The classical limit of quantum transport

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Interference effects in transport

Weak Localization:
Small negative correction for the average conductance at $B = 0$

Conductance fluctuations:
Reproducible fluctuation of the conductance when a parameter is varied.

First studied for disordered samples.

Anderson, Abrahams, Ramakrishnan (1979)
Gorkov, Larkin, Khmelnitskii (1979)
Altshuler (1985)
Lee and Stone (1985)
Ballistic quantum dots

No impurities, smooth boundaries:

Chaotic classical dynamics

Exhibit interference effects:

Marcus group
RMT, Universality

Random matrix theory \( N = k_F W \gg 1 \)

\[
\delta G_{\text{RMT}} = \frac{1}{4} \left( \frac{2e^2}{h} \right)
\]

\[
\text{var } G_{\text{RMT}} = \frac{1}{16} \left( \frac{2e^2}{h} \right)^2
\]

Jalabert, Pichard, Beenakker (1994)
Baranger and Mello (1994)

RMT: \( \delta G \) and var \( G \) do not depend on dot size \( L \)
Semiclassical theory

\[ \delta G = \delta G_{\text{RMT}} (k_F L)^{-2/\lambda \tau_D} \]

Aleiner, Larkin (1996)

\[ \lambda \sim \frac{v_F}{L} \quad \text{Lyapunov exponent} \]

\[ \tau_D \sim \frac{L^2}{W v_F} \quad \text{dwell time} \]

\[ \frac{1}{\lambda \tau_D} \sim \frac{W}{L} \]
RMT, Universality ???

Semiclassical theory

\[ \delta G = \delta G_{\text{RMT}} (k_F L)^{-2/\lambda \tau_D} \]
\[ \delta G = \delta G_{\text{RMT}} (k_F L)^{-1/\lambda \tau_D} \]

Aleiner, Larkin (1996)
Adagideli (2003)

Numerical simulations

\[ \text{var } G = \text{var } G_{\text{RMT}} \]
\[ \delta G = \delta G_{\text{RMT}} \]

Tworzydlo, Tajic,Beenakker (2004)
Jacquod and Sukhorukov (2004)

Theory and numerics disagree!
Semiclassical theory

\[ \delta G = \delta G_{RMT} \left(k_F L\right)^{-2/\lambda \tau_D} \]

\[ \delta G = \delta G_{RMT} \left(k_F L\right)^{-1/\lambda \tau_D} \]

Numerical simulations

\[ \text{var } G = \text{var } G_{RMT} \]

\[ \delta G = \delta G_{RMT} \]

This talk: Classical correlations for

- Weak localization - \( \delta G \)
- Conductance fluctuation - \( \text{var } G \)

Remark: Parametric correlations
Semiclassical picture of weak localization

Disordered systems:

Ballistic systems:

Anderson, Abrahams, Ramakrishnan (1979), Gorkov, Larkin, Khmelnitskii (1979)


A ‘small region’ responsible for phase difference
Quantum uncertainty in position or direction of incoming wavepacket is magnified by chaotic boundary scattering.  

Aleiner and Larkin (1996)

\[ \delta r(t) = \delta r(0) e^{t\lambda} \]
`Ehrenfest time’ $\tau_E$: Time until initial uncertainty $1/k_F$ has reached dot size $L$:

$$L = \frac{1}{k_F} e^{\lambda \tau_E}$$

$$\tau_E = \frac{1}{\lambda} \ln k_F L$$

$\lambda$: Lyapunov exponent

Random Matrix Theory valid if $\tau_E \ll \tau_D$
Theory for $\tau_E \ll \tau_D$

\[ G = \frac{2e^2}{\hbar} \sum_{m,n} |t_{mn}|^2 \]

Landauer formula

\[ |t_{mn}|^2 \sim \sum_{\alpha,\beta} A_\alpha A_\beta e^{i(S_\alpha - S_\beta)/\hbar}, \]

Jalabert, Baranger, Stone (1990)

- $S_\alpha, S_\beta$: classical action
- angles of $\alpha, \beta$ consistent with transverse momentum in lead,

\[ p_\perp(m) = \pm \pi \hbar m/W_j, \quad m = 1, \ldots, N_j, \]

- $A_\alpha, A_\beta$: stability amplitudes
Action difference $S_\alpha - S_\beta = su$

$$t_{\text{enc}} = \frac{1}{\lambda} \left( \ln \frac{c}{|s|} + \ln \frac{c}{|u|} \right)$$

$s, u$: distances along stable, unstable phase space directions

encounter region: $|s|, |u| < c$

c: classical cut-off scale

Richter and Sieber (2003)
Spehner (2003)
Turek and Richter (2003)
Müller et al. (2004)
Heusler et al. (2006)
Theory for $\tau_E \sim \tau_D$

Encounters have ‘size’:

$$S_\alpha - S_\beta \simeq \hbar$$

$$t_{\text{enc}} \simeq \frac{1}{\lambda} \ln \frac{c^2}{\hbar} = \tau_E$$

Uncorrelated escape:

$$P(t) = e^{-t/\tau_D}$$

$$P_{\text{stay}} = ??$$
Theory for $\tau_E \sim \tau_D$

Correlated motion means correlated escape!

