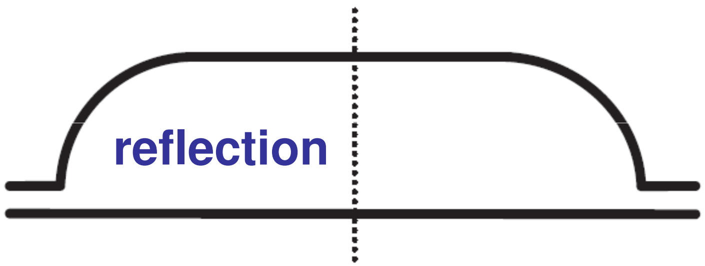
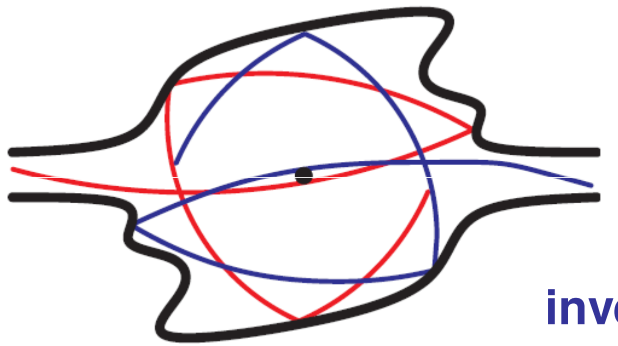
Staggered level repulsion in lead-symmetric transport





