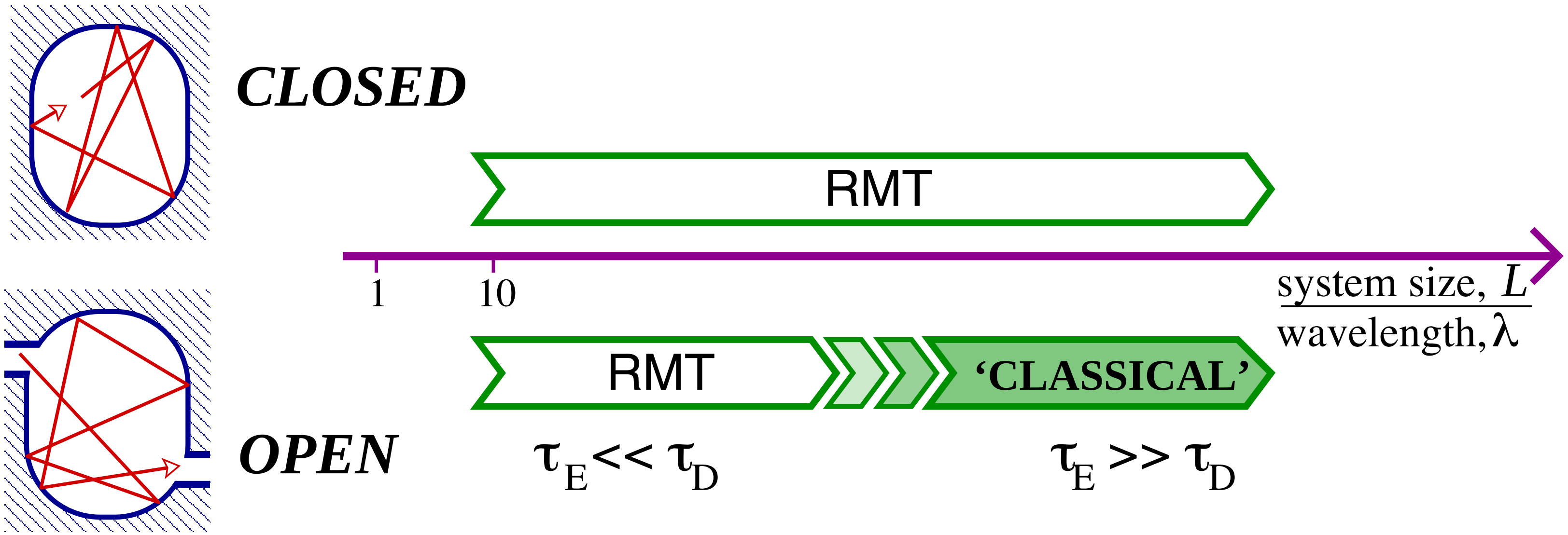
Institut Laue Langevin, Grenoble, France.

Banff, February 2008

Introduction – Ehrenfest time in scattering problems

Robert S Whitney



Collaborations & Discussions: P. Jacquod, C. Petitjean, P. Brouwer, E. Sukhorukov

When is quantum = classical ?