$P_{\text{stay}} = e^{-t_{\text{enc}}/\tau_D} \sim e^{-\tau_E/\tau_D}$

Not $e^{-2\tau_E/\tau_D}$
Theory for $\tau_E \sim \tau_D$

$P_1, P_2$: probabilities to enter, exit through left, right contacts

$$\delta G = -\frac{2e^2}{\hbar} P_1 P_2 \int d\tau' \int_{-c}^{c} dsdu \frac{e^{isu/\hbar-(t_{enc}+\tau')/\tau_D}}{2\pi \hbar t_{enc}}$$

$$= \frac{2e^2}{\hbar} P_1 P_2 e^{-\tau_E/\tau_D}$$

Theory for $\tau_E \sim \tau_D$

$\delta G = \delta G_{RMT} e^{-\tau_E/\tau_D}$

$= \delta G_{RMT} (k_F L)^{-1/\lambda \tau_D}$

$P_1, P_2$: probabilities to enter, exit through left, right contacts

Unitarity

$\delta G$ can also be calculated through the dot’s reflection

\[
\delta G = -2 \frac{e^2}{h} P_1^2 e^{-\frac{\tau_E}{\tau_D}} + 2 \frac{e^2}{h} P_1 e^{-\frac{\tau_E}{\tau_D}} \\
= 2 \frac{e^2}{h} P_1 (1 - P_1) e^{-\frac{\tau_E}{\tau_D}} \\
= \delta G_{RMT} e^{-\frac{\tau_E}{\tau_D}}
\]

$P_1, P_2 = 1 - P_1$: probabilities to enter, exit through left, right contacts

Weak localization

Coherent backscattering
Conductance fluctuations

Two classes of trajectories:

“Diffusion constant”

“Density of states”

RMT: 1 : 0
Conductance fluctuations

Semiclassical theory:

\[ \text{var } G \text{ RMT} \times e^{-2\tau_E/\tau_D} \]

\[+\]

\[ \text{var } G \text{ RMT} \times (1 - e^{-2\tau_E/\tau_D}) \]

\[\text{var } G = \text{var } G \text{ RMT}\]


Conductance fluctuations independent of Ehrenfest time!
The ‘density-of-states’ configurations:

Periodic trajectory $\tau_p$

Encounter with the periodic orbit

Second encounter.
Need to sum contributions from all possible encounters
Need to sum contributions from all possible encounters
Need to sum contributions from all possible encounters

\[ P_{stay} = e^{-\frac{\tau_s}{\tau_P}} \]

Systematic, non-oscillatory contribution from trajectories correlated away from the periodic orbit
Survival probability $t_{enc} < \tau_P$

$$P = \exp \left[-\frac{t_{\text{enc}}}{\tau_D}\right]$$
Survival probability: \[ t_{\text{enc}} > \tau_P \]

\[ t_{\text{enc}} = t' + 2\tau_P \]

\[ P = \exp \left[ -\frac{\tau_P}{\tau_D} \right] \]

Can encircle many times without affecting the survival probability!
Numerical results

\[ \delta G = \delta G_{\text{RMT}} e^{-\tau_E/\tau_D} \]
\[ = \delta G_{\text{RMT}} (k_F L)^{-1/\lambda \tau_D} \]

\[ \ln|\delta G| = -\frac{1}{\lambda \tau_D} \ln N + \text{const} \]

\( \lambda, \tau_D \) are known

Numerical results consistent with theory !

SR and Brouwer (2005)
Numerical results

\[ \text{var } G = \text{var } G_{\text{RMT}} \]
Parametric correlations

Is there a deep reason for $\tau_E$ independence of $\text{Var}(G)$?

Study:

$$\langle \delta G(E_1, B_1) \delta G(E_2, B_2) \rangle$$

$$\langle \delta S \rangle \propto i(E_1 - E_2)t$$

$$\langle \delta S^2 \rangle \propto (B_1 - B_2)^2 t$$
Parametric correlations

Is there a deep reason for $\tau_E$ independence of $\text{Var}(G)$?

Study:

$$\langle \delta G(E_1, B_1) \delta G(E_2, B_2) \rangle$$

The parametric correlations depend on $\tau_E$. New, non-RMT, universal regime.

Brouwer and SR (2007)
Summary:

Wave phenomena in ballistic quantum dots appear only after the Ehrenfest time!

Summary of results:

- Weak localization
  \[ \delta G = \delta G_{\text{RMT}} \times e^{-\tau_E/\tau_D} \]

- Conductance fluctuations
  \[ \text{var } G = \text{var } G_{\text{RMT}} \]

- Many other: shot noise, dephasing, coherent backscattering, quantum pumping …