Diffractive Paths for Weak Localization in Quantum Billiards

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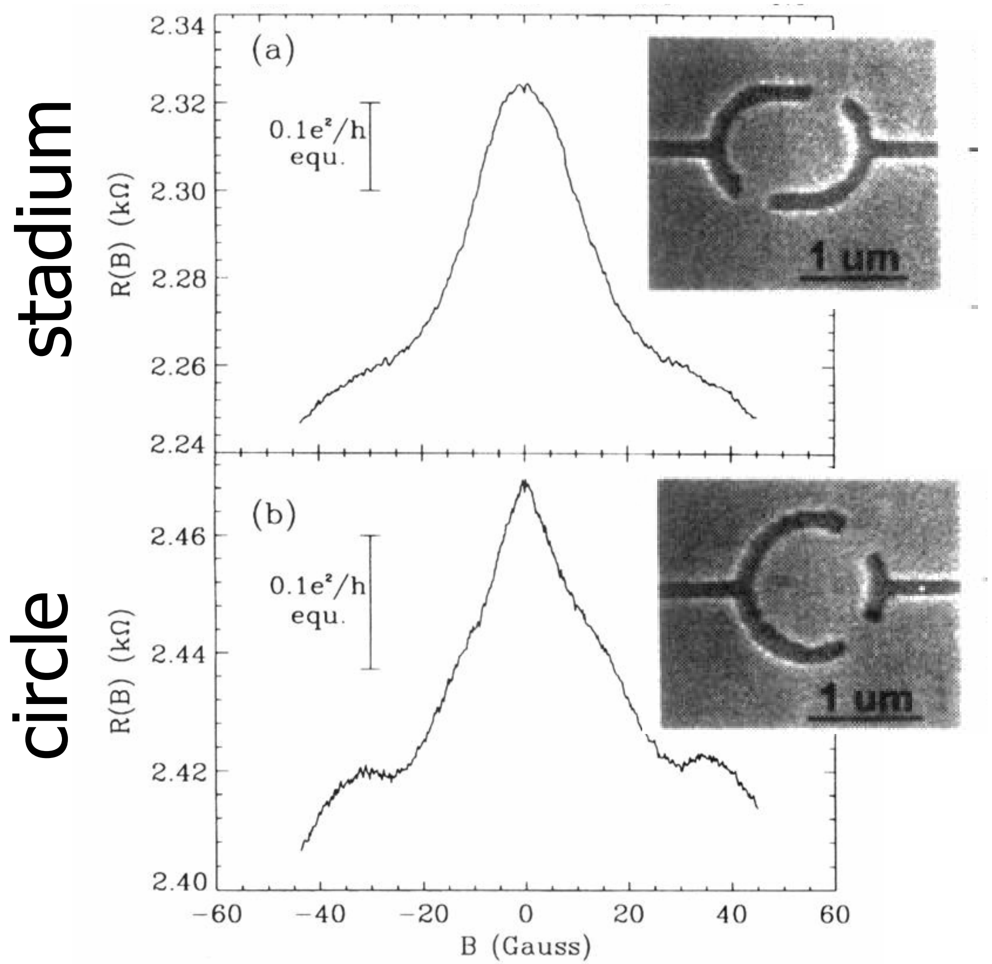
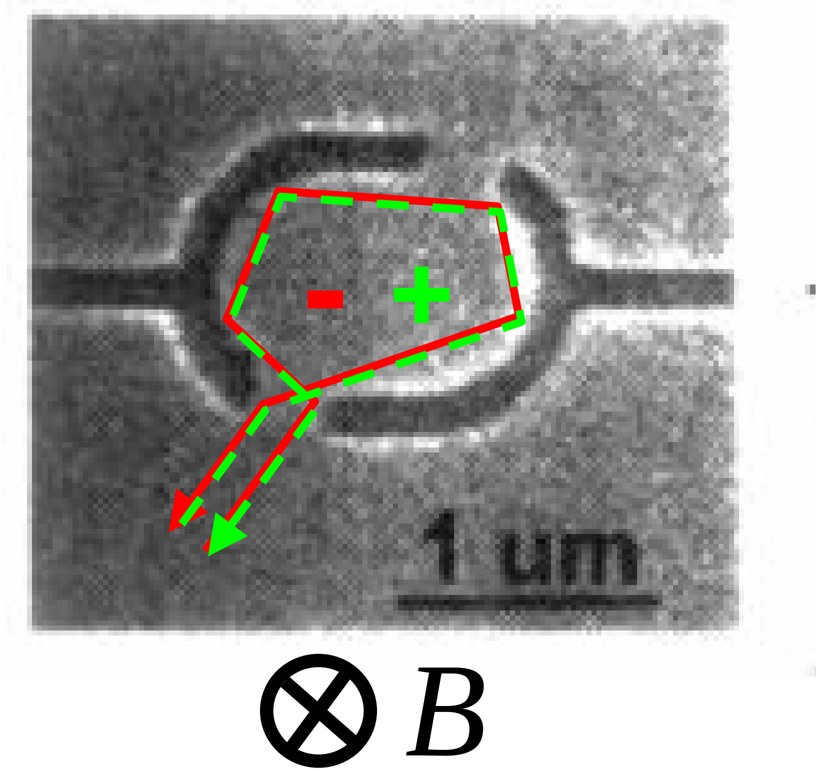
WL in Ballistic Transport