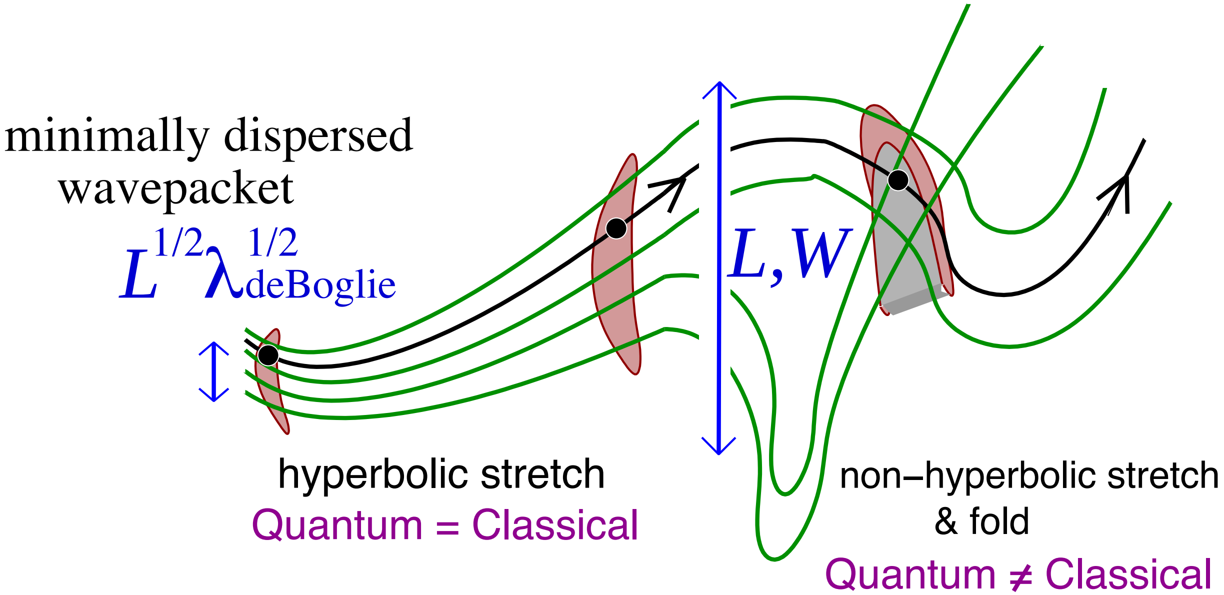
QUANTUM Ehrenfest theorem

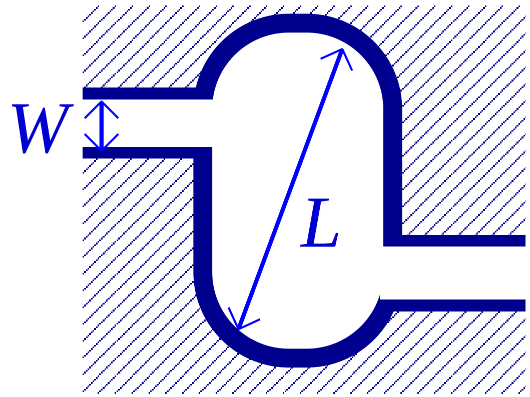
$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho} = -\mathrm{i}\hbar[\hat{\mathcal{H}}, \hat{\rho}]$$

CLASSICAL Liouville theorem

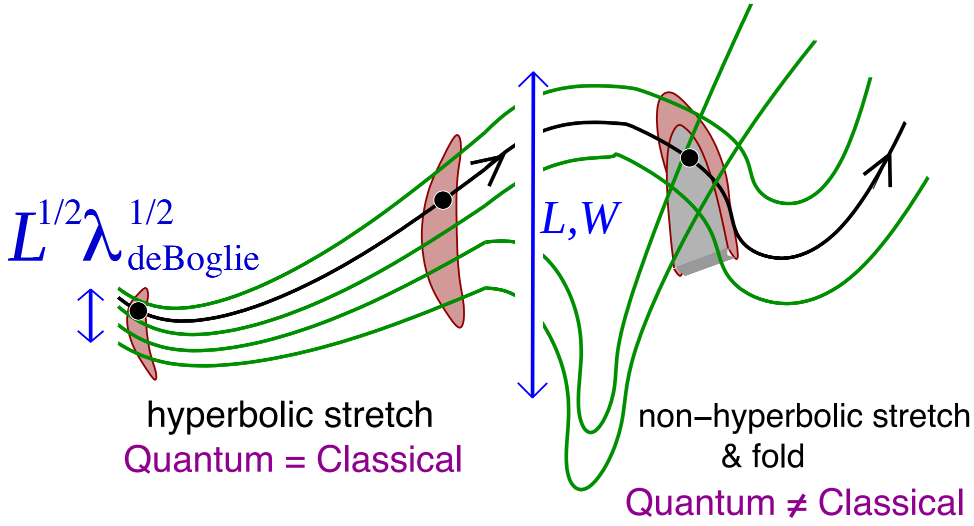
$$\frac{\mathrm{d}}{\mathrm{d}t} f(r,p) = -\{\mathcal{H}, f(r,p)\}$$

commutator —(phase−space, Wigner function)→ Moyal bracket $\neq$ Poisson bracket





Ehrenfest time (log time)