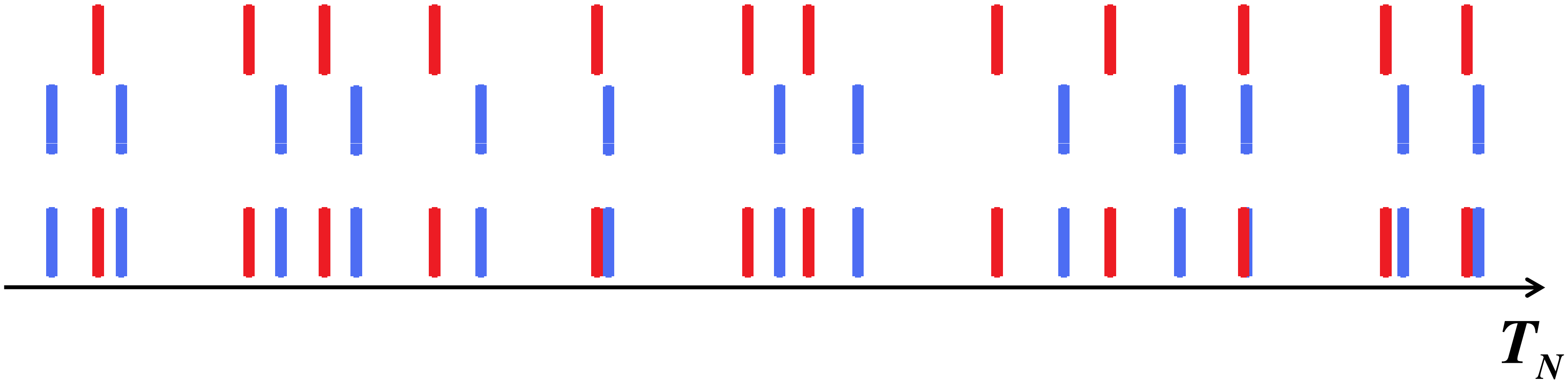
inversion

Henning Schomerus
with: M Kopp, S Rotter

Banff, 28 February 2008

MARIE CURIE ACTIONS

Staggered level repulsion in lead-symmetric transport



Henning Schomerus

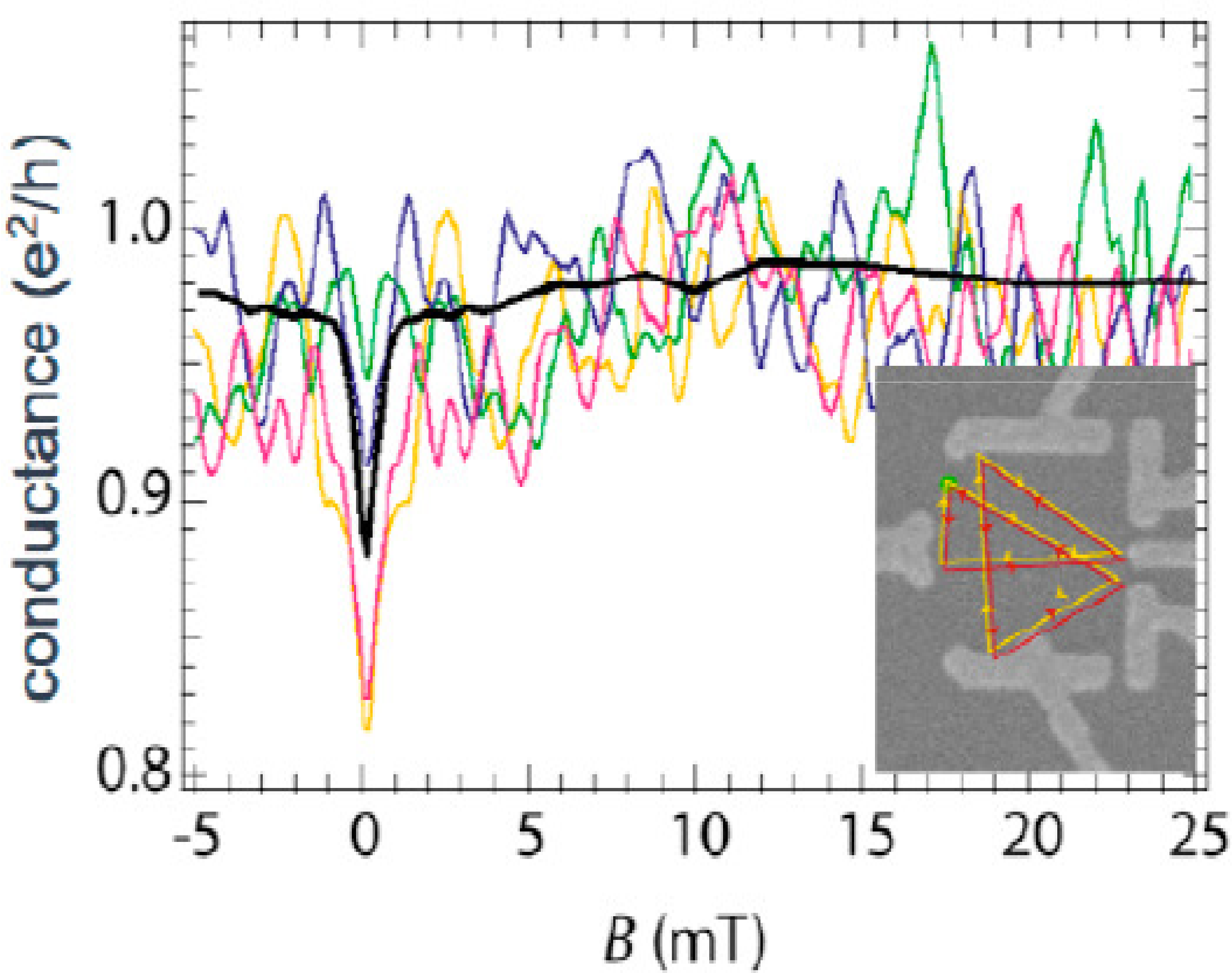
with: M Kopp, S Rotter

Banff, 28 February 2008

Overview

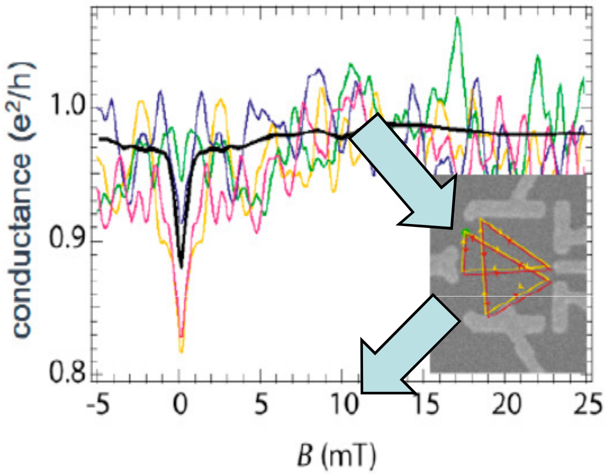
- Motivation: Transport in mesoscopic systems
- Symmetric systems
- RMT: staggered level repulsion
- Large number of channels
- *Appendix: details of the calculation*

Transport in mesoscopic systems



Marcus group

Transport in mesoscopic systems



S matrix $S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$

evals T_n of $t^\dagger t$

- conductance $G = \frac{e^2}{h} \sum_{n=1}^{N} T_n$
- shot noise $P = \frac{2e^2}{h} V \sum_{n=1}^{N} T_n (1 - T_n)$

RMT

Scattering matrix from circular ensemble
(COE: β=1; CUE: β=2; CSE: β=4)

➡ Joint pdf of transmission eigenvalues

$$P(\{T_n\}) \propto \prod_{n<m} |T_n - T_m|^{\beta} \times \prod_k T_k^{\frac{1}{2}(\beta-2)}$$

level repulsion
(UCF)

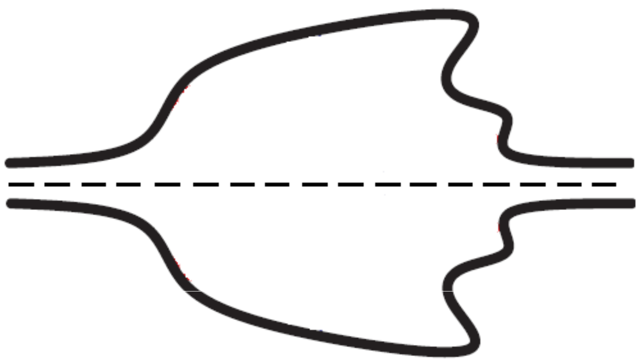
1-point density
(WL)

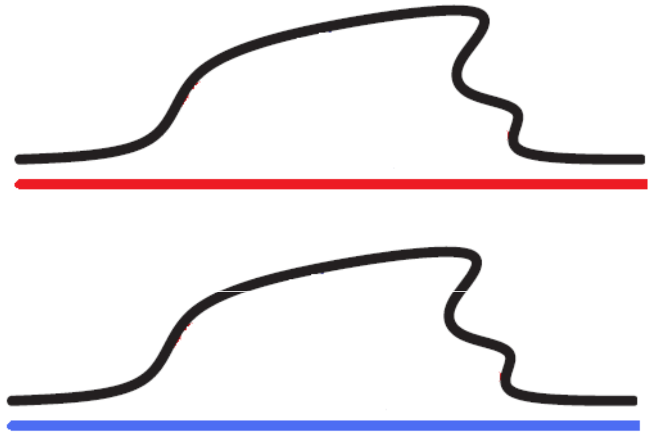
(Baranger & Mello 1994; Jalabert, Pichard & Beenakker 1994)

lead-preserving symmetries

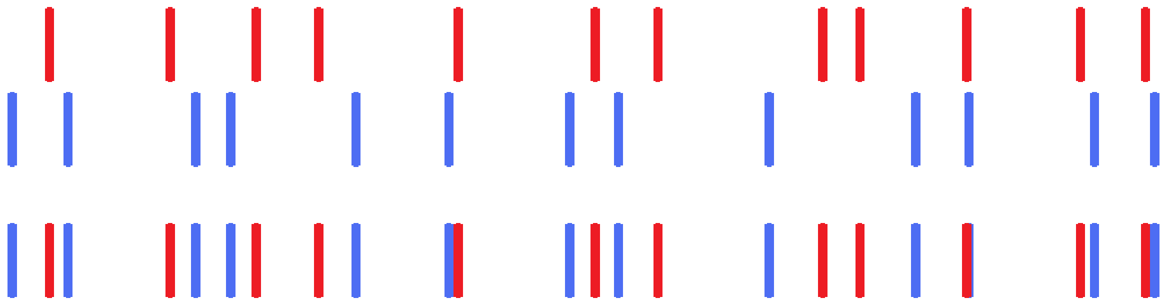
- desymmetrization

(Baranger & Mello 1996)