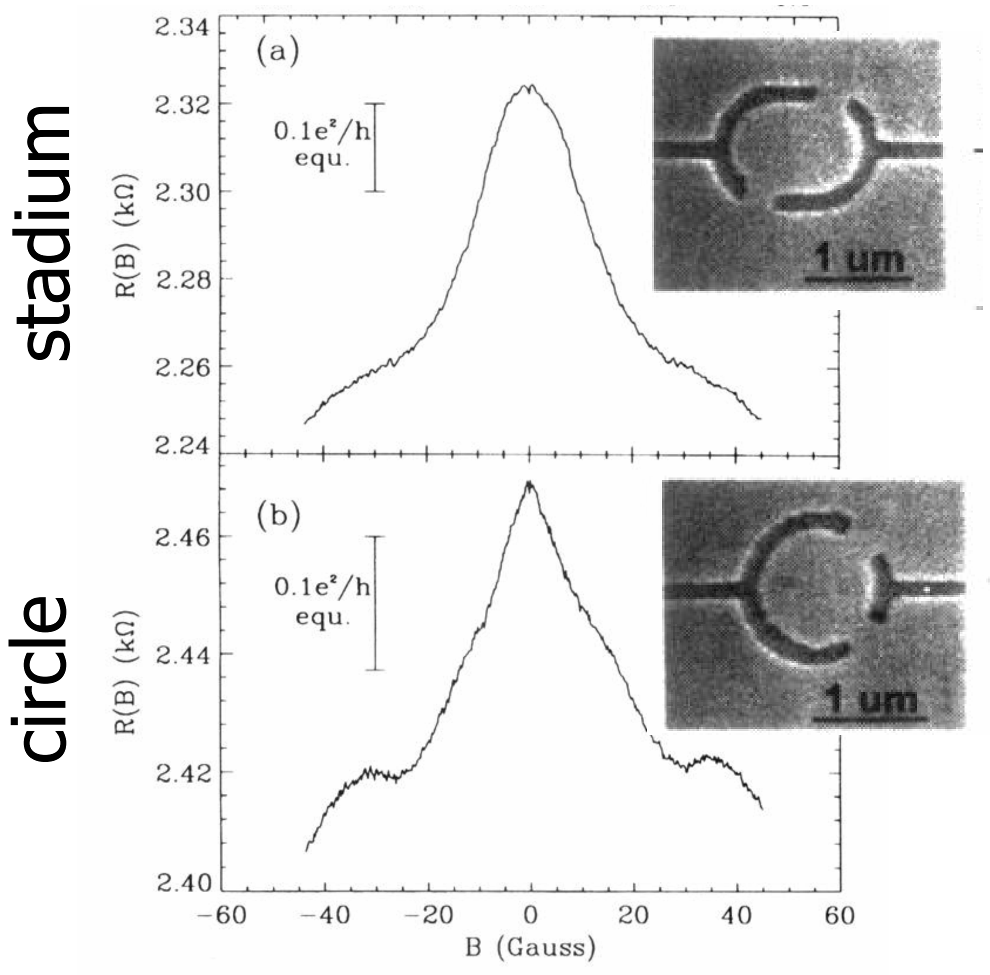


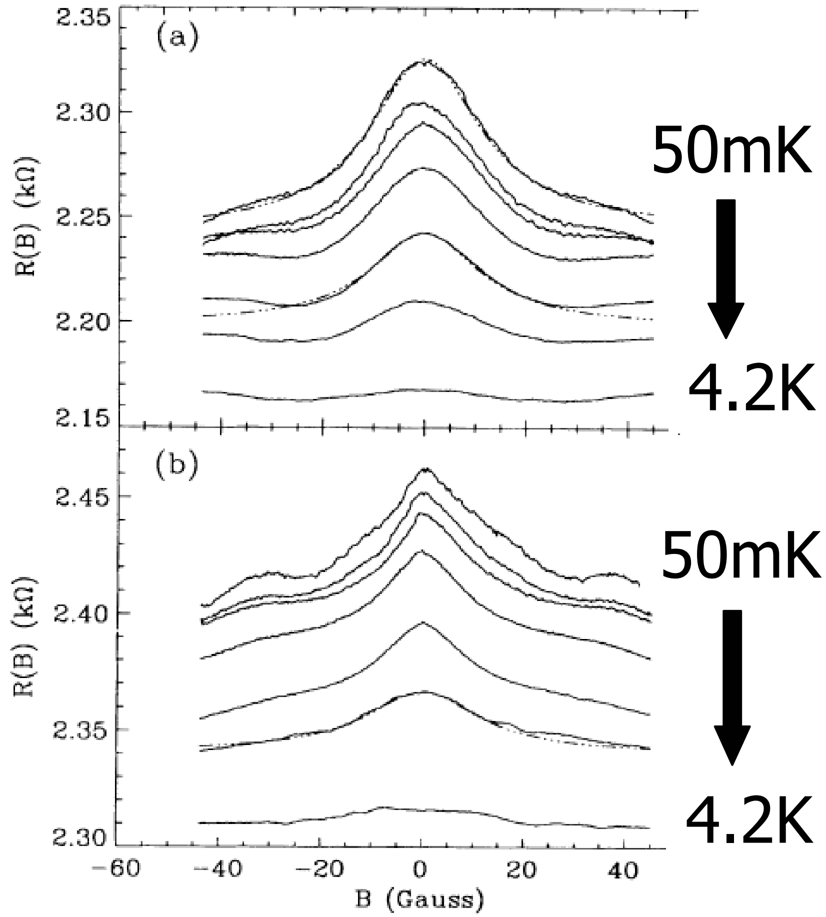


Chang, Baranger, Pfeiffer, West, PRL 73, 2111 (1994).

see also: Marcus, Rimberg, Westervelt et al., PRL 69, 506 (1992).

WL in Transport through Quantum Dots





Chang, Baranger, Pfeiffer, and West, PRL 73, 2111 (1994).

Ballistic transport

Landauer-Büttiker:

$$G = \frac{e^2}{h} T = \frac{e^2}{h} \sum_{m,n}^{N} |t_{mn}|^2$$

$$T + R = N$$

Quantum calculation:
Good agreement
with experiment!

Chang, Baranger, Pfeiffer, West,
PRL 73, 2111 (1994).

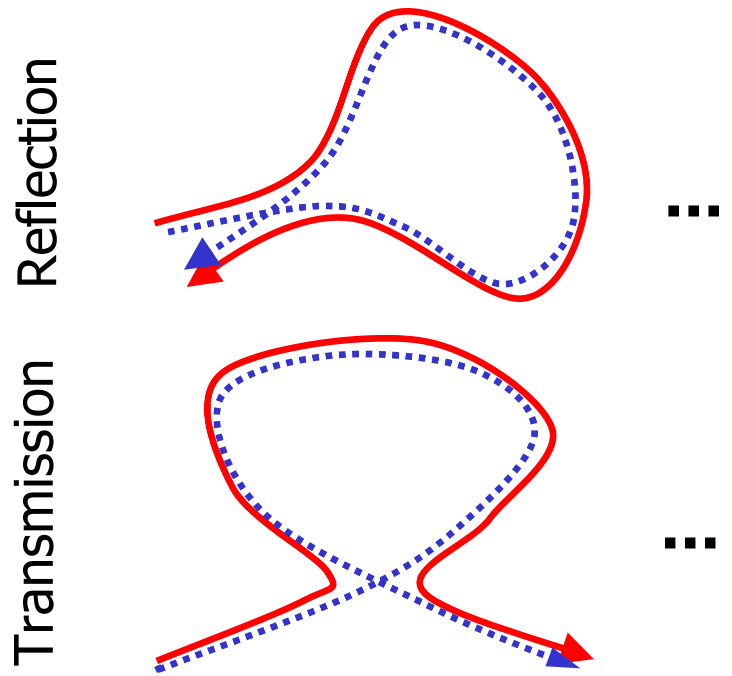
"*...interference between trajectories which are not exactly time-reversed must be included.*"

Baranger, Jalabert, and Stone, PRL 70, 3876 (1993).

➡ Additional contributions required for reflection.

➡ Which paths for transmission?

Trajectory pairs



- WL-correction in agreement with RMT
- Current conservation

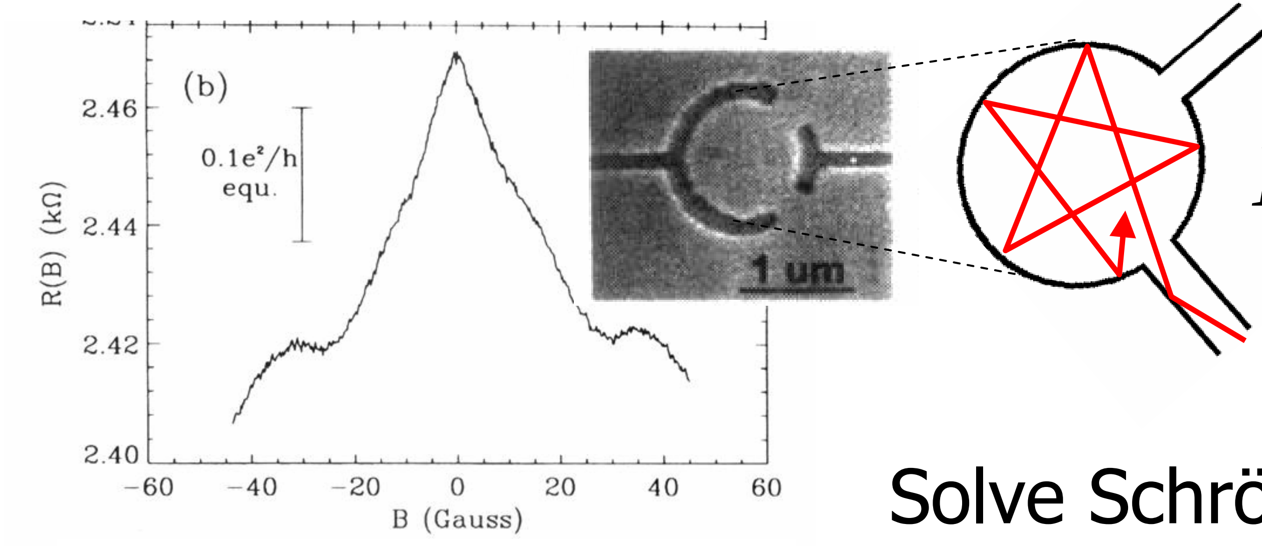
Assumptions:

- Ensemble Average
- $\lambda/D \rightarrow 0$
- Chaotic dynamics
- Smooth boundaries

Aleiner & Larkin, PRB 54, 14423 (96).
Takane & Nakamura, J. Phys. Soc. Jpn. 66, 2977 (97).
Sieber & Richter, PRL 89, 206801 (02).
Rahav & Brouwer, PRL 95, 056806 (05).
Jacquod & Whitney, PRB 73, 195115 (06).
Heusler, Müller, Braun, Haake, PRL 96, 066804 (06).