WAVEPACKET growth $\propto \exp[-\Lambda t]$

Ehrenfest Time, τ_{E}:

time during which

quantum $=$ *classical*

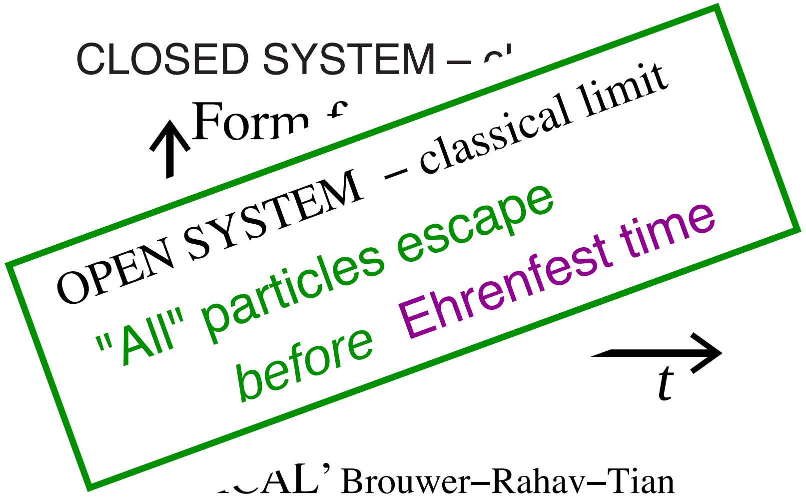
$$\tau_{\mathrm{E}} = 2\Lambda^{-1}\ln[\mathcal{L}/\lambda_{\mathrm{deBroglie}}]$$

with $\mathcal{L}{=}L,\,(W^2/L)$

TIMESCALES:

- ♣ $\big(\text{Level-spacing}\big)^{-1} \;\propto (\mathcal{L}/\lambda_{\mathrm{deBroglie}})$
- ♣ Ehrenfest time $\;\propto \ln[\mathcal{L}/\lambda_{\mathrm{deBroglie}}]$
- ♣ Classical scales:

  dwell time = $\lambda_{\mathrm{deBroglie}}$-independent



A short and *inaccurate* history of the Ehrenfest time!

LAST CENTURY

Ehrenfest: Ehrenfest theorem. $\neq$ Liouville theorem — Z. Physik 45 455 (1927)

??? Larkin-Ovchinnikov: superconducting fluct. in metals — Sov. Phys. JETP 28 1200 (1969)

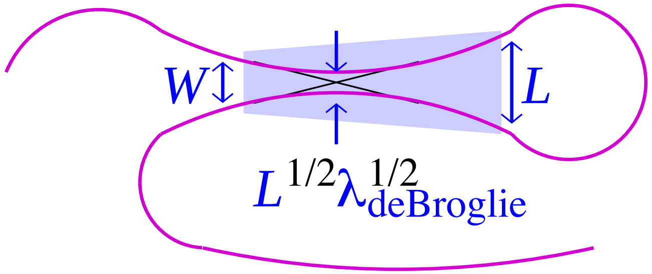
Berman-Zaslavsky: stochasticity — Physica A 91, 450 (1978), Phys. Rep. 80, 157 (1981)

Aleiner-Larkin weak-loc. in transport & level-statistics — PRB 54, 14423 (1996); 55, R1243 (1997).

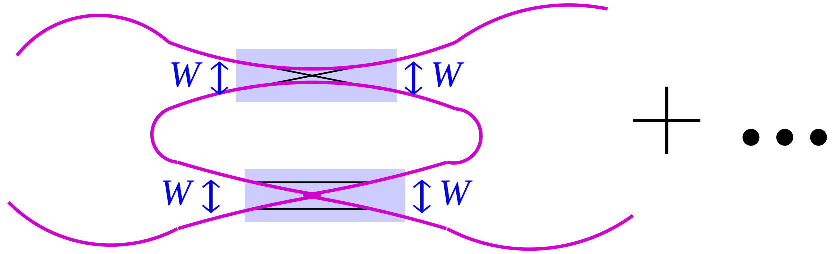
THIS CENTURY Semiclassics ... see other talks this week.