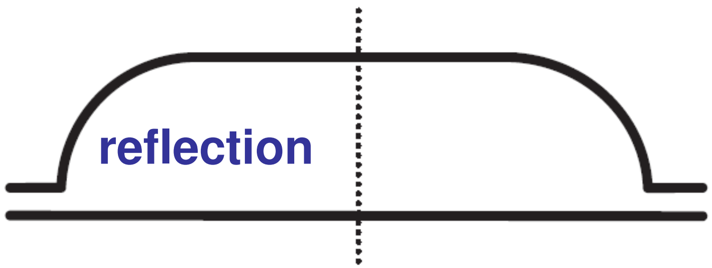


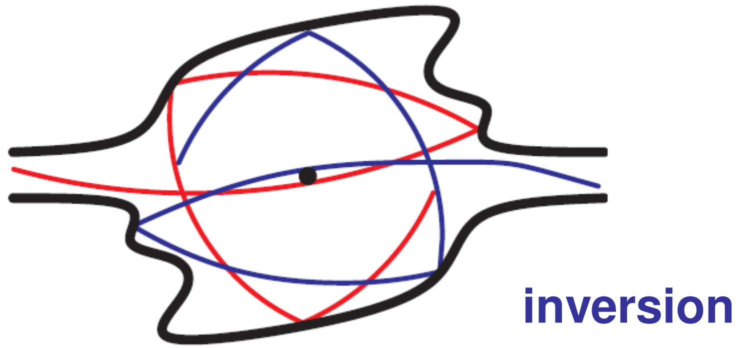
Superposition of transmission eigenvalues



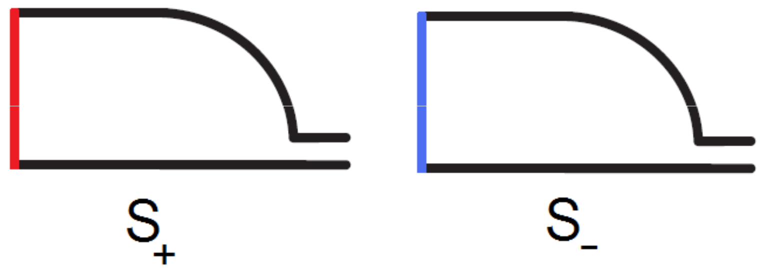
(reduced repulsion)

lead-transposing symmetries





- desymmetrization



- transmission matrix $t = \frac{1}{2}(S_{+} - S_{-})$

$$\Rightarrow \quad t^{\dagger}t = \frac{1}{4}(2 - U - U^{\dagger}), \quad U = S_{+}S_{-}^{\dagger}, \quad T_n = \sin^2\frac{\Theta_n}{2}$$

- mixes parities ([current,symmetry] ≠ 0)

previous observations

RMT: U from COE

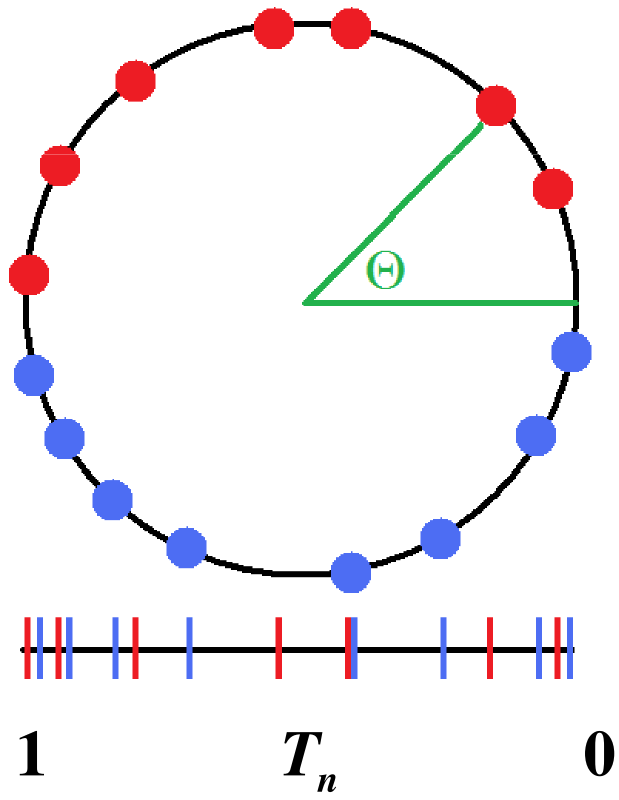
- $\mathrm{var}\,G$ **increases by factor 2** (Baranger & Mello)
- **no WL corrections** (S Rotter & co: Numerics)
- **one-point function** (Gopar, Rotter & HS)

$$T_n = \sin^2 \frac{\Theta_n}{2}, \quad \Theta_n \ \text{uniform} \Rightarrow \rho(T) = \frac{1}{\pi\sqrt{T(1-T)}}$$

here: complete statistics (β=1)

- $U = S_+ S_-^\dagger$ from COE
- $P(\Theta_n) \propto \prod_{n<m} \left| \sin\left(\frac{\Theta_n - \Theta_m}{2} \right) \right|$
- $T_n = \sin^2(\Theta_n / 2)$
- can be realized by

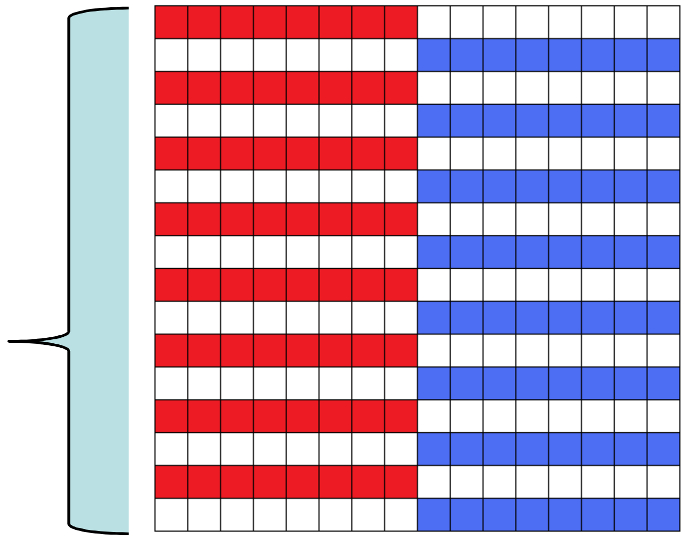
  $\Theta_n = \pm 2 \arcsin \sqrt{T_n}$



here: complete statistics (β=1)