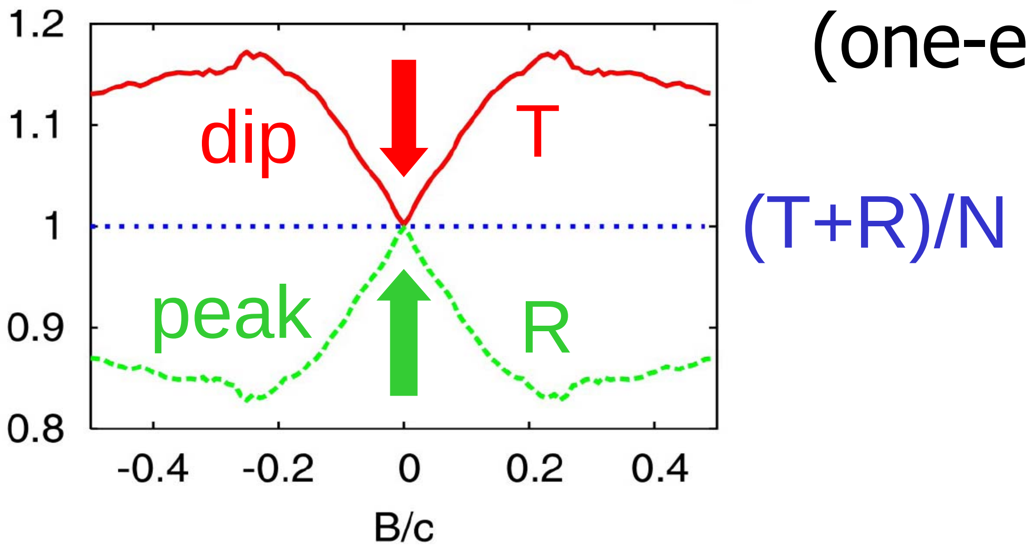
Very long paths!

Circular billiard



$$H = -\frac{\hbar^2}{2m}\Delta + V_0$$

Solve Schrödinger equation
(one-electron, flat potential)



Unitarity:

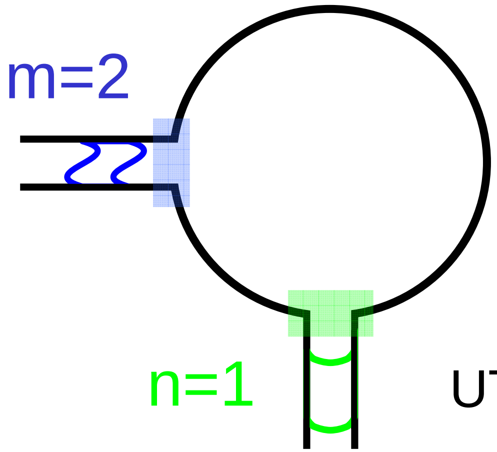
$$T + R = N = 2$$

Semiclassical Transport

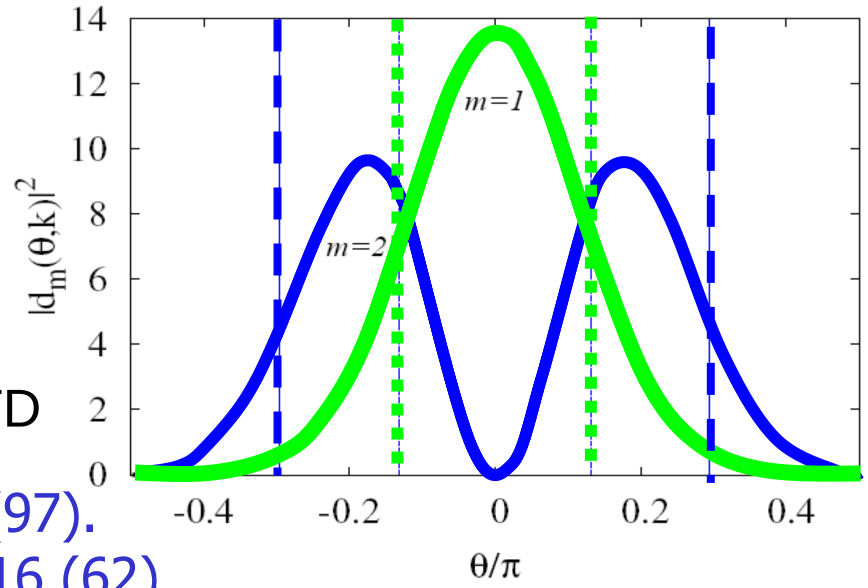
Transmission and reflection amplitudes

$$t_{nm} \propto \langle n | G^{\mathrm{SC}}_{2\leftarrow 1} | m \rangle \qquad r_{nm} \propto \langle n | G^{\mathrm{SC}}_{1\leftarrow 1} | m \rangle$$

$$t_{nm} = \frac{1}{\sqrt{8\pi i}} \sum_{class.\,path:\,q} d_n(\theta',k)\sqrt{|D_q|}\, e^{i\left(\frac{S_q}{\hbar}-\frac{\pi}{2}\mu_q\right)} d_m(\theta,k)$$



