TEXAS A&M UNIVERSITY
DEPARTMENT OF MATHEMATICS

MATH 425-500

Midterm test version A, 9 Mar 2017

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this work.

Name (print):

No detailed analytical work — no points.

Each question is worth 10 points unless noted otherwise

1. (Based on a real story) You observe that almost every time a certain Twitter account issues a tweet, the market moves strongly. Sometimes it moves up and sometimes it moves down. To make money from either move, you write a program that tracks the updates to this Twitter account and buys a straddle spread on SPY. A straddle consists of one (long) call and one (long) put with the same strike prices. SPY is a fund that tracks the stock-market index. For your strike you use the current (spot) price of SPY.
At 4pm on 2017-03-07, the twitter account becomes active and your program makes a purchase. The spot price is $S_0 = 237$, so your program buys 100 calls with strike $E = 237$ and expiration in 3 days at the premium \$0.73 per call and also 100 puts with the same strike and expiration at the premium \$0.72 per put.

1. Draw the payoff diagram of your spread.
2. Draw the profit diagram of your spread. Label your axes and choose the appropriate scale for them. For simplicity, take $r = 0$.
3. By how much does the SPY price have to swing so that your purchase generates profit. Does the direction of the swing matter?
4. Is it a coincidence that the option prices quoted above are so close? Support your answer with equations.

2. **(20 points)** Calculate the prices for the spread in the previous question using the following settings: 3 level tree (one level per day), daily move is $u = 1.01$ or $d = 0.99$, $S_0 = 237$, $E = 237$, $r = 0$. Assume the options are European. (Hint: you know the payoff for the spread at the expiration time, use these values in the last level and propagate backwards as usual).

Suppose the stock moves up, then up again and then down. Describe the delta-hedging procedure the writer undertakes, calculate cash balances at every node. What is the final balance? (If you round to cents every number you use, numerical error is not more than a few cents).

Bonus +1 pt: Compare the price for the spread you got here with the prices in Q.1. Who assumes higher volatility, you or the market?

3. In finance, it is often important to know the distribution of $Y = Z^2$, where Z is standard normal. Calculate the probability density function of Y.

Hints: use the formula $f_X(x) = \frac{d}{dx}\mathbb{P}(X < x)$ and the Leibnitz rule

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(s)ds = f(b(x))\, b'(x) - f(a(x))\, a'(x);$$

the density of the standard normal is $f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$; be careful with $\sqrt{x^2} \neq x$.

Points:	/40