- $U = S_+ S_-^\dagger$ from COE
- $P(\Theta_n) \propto \prod_{n<m} \left| \sin\left(\frac{\Theta_n - \Theta_m}{2} \right) \right|$
- $T_n = \sin^2(\Theta_n / 2)$
- combinatorics over $\operatorname{sgn}\Theta_n$:
  - order $T_1 \le T_2 \le \cdots \le T_N$
  - pdf as a Vandermonde det.
  - sum over $\operatorname{sgn}\Theta_n$

➢ determinant factorizes (odd indices vs even indices)



For details see appendix

Final result

$$P(\{T_n\}) \propto \left(\prod_{\substack{m>n \\ \text{both odd}}} (T_m - T_n) \right) \cdot \left(\prod_{\substack{m>n \\ \text{both even}}} (T_m - T_n) \right) \cdot \left(\prod_{l \text{ odd}} \frac{1}{\sqrt{T_l}} \right) \cdot \left(\prod_{l+N \text{ even}} \frac{1}{\sqrt{1-T_l}} \right)$$

Reduced level repulsion

➡ enhanced fluctuations

Symmetric weight

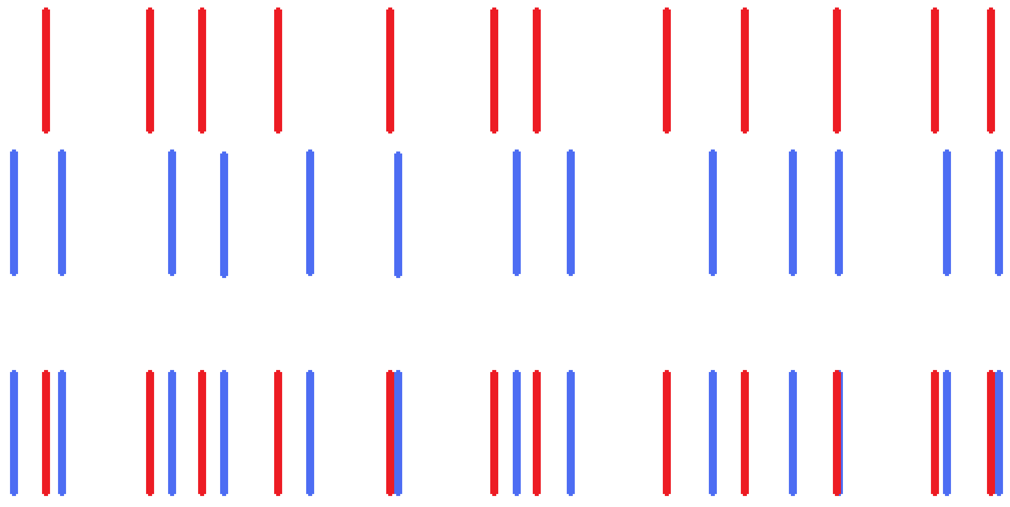
$$\rho(T) = \frac{1}{\pi\sqrt{T(1-T)}}$$

no 1/N corrections (WL)

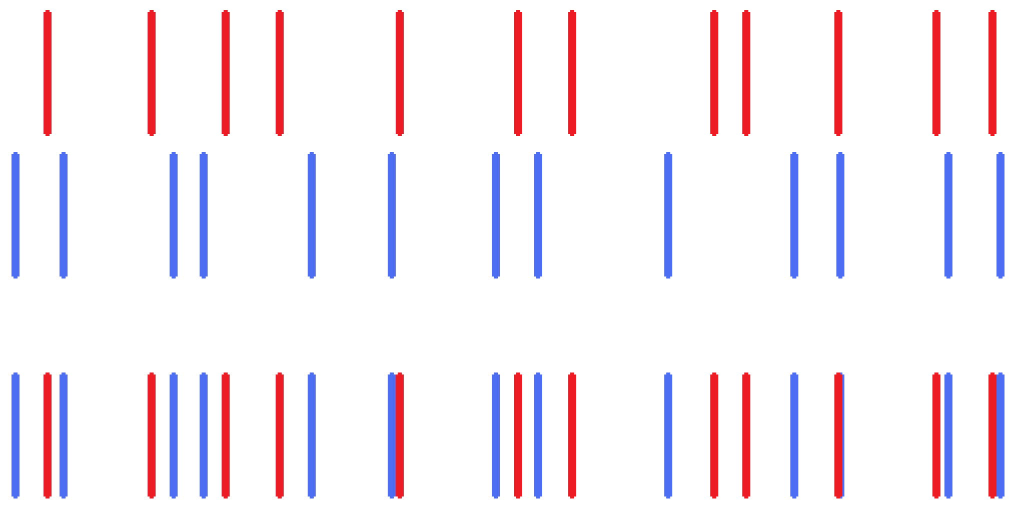
$T_1 \leq T_2 \leq \cdots \leq T_N$ ➡ **Staggered level sequence**

(magnitude, not: parity/ill defined)