UTD-GTD

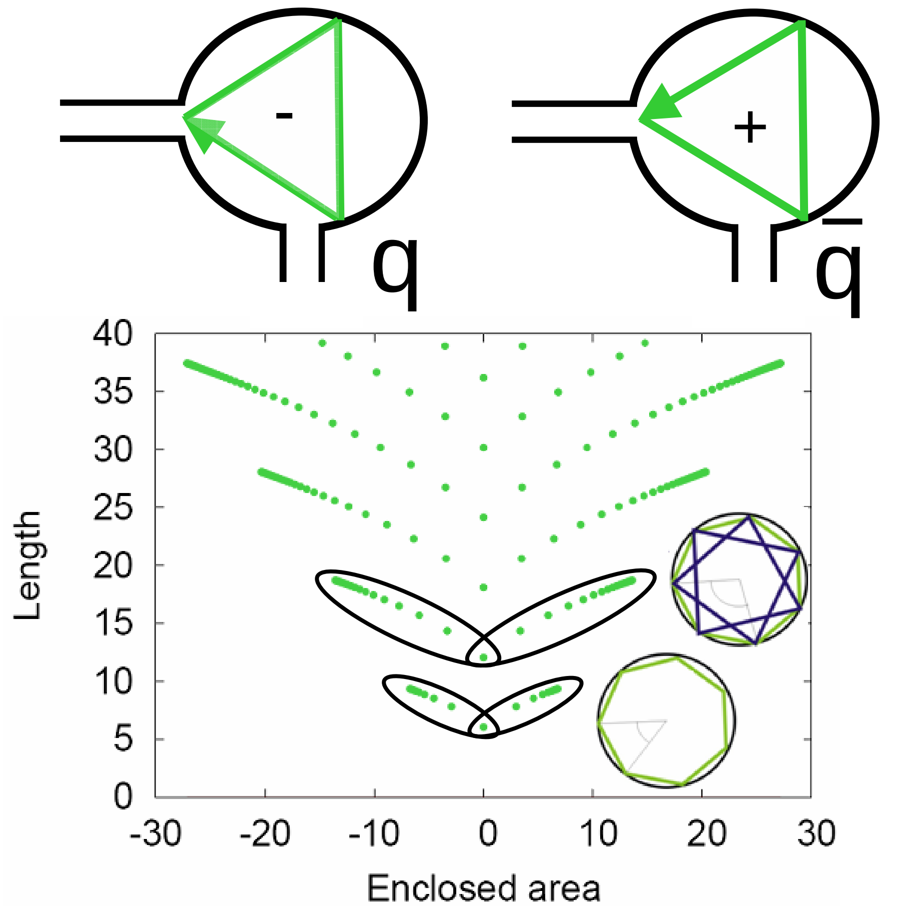


$$k \to \infty$$

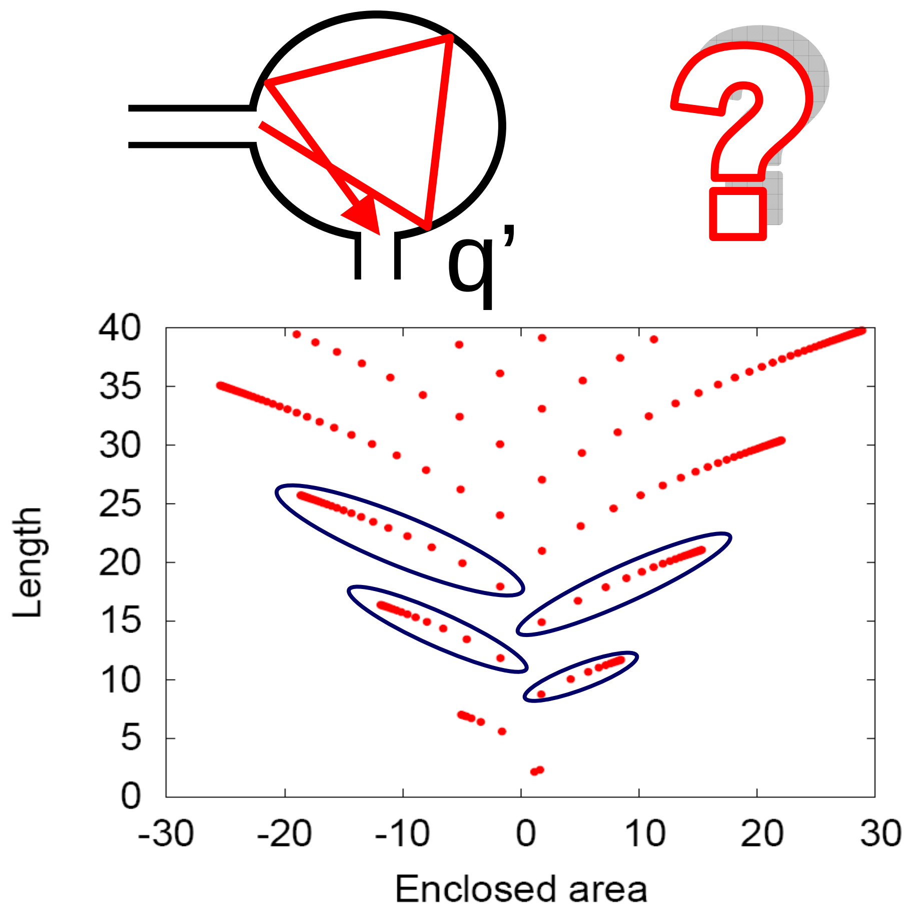
Sieber et al., PRE 55, 2279 (97).
Keller, J. Opt. Soc. Amer. 52, 116 (62).

