TEXAS A&M UNIVERSITY
DEPARTMENT OF MATHEMATICS

MATH 425-500

Midterm test, 11 Mar 2015

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this work.

Name (print):

No detailed analytical work — no points.

Each question is worth 10 points

1. Matlab offers a black-box function to evaluate the *error function*, defined by

$$\operatorname{erf}(x) := \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

Express the distribution function $N(x) = \mathbb{P}(X \leq x)$ of the standard normal variable X in terms of the function erf. The density of the standard normal is

$$p(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

2. You go to lunch with financial guru W.B. In strictest confidence, he tells you that in 1 year's time the E&R 500 stock index will be either at 2000 or at 2300 points. The index is currently at 2100. Acting upon this information and using the "dichotomy" (aka "1-level binomial tree") model, you price and sell 100 calls for the index at the strike $E = 2180$. In the call price, you include a \$10 mark-up. Assume, for simplicity, that you have access to interest-free borrowing, i.e. $r = 0$.

1. What is the price at which you sell the calls (including the mark-up)?
2. Describe the hedging procedure you undertake. What is you total profit/loss if the index ends up at 2000? What is you total profit/loss if the index ends up at 2300?
3. If W.B. is wrong and the index ends up at 2320, what is your total profit/loss?
4. What do you think E&R stand for?

3. The Black–Scholes–Merton formula for the price of a put with strike E is

$$P(S,t) = Ee^{-r(T-t)}N(-d_2) - SN(-d_1),$$

where

$$d_{1,2} = \frac{\ln(S/E) + \left(r \pm \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}.$$

1. Show that the Delta for the put is given by

$$\Delta := \frac{\partial P}{\partial S} = -N(-d_1). \tag{1}$$

   (Hint: you may use any of the formulas in the back, including the expression for Delta of a call and the put-call parity.)

2. What's the approximate value of Delta for a put that is deep in the money ($S \ll E$)? For a put that is far out of the money ($S \gg E$)? Derive/justify your answers from equation (1). Give an interpretation for your answers from the hedging perspective.

4. The so-called "box spread" consists of four options: long E_1 call, short E_1 put, short E_2 call and a long E_2 put.

1. Calculate the payoff from a box spread at expiration, in terms of E_1 and E_2.

2. Use put-call parity to calculate the price of the box spread at time $\tau = T - t$ before expiration, if the risk-free rate is $r > 0$. You do not need to know the prices of the individual options to price the box spread!

3. Give an explanation for your answer from the "no-arbitrage" point of view.

4. Profits from option trading are often taxed at a reduced rate (because the investor undertakes risk), when compared to tax rate on wage or (risk-free) interest earnings. Why do you think the IRS takes a dim view of using box spreads?

Points: /40

Formulas that might be useful

- Call payoff
$$C(T, S, E) = \max(0, S_T - E)$$
- Put payoff
$$P(T, S, E) = \max(0, E - S_T)$$
- Put-call parity
$$C(t, S, E) + Ee^{-r(T-t)} = P(t, S, E) + S_t.$$
- Dichotomy model with $r = 0$
$$C = (S_0 - S_d)\Delta, \qquad \text{where } \Delta = \frac{S_u - E}{S_u - S_d}.$$
- Black–Scholes–Merton model for call price
$$C(t, S, E) = SN(d_1) - Ee^{-r(T-t)}N(d_2),$$
where
$$d_{1,2} = \frac{\ln(S/E) + \left(r \pm \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}.$$
- Delta for the call is given by
$$\Delta := \frac{\partial C}{\partial S} = N(d_1),$$
derived using the identity
$$SN'(d_1) - Ee^{-r(T-t)}N'(d_2) = 0.$$