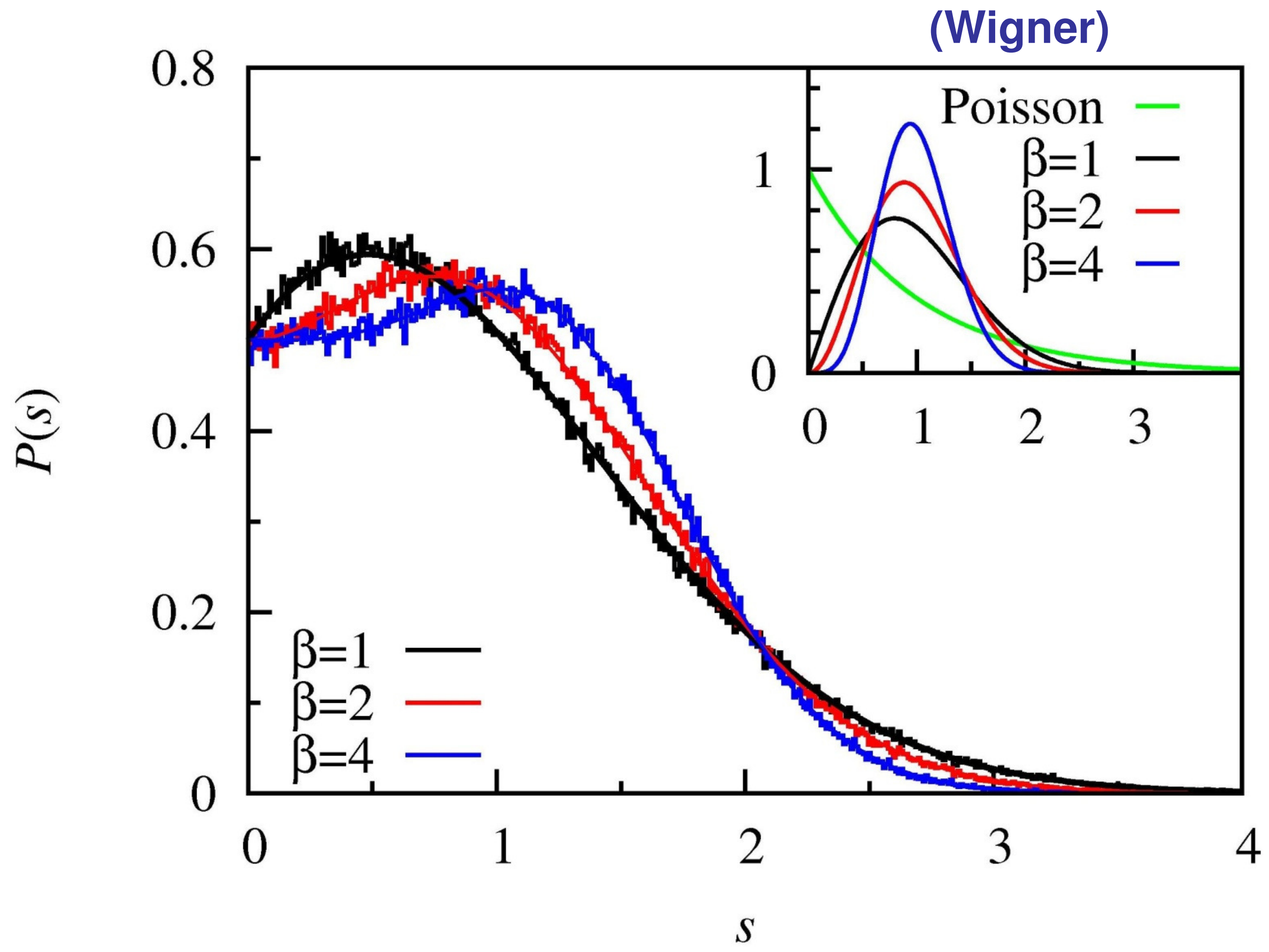
staggered level sequences (lead-transposing)



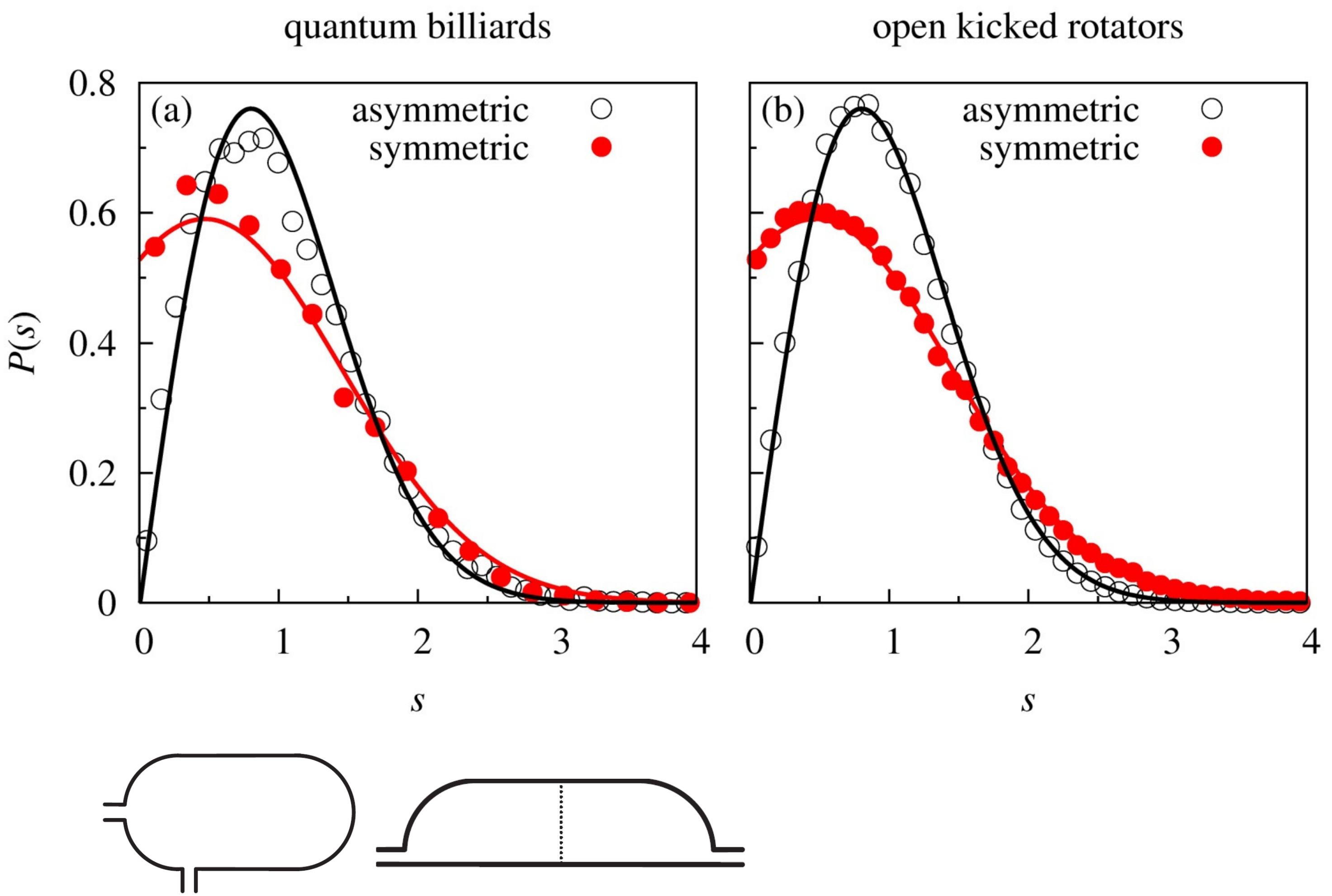
uncorrelated level sequences (lead-preserving)

