

TEXAS A&M UNIVERSITY
DEPARTMENT OF MATHEMATICS

MATH 425-500

Midterm test, 10 Mar 2016

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this work.

Name (print):

No detailed analytical work — no points.

Each question is worth 10 points

1. You purchase a “long call ladder” spread by buying one 90 Call for \$12.35, selling one 100 Call for \$3.15 and selling a 110 Call for \$1.20.
 1. Plot the payoff diagram of your spread. Label your axes!
 2. Plot the profit diagram of your spread (assume, for simplicity, that $r = 0$).
 3. Would the spread be worth buying if the 90 Call was priced at \$15.00 while the other prices remained the same?

2. Let X be a continuous random variable and $Y = aX + b$ for some $a, b \in \mathbb{R}$.
 1. Compute $\mathbb{E}(Y)$ in terms of $\mathbb{E}(X)$.
 2. Compute $\text{var}(Y)$ in terms of $\text{var}(X)$.
 3. Compute the probability density function $f_Y(y)$ of Y in terms of the probability density function $f_X(x)$, starting from the definition of the p.d.f. (see attached formulae).

3. You buy a Call and sell a Put with the same strike $E = S_0 e^{rT}$.
 1. Plot the payoff diagram of your spread; calculate the formula for it and simplify it.
 2. Derive the price for your spread from first principles (e.g. using the Portfolio Lemma).
 3. Derive the price for your spread from the put-call parity.
 4. Why is such a spread called “synthetic forward”?

4. Price the following European call option using the 3-level binomial tree with $u = 1.2$ and $d = 0.9$: time to expiration $\tau = 3$ months, interest rate is $r = 0.06$, spot price $S_0 = 100$, strike is $E = 115$.

Suppose the stock moves up, then down and then up again. Describe the delta-hedging procedure you undertake, calculate cash balances at every node (assume no margins, i.e. everything is bought and sold for its fair value; if you round to cents, arithmetical errors accumulate to about 0.1).

Points: /40