- ♣ Cut-off on loops Sieber, Müller-Braun-Heusler-Haake
- ♣ Off-diagonal (Noise, weak-loc., UCFs, etc) suppressed (or not) Rahav-Brouwer & Jacquod-Whitney

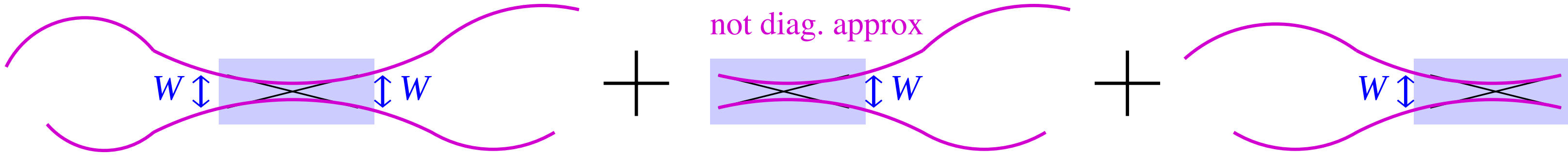
Weak localization and back−scattering



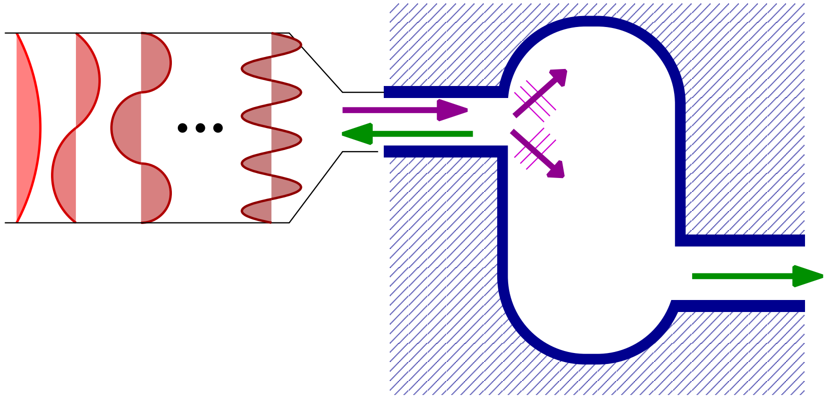
Conductance fluctuations



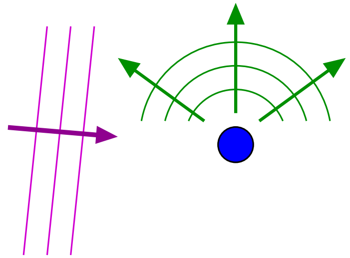
Fano factor of shot noise



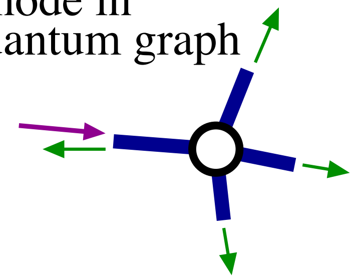
Open systems – scattering matrix



cf. scattering from atom



or node in
quantum graph



Scattering matrix, $\mathcal{S} = \begin{pmatrix} \mathbf{r} & \mathbf{t} \\ \mathbf{t}' & \mathbf{r}' \end{pmatrix}$

Landauer-Büttiker: eigenvalues of $\mathbf{t}^\dagger \mathbf{t} \rightarrow$ *all* transport properties

- ♣ Electrical conductance $\propto \sum_n T_n$
- ♣ quantum noise $\propto \sum_n T_n(1 - T_n)$ & higher moments