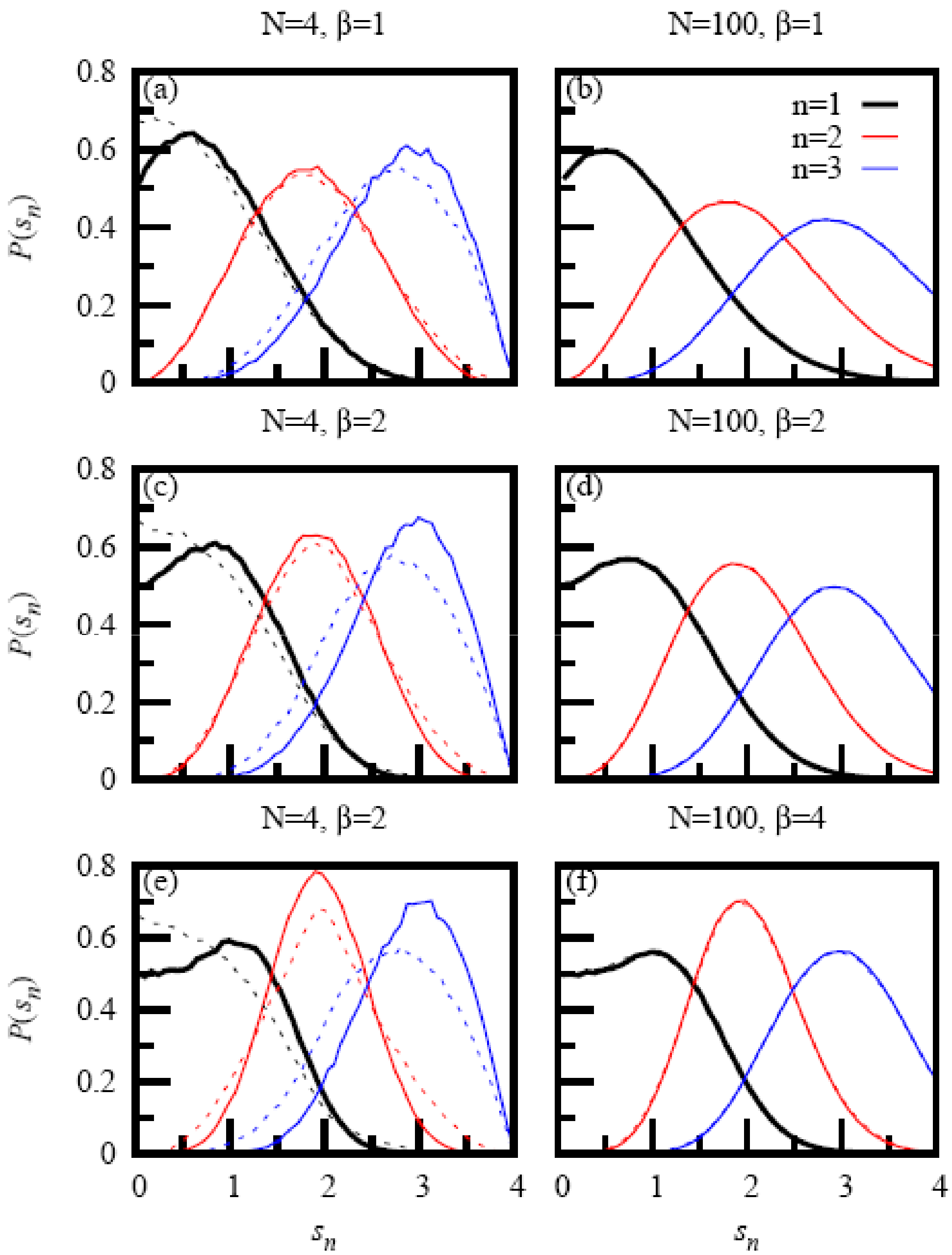


Nearest-neighbour spacing $s = T_{n+1} - T_n$



Test: Model systems





n th-nearest neighbour statistics

- large N: statistics of staggered & independent superpositions *converge*

large-N asymptotics

Observation: ***ignore weights*** $\left(\prod_{l\text{ odd}} \frac{1}{\sqrt{T_l}}\right)\cdot\left(\prod_{l+N\text{ even}} \frac{1}{\sqrt{1-T_l}}\right)$

uncorrelated superposition (2+2 levels)

$$(T_2-T_1)(T_4-T_3)+(T_3-T_1)(T_4-T_2)+(T_4-T_1)(T_3-T_2)$$
$$=2(T_3-T_1)(T_4-T_2)$$

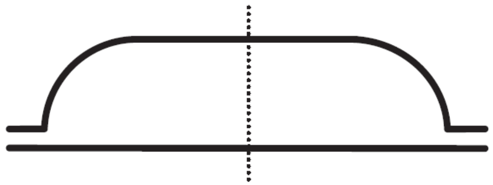
staggered superposition

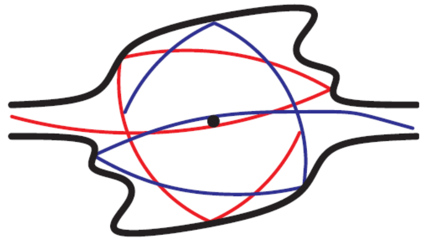
- holds for all N
- large N: continuum approx: weights → constant

