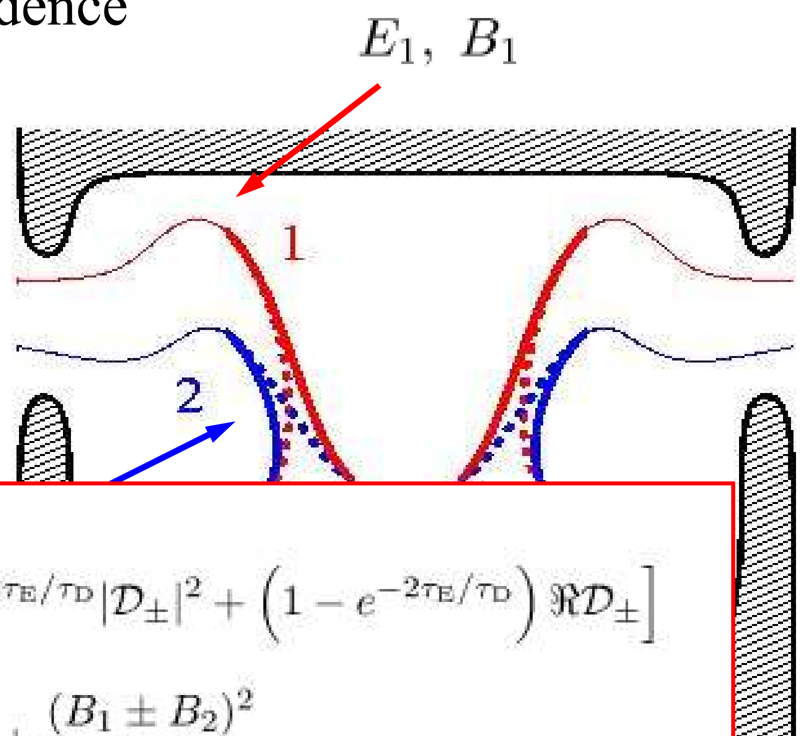
$$\langle \delta G(E_1, B_1)\delta G(E_2, B_2)\rangle$$



$$\begin{aligned} \langle \delta S \rangle &\propto i(E_1 - E_2)t \\ \langle \delta S^2 \rangle &\propto (B_1 - B_2)^2 t \end{aligned}$$

Parametric correlations

Is there a deep reason for τ_E independence of Var(G)?



Study:

$$\langle \delta G(E_1, B_1)\delta G(E_2, B_2)\rangle$$

$$\langle \delta G(E_1, B_1)\delta G(E_2, B_2)\rangle = P_1^2 P_2^2 \sum_{\pm} \left[e^{-2\tau_E/\tau_D} |\mathcal{D}_\pm|^2 + \left(1 - e^{-2\tau_E/\tau_D}\right) \Re \mathcal{D}_\pm \right]$$

$$\mathcal{D}_\pm^{-1} = 1 - i\frac{E_1 - E_2}{E^*} + \frac{(B_1 \pm B_2)^2}{2(B^*)^2}$$

The parametric correlations depend on τ_E. New, non-RMT, universal regime.

Brouwer and SR (2007)

Summary:

Wave phenomena in ballistic quantum dots appear only after the Ehrenfest time!

Summary of results:

- **Weak localization** $\delta G = \delta G_{\mathrm{RMT}} \times e^{-\tau_{\mathrm{E}}/\tau_{\mathrm{D}}}$

- **Conductance fluctuations** $\mathrm{var}\, G = \mathrm{var}\, G_{\mathrm{RMT}}$

- Many other: shot noise, dephasing, coherent backscattering, quantum pumping …