Classical paths

Reflection:
Time reversal symmetry

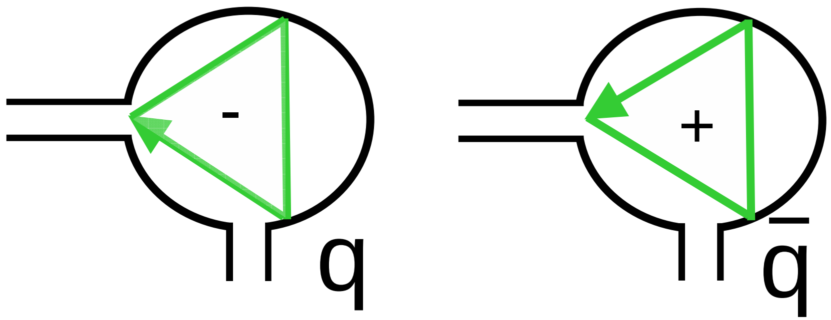


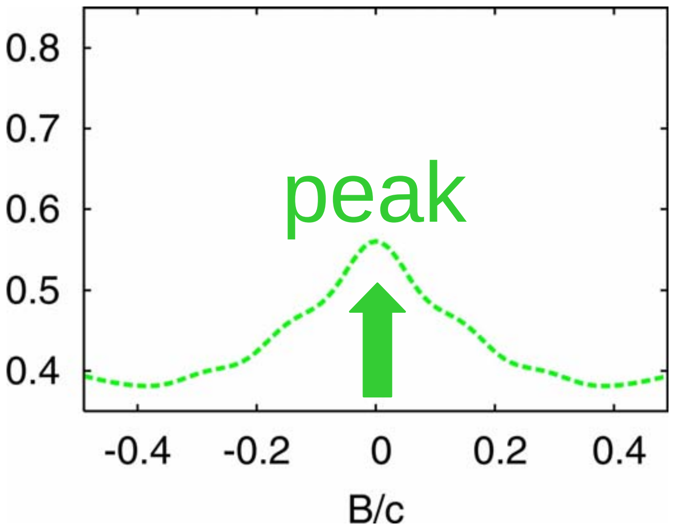
Transmission:
No left-right symmetry



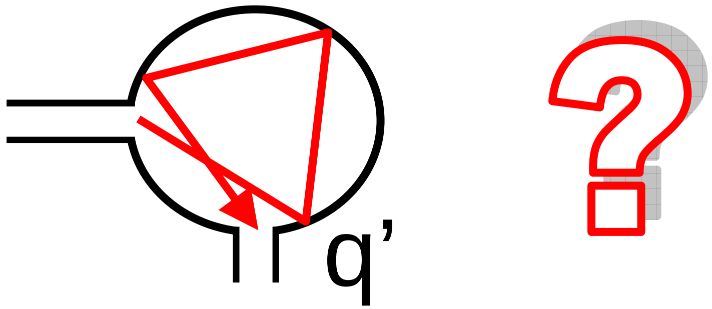
Classical paths insufficient

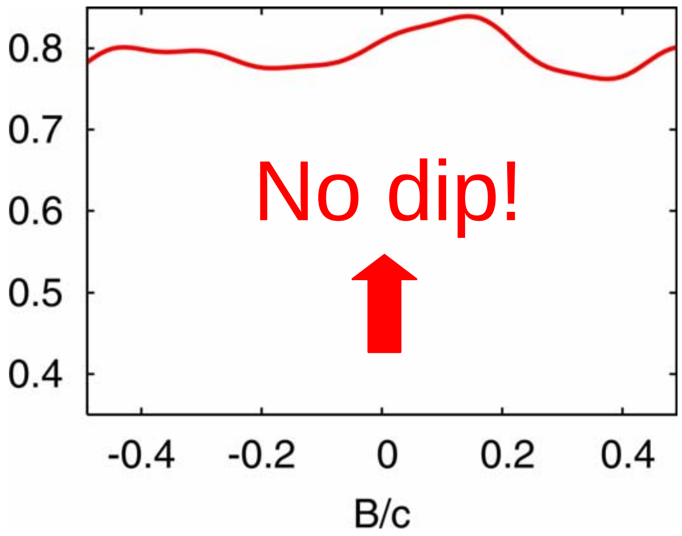
Reflection:
Time reversal symmetry





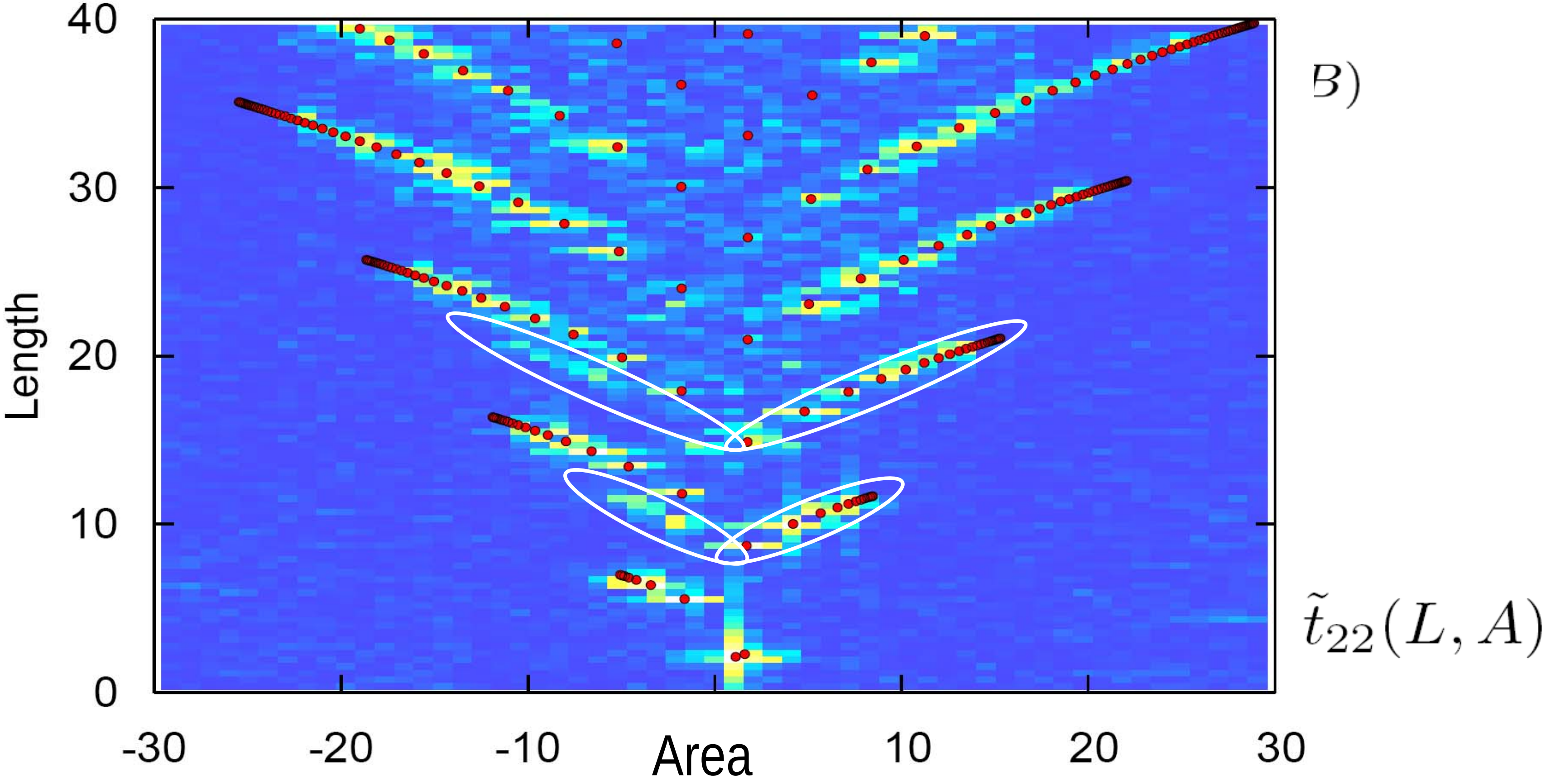
Transmission:
No left-right symmetry



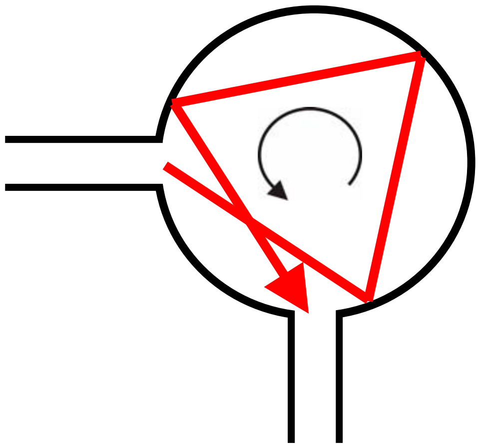


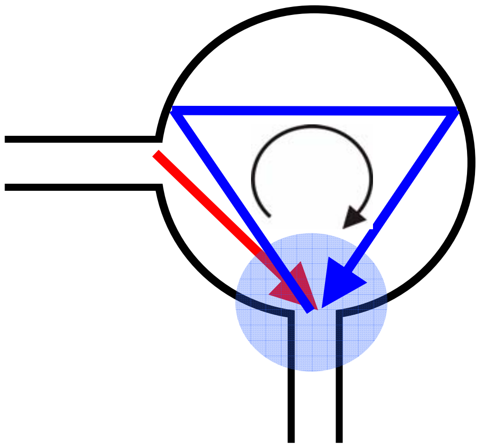
What's missing?

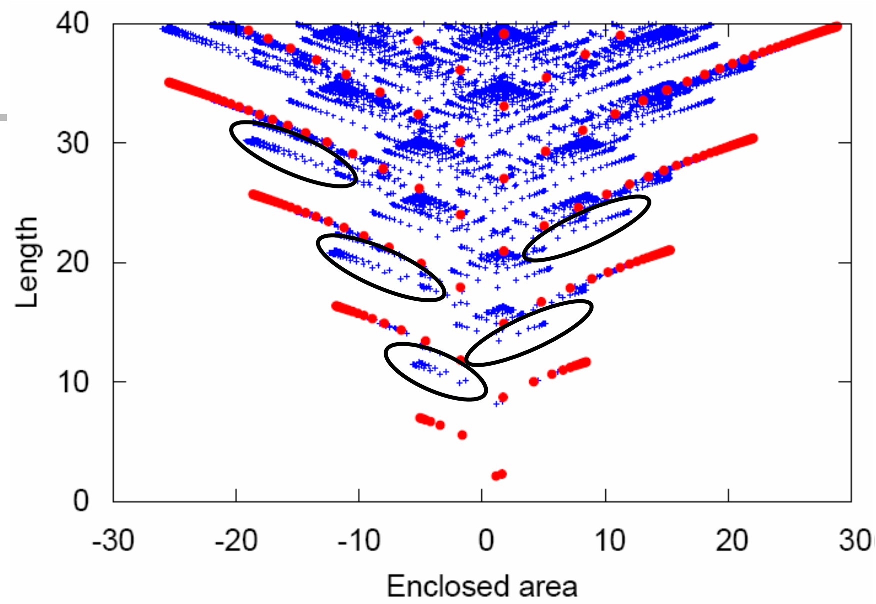
$$\tilde{S}_{nm}(L,A) = \int dk \int dB\; e^{-i(kL+\frac{B}{c}A)}\; S_{nm}(k,B)$$