(low-order) correlation functions all converge to superposition of two uncorrelated level sequences (w/o WL)

Summary

Transport in systems with lead-transposing symmetry:





- **Mixes parities**
- **Joint pdf: staggered levels, no direct repulsion**
- **For many channels: like uncorrelated level sequences (as if system could be desymmetrized)**

- **Dynamical mechanism? (semiclassics?)**

preprint: arxiv:0708.0690

Appendix: Details of the calculation

- **Order eigenphases**

$$|\Theta_1| \leq |\Theta_2| \leq |\Theta_3| \leq \ldots \leq |\Theta_N| \leq \pi \qquad \sigma_n = \operatorname{sgn}\Theta_n$$

$$P_\Theta(\{\Theta_n\}) \propto \prod_{m>n}\left[\sigma_m \sin\frac{\Theta_m - \Theta_n}{2}\right] \propto \prod_{l\ \mathrm{even}} \sigma_l \prod_{m>n} \sin\frac{\Theta_m - \Theta_n}{2}$$

- **Vandermonde determinant**

$$\prod_{m>n} \sin\frac{\Theta_m - \Theta_n}{2} = (-i)^{N(N-1)/2} \det B(\{\sigma_n \theta_n\})$$

$$B_{ml}(\{\Theta_n\}) = \exp(i\Theta_m l),\ m = 1, 2, 3, \ldots, N$$

l runs in integer steps from $-(N-1)/2$ to $(N-1)/2$

- **sum over** $\mathrm{sgn}\,\Theta_n$

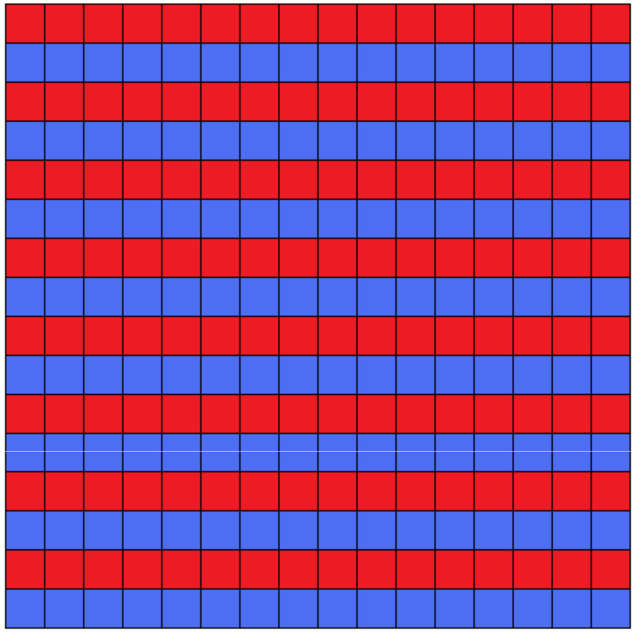
$$P_\theta(\{\theta_n\}) = \sum_{\{\sigma_n\}} P_\Theta(\{\sigma_n\theta_n\})$$

multilinearity of the determinant

$$P_\theta(\{\theta_n\}) \propto (-i/2)^{N(N-1)/2} \det C,$$

$$C_{ml} = 2\cos(\theta_m l) \text{ for odd } m$$

$$C_{ml} = 2i\sin(\theta_m l) \text{ for even } m$$



- **sum over** $\mathbf{sgn}\,\Theta_n$

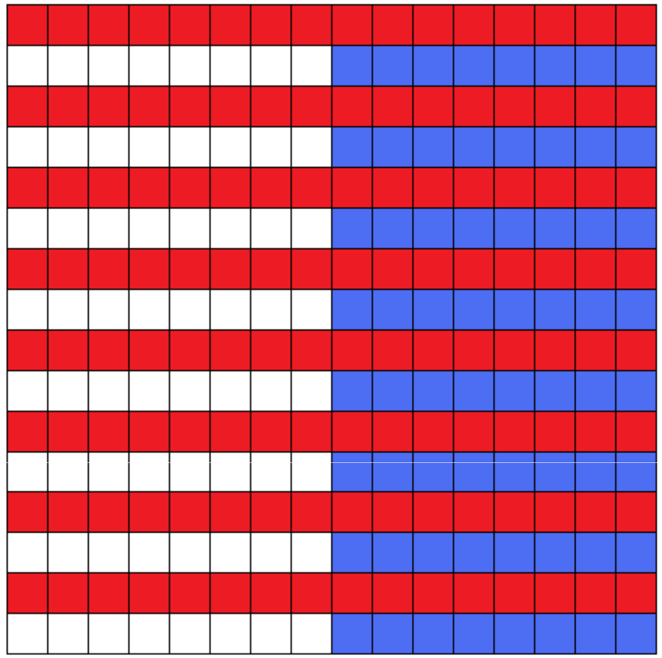
$$P_\theta(\{\theta_n\}) = \sum_{\{\sigma_n\}} P_\Theta(\{\sigma_n\theta_n\})$$

