TEXAS A&M UNIVERSITY
DEPARTMENT OF MATHEMATICS

MATH 425-500

Midterm test version A, 8 Mar 2018

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this work.

Name (print):

No detailed analytical work — no points.

Each question is worth 10 points unless noted otherwise

1. You assumed the 1-level tree model, priced and sold a call with strike E, $S_d < E < S_u$, followed the hedging procedure but the market ended up at S_1,

$$S_d < S_1 < S_u.$$

Did you lose or make money? Calculate how much profit or loss do you make and plot (sketch) your answer as a function of S_1. You may assume $r = 0$ for