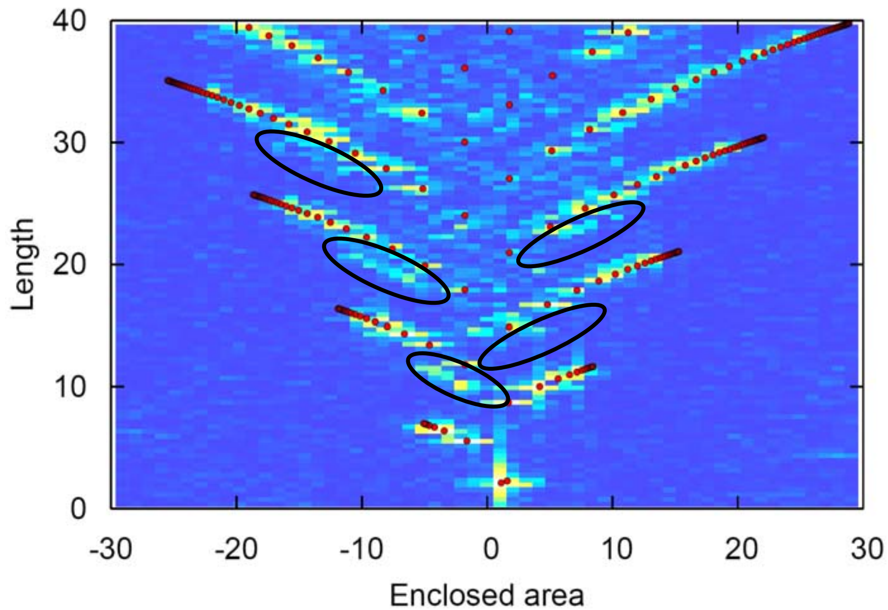


Identify diffractive paths

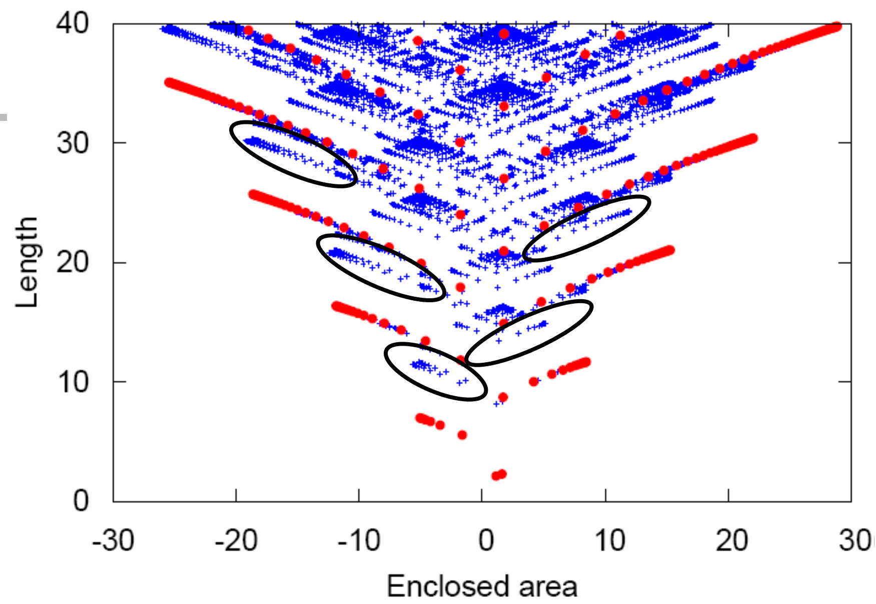


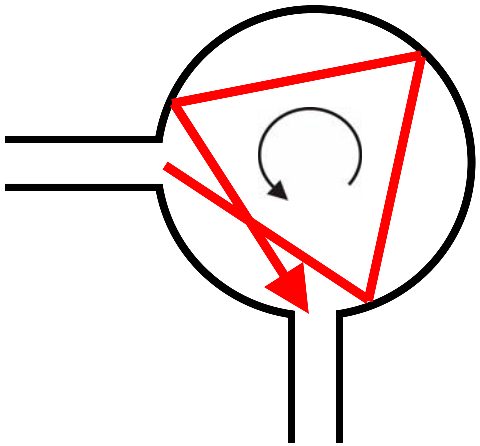


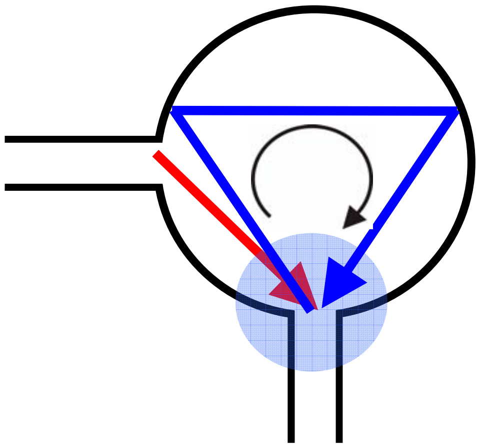


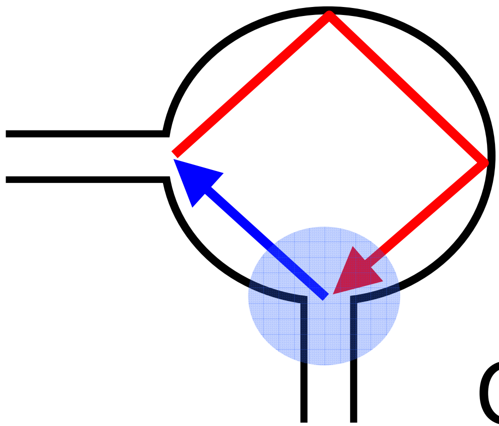


Identify diffractive paths









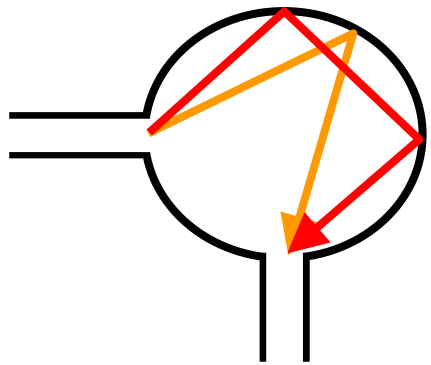
"Non-local Correlation"

Pseudo-path semiclassical approximation (PSCA)

Dyson equation: $G^{PSCA} = G^{SC} + G^{SC} V G^{PSCA}$

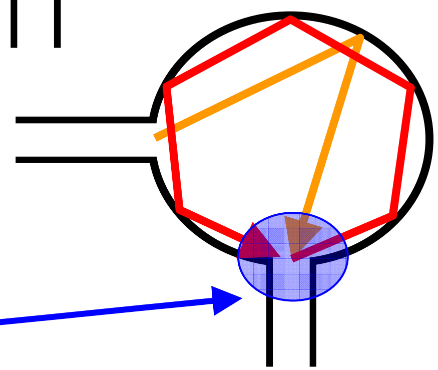
$$G^{PSCA} = \sum_{i=0}^{\infty} \left(G^{SC} V \right)^i G^{SC}$$ Perturbative expansion

- i=0 $\quad G^{SC}: \quad \sum_{class.paths\ q} [\ldots]$



- i=1 $\quad G^{SC} V G^{SC}:$

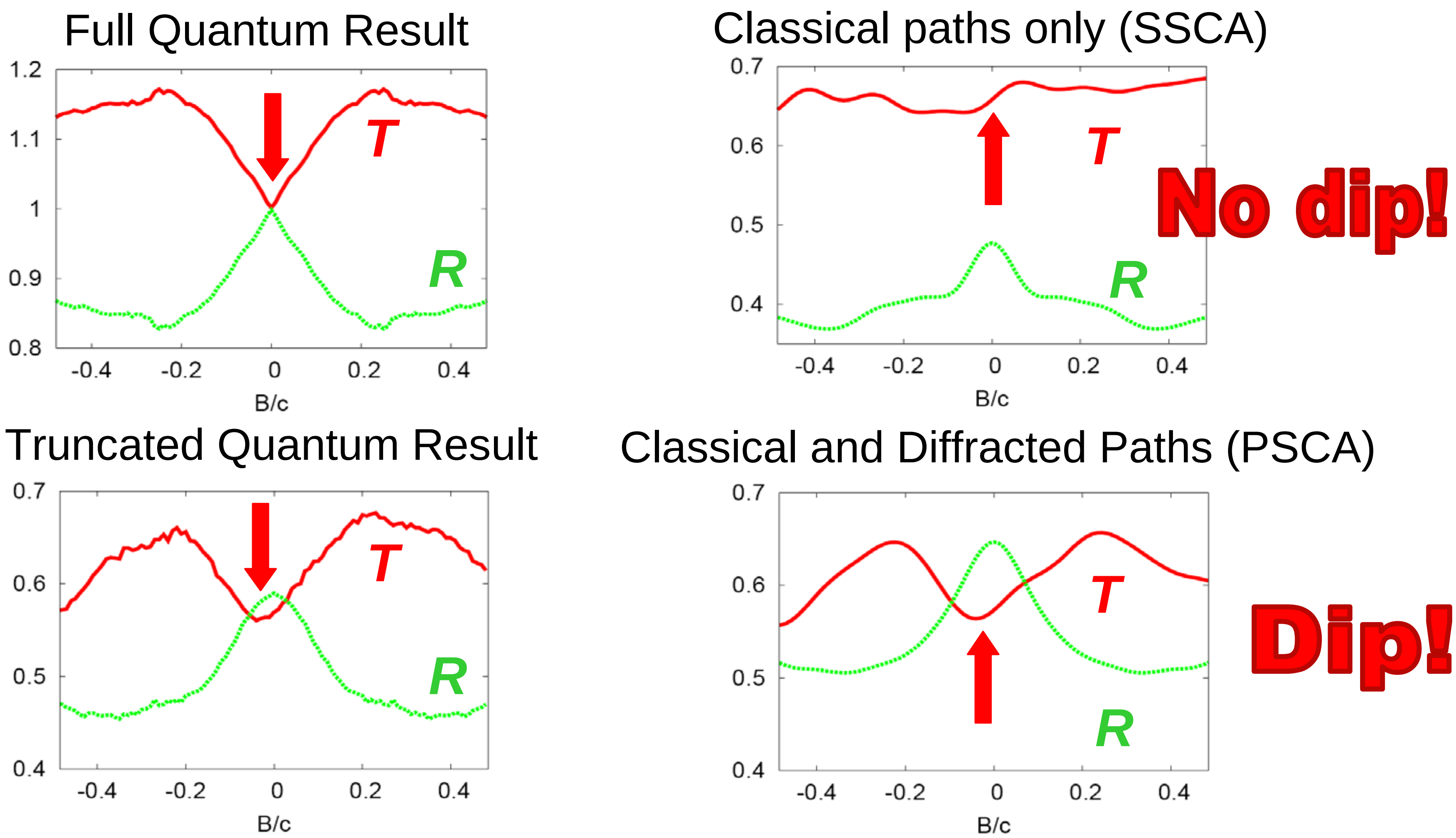
$$\sum_{class.paths\ q'} [\ldots] \, v(\theta', \theta'') \sum_{class.paths\ q''} [\ldots]$$