multilinearity of the determinant

$$P_\theta(\{\theta_n\}) \propto (-i/2)^{N(N-1)/2} \det C,$$

$$C_{ml} = 2\cos(\theta_m l) \text{ for odd } m$$

$$C_{ml} = 2i\sin(\theta_m l) \text{ for even } m$$



- for every $l > 0$: add lth column to $-l$the column

➡ cancels all sine terms in the latter columns

- sum over $\mathrm{sgn}\,\Theta_n$

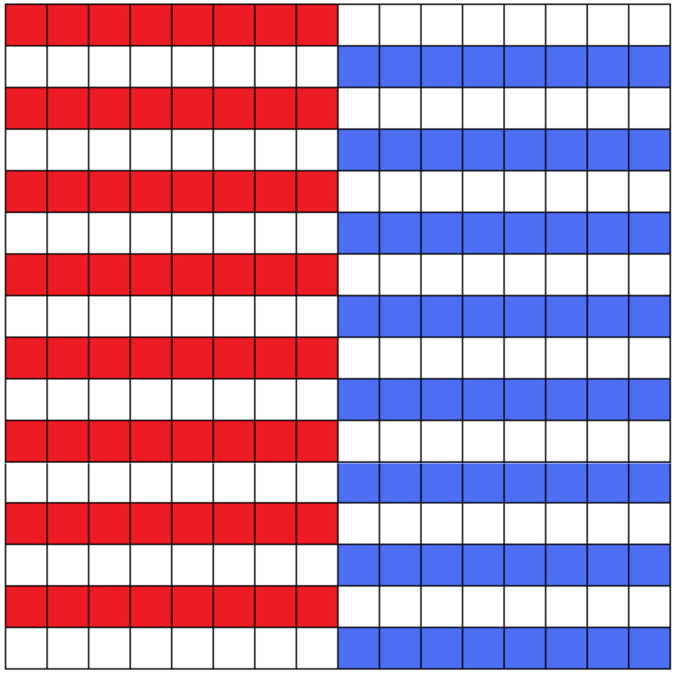
$$P_\theta(\{\theta_n\}) = \sum_{\{\sigma_n\}} P_\Theta(\{\sigma_n \theta_n\})$$

multilinearity of the determinant

$$P_\theta(\{\theta_n\}) \propto (-i/2)^{N(N-1)/2} \det C,$$

$$C_{ml} = 2\cos(\theta_m l) \text{ for odd } m$$

$$C_{ml} = 2i\sin(\theta_m l) \text{ for even } m$$

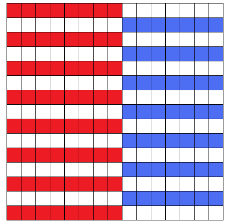


- for every $l > 0$: add lth column to $-l$the column

➡ cancels all sine terms in the latter columns

- **determinant factorises** $\det C = \det D \det E$

N odd



$$\det C = \det D \det E$$

$D_{ml} = \cos\theta_m l,\ m$ odd $\qquad l = 0, 1, 2, \ldots, (N-1)/2$ for the matrix D

$E_{ml} = \sin\theta_m l,\ m$ even $\qquad l = 1, 2, \ldots, (N-1)/2$ for the matrix E

D_{ml}: polynomial in $\cos\theta_m$ of degree l

E_{ml}: $\sin\theta_m$ times polynomial of degree $l-1$

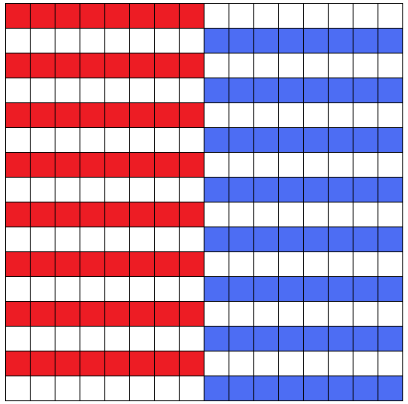
only need to keep highest monomial $\cos^l\theta_m$

➡ Vandermonde determinant

$$\det D \propto \prod_{m>n,\ \text{both odd}} (\cos\theta_n - \cos\theta_m),$$

$$\det E \propto \prod_{l\ \text{even}} \sin\theta_l \prod_{m>n,\ \text{both even}} (\cos\theta_n - \cos\theta_m)$$

N even



$$\det C \;=\; \det D \det E$$

$$D_{ml} = \cos\theta_m l,\; m \text{ odd}$$

$$E_{ml} = \sin\theta_m l,\; m \text{ even}$$

$$l = 1/2, 3/2, \ldots, (N-1)/2$$

D_{ml}: $\cos(\theta_m/2)$

E_{ml}: $\sin(\theta_m/2)$

} times polynomial in $\cos\theta_m$ of degree $(l-1)/2$

only need to keep highest monomial $\cos^l\theta_m$

➡ Vandermonde determinant

$$\det D \;\propto\; \prod_{l \text{ odd}} \cos(\theta_l/2) \prod_{m>n,\,\text{both odd}} (\cos\theta_n - \cos\theta_m)$$

$$\det E \;\propto\; \prod_{l \text{ even}} \sin(\theta_l/2) \prod_{m>n,\,\text{both even}} (\cos\theta_n - \cos\theta_m)$$

- **transformation to** $T_n = \sin^2 \frac{\Theta_n}{2}$

$$\cos\theta_n - \cos\theta_m = 2(T_m - T_n)$$

$$\sin(\theta_l/2) = \sqrt{T_l}$$

$$\cos(\theta_l/2) = \sqrt{1 - T_l}$$

$$\sin(\theta_l) = 2\sqrt{T_l(1 - T_l)}.$$

Jacobian $d\theta_n/dT_n = 1/\sqrt{T_n(1 - T_n)}$

Final result

N even:

$$P(\{T_n\}) \propto \left(\prod_{\substack{m>n \\ \text{both odd}}} (T_m - T_n) \right) \cdot \left(\prod_{\substack{m>n \\ \text{both even}}} (T_m - T_n) \right) \cdot \left(\prod_{l \text{ odd}} \frac{1}{\sqrt{T_l}} \right) \cdot \left(\prod_{l \text{ even}} \frac{1}{\sqrt{1-T_l}} \right)$$

N odd:

$$P(\{T_n\}) \propto \left(\prod_{\substack{m>n \\ \text{both odd}}} (T_m - T_n) \right) \cdot \left(\prod_{\substack{m>n \\ \text{both even}}} (T_m - T_n) \right) \cdot \left(\prod_{l \text{ odd}} \frac{1}{\sqrt{T_l (1-T_l)}} \right)$$

or:

$$P(\{T_n\}) \propto \left(\prod_{\substack{m>n \\ \text{both odd}}} (T_m - T_n) \right) \cdot \left(\prod_{\substack{m>n \\ \text{both even}}} (T_m - T_n) \right) \cdot \left(\prod_{l \text{ odd}} \frac{1}{\sqrt{T_l}} \right) \cdot \left(\prod_{l+N \text{ even}} \frac{1}{\sqrt{1-T_l}} \right)$$