Quantum noise : measures ***determinism***

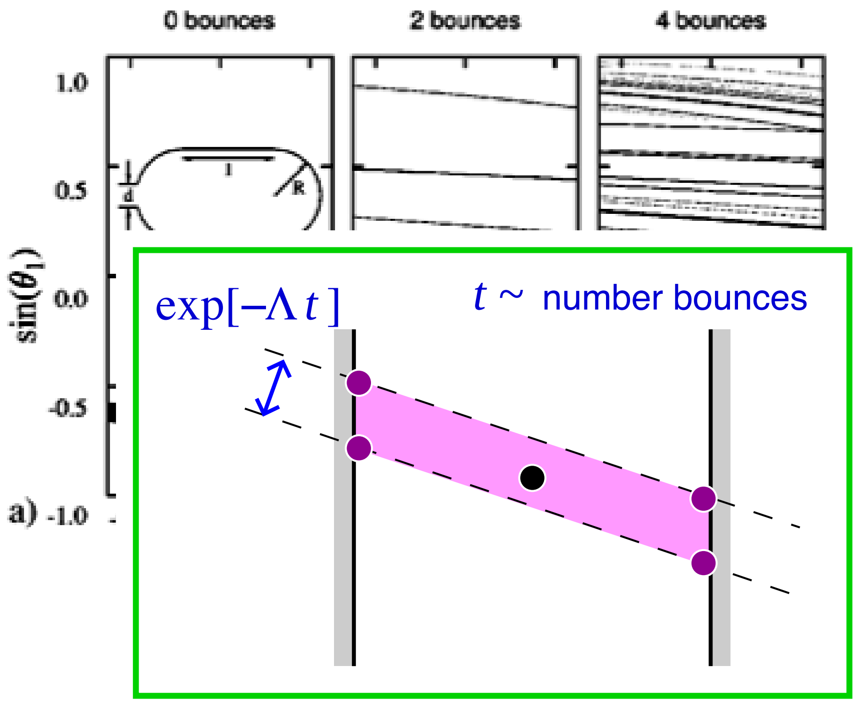
Bands in classical phase-space

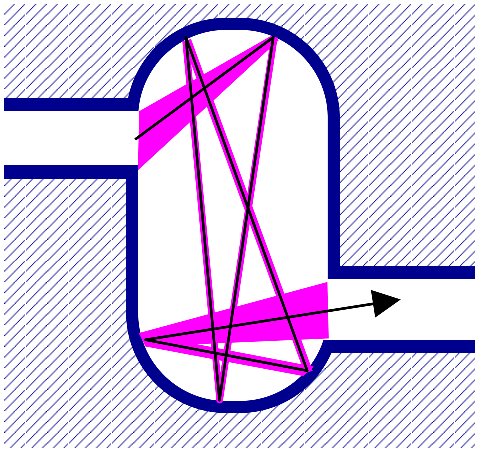
Silvestrov-Goorden-Beenakker (2003)
Tworzydlo-Tajic-Schomerus-Beenakker (2003)
Whitney-Jacquod (2005)

Open systems:

Paths in *families* (bands)

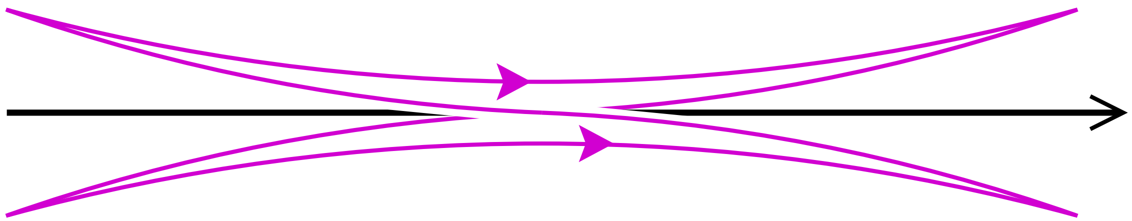
cf. integrable systems



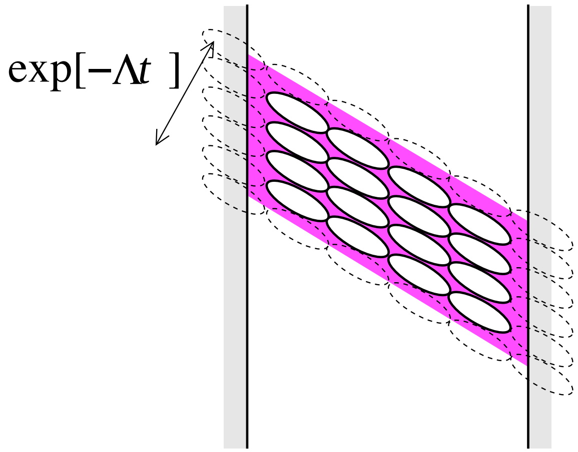


Paths associated with 4 band corners

dynamics relative to black path

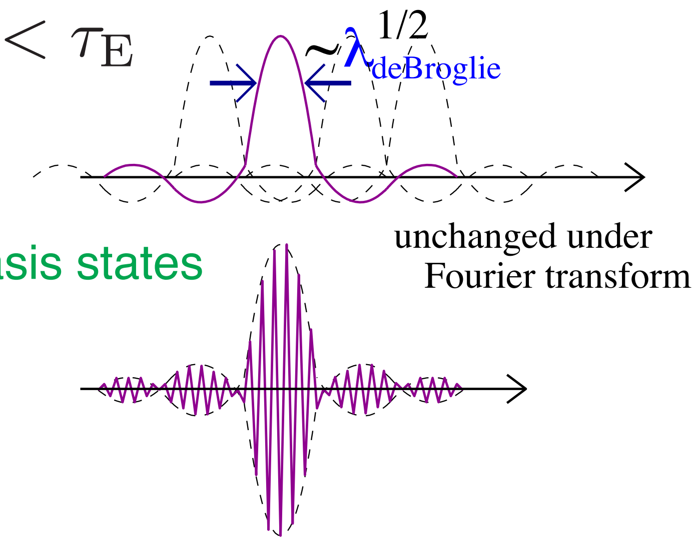


Phase-space basis : orthogonal basis of wavepackets



Cover: bands with area $> 2\pi\hbar$

$\Rightarrow$ paths with $t < \tau_{\mathrm{E}}$



$$\mathbf{t} = \begin{pmatrix} \spadesuit & 0 & 0 & \cdots & 0 & 0 \\ 0 & \spadesuit & 0 & \cdots & 0 & 0 \\ 0 & 0 & \ddots & & \vdots & \vdots \\ \vdots & & & \spadesuit & 0 & 0 \\ 0 & 0 & \cdots & 0 & \diamondsuit & \diamondsuit \\ 0 & 0 & \cdots & 0 & \diamondsuit & \diamondsuit \end{pmatrix}$$

$\Longleftarrow$ CLASSICAL block: ***diagonal***

| eigenvalues | = 0,1

$\Longleftarrow$ QUANTUM block: ***not diagonal***

semiclassics shows *non-RMT.*

Eigenvalues of $\mathbf{t}^\dagger\mathbf{t} = 0, 1 \;\Rightarrow\; T_n(1-T_n)=0 \;\Rightarrow$ Noise-less Modes

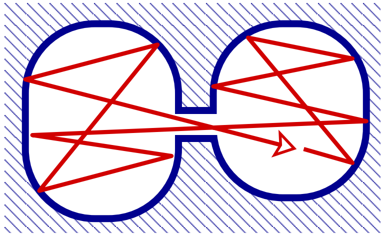
Conclusion

Open systems: Ehrenfest time controls cross-over to
NEW universal regime (*non-RMT*) as $\lambda_{\rm deBroglie} \to 0$.

Open questions

[A] Ehrenfest time relevant for closed systems?

- need classical timescale; i.e. inter-billiard transfer time

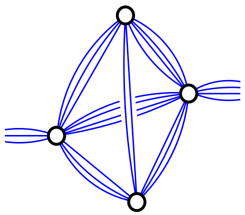


[B] Re-summing loop expansion for finite Ehrenfest time?

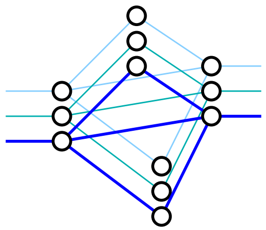
[C] Negative Ehrenfest time $\tau_{\rm E} = \Lambda^{-1} \ln[\mathcal{L}/\lambda_{\rm deBroglie}]$??
... for noise $\mathcal{L} = W^2/L \to 0$ for $W \ll L$.

[D] Ehrenfest time in graphs?

- require correlated actions?



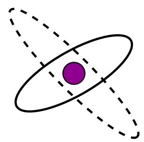
or



Typical Ehrenfest times

Zurek (2003)

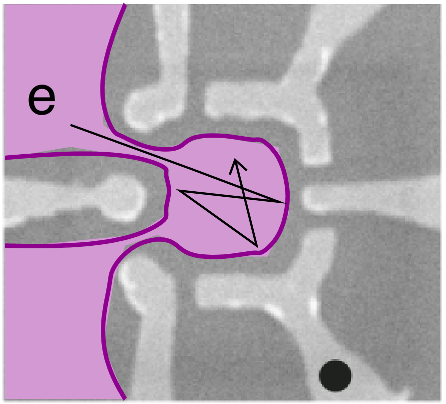
QUANTUM ⟷ CLASSICAL



He

$\mathcal{L} \sim \lambda_{\mathrm{deBroglie}}$

$$\tau_E = \Lambda^{-1} \ln[1] \sim 0$$



Quantum dot

Marcus Group, Harvard (2003)

$\mathcal{L} \sim 10^3\ \lambda_{\mathrm{deBroglie}}$

$$\tau_E = \Lambda^{-1} \ln[10^3] \sim 7\tau_0$$

Double pendulum

www.chaoticpendulums.com

$\mathcal{L} \sim 10^{34}\ \lambda_{\mathrm{deBroglie}}$

$$\tau_E = \Lambda^{-1} \ln[10^{34}] \sim 80\tau_0$$

Schrodinger cat state after 2 minutes !!

Decoherence??