- …

C.Stampfer et al. PRE 72, 036223 (05)

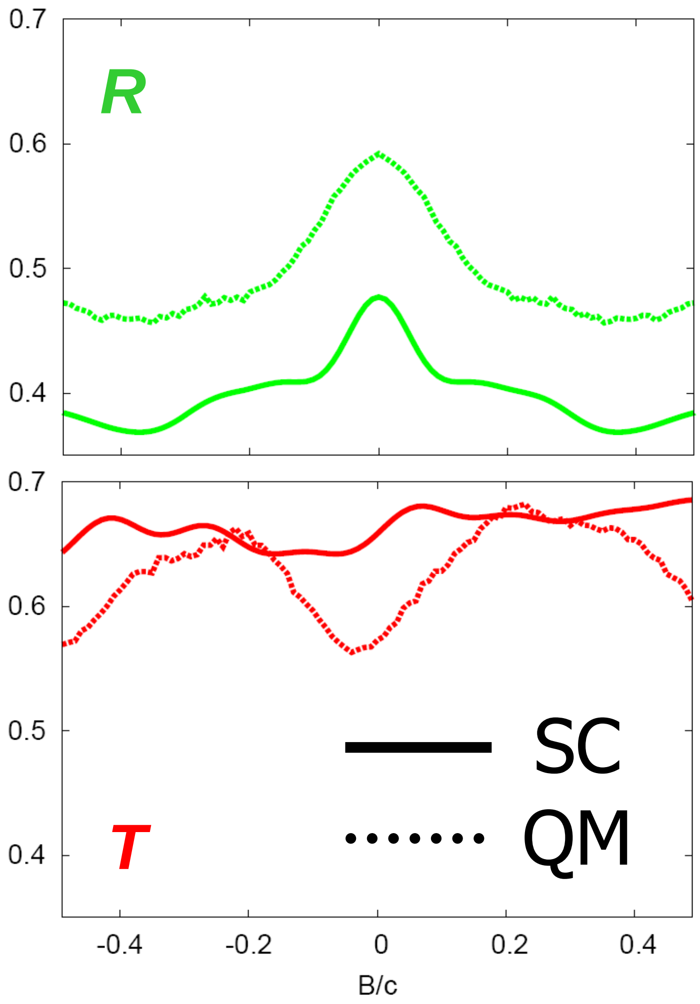
Transmission Dip Reproducible



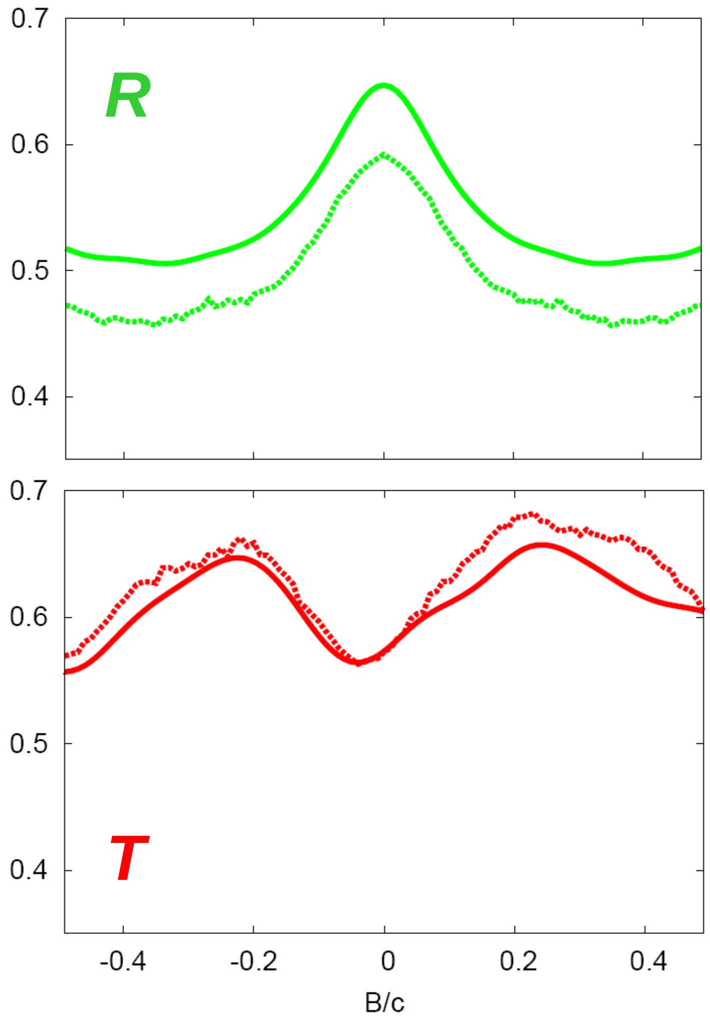
Brezinova, Stampfer, Wirtz, Rotter, Burgdörfer, arXiv:0709.3210

Transmission Dip Reproducible

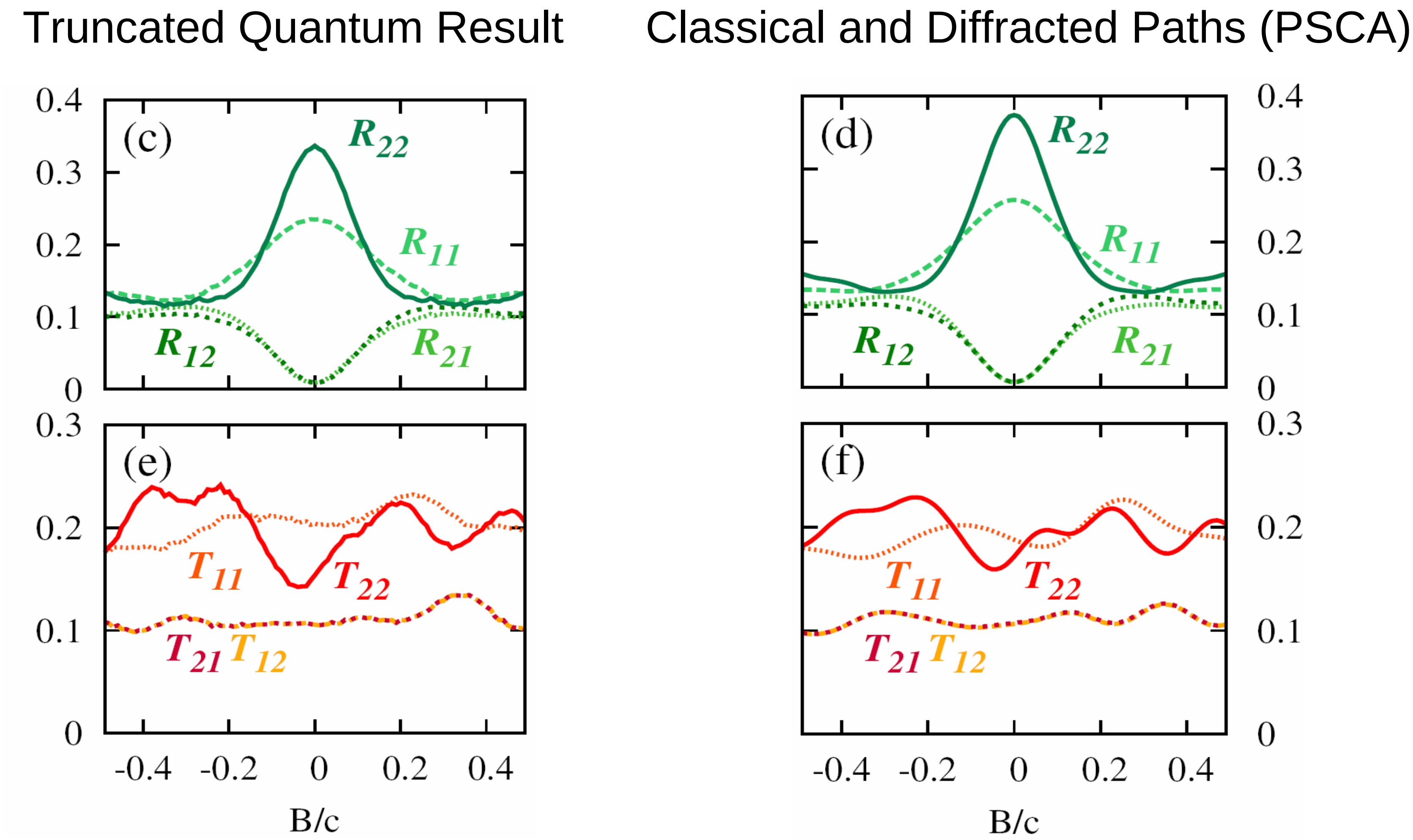
Truncated QM, Classical Paths



Truncated QM, Classical & Diffr. Paths



Comparison: S-matrix



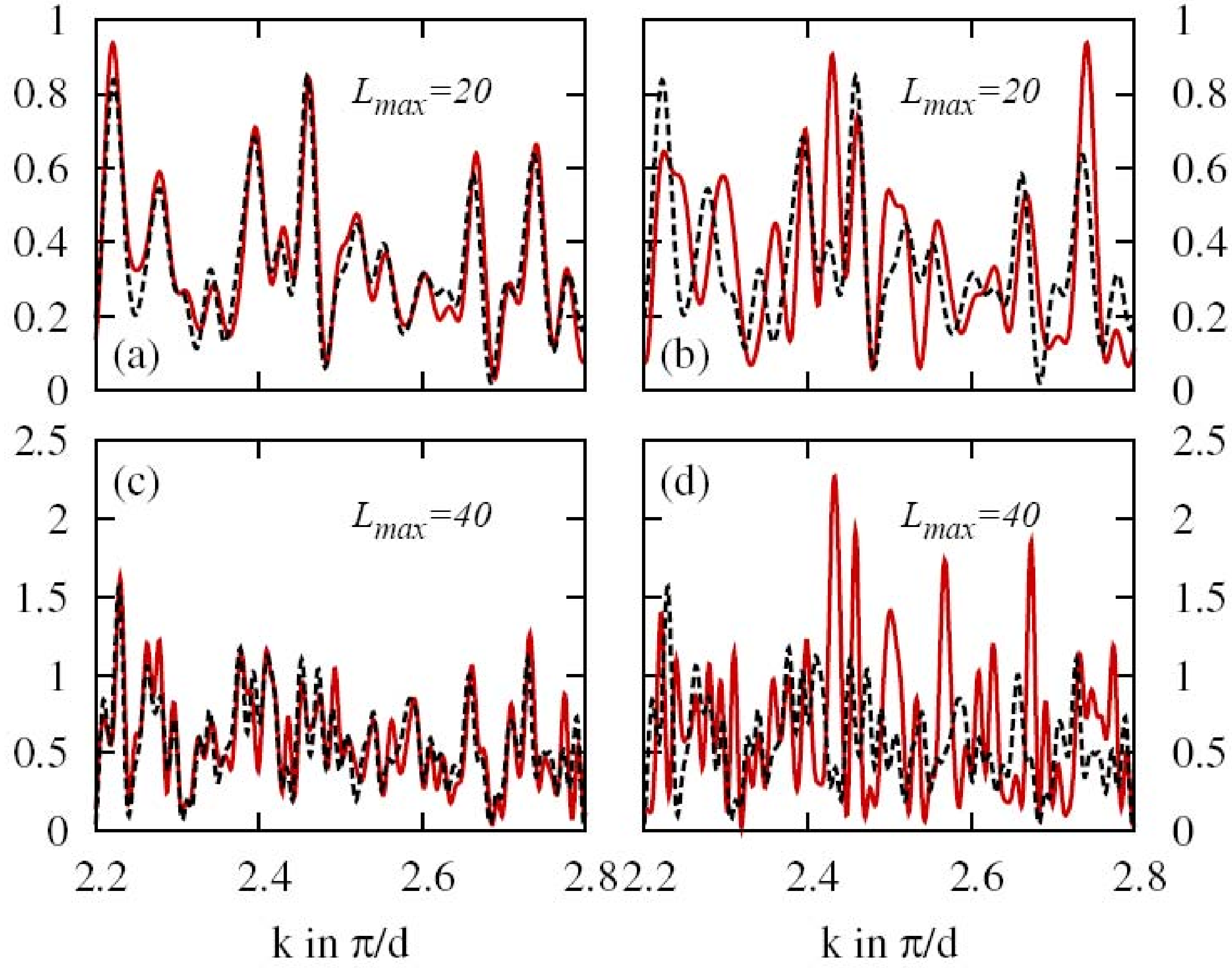
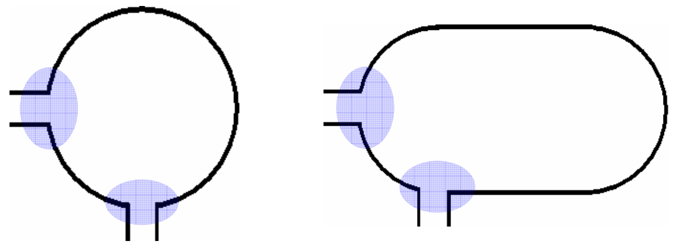


FIG. 8: Conductance fluctuations $T(k, B = 0)$ within truncated quantum mechanics (dashed line) and semiclassics (red solid line): PSCA (left column (a), (c)) and SSCA (right column (b),(d)).

Discussion



- New mechanism for WL in regular billiards
- Paths with corner-diffraction necessary for WL & Unitarity
- System-specific semiclassical theory which reproduces WL-dip in Conductance
- Paths shorter than L_E, L_φ (experiment!)
- Especially important for low N
- Effect persists for $\lambda > r_c$
- Diffractive paths: regular & chaotic cavities

Outlook

- Contributions to shot-noise, conductance fluctuations
- Diffractive paths in chaotic systems
- Relation to “universal” path pairs
- Crossover diffractive-chaotic orbits

The End

Thank you for your attention!

Please check preprint:
arXiv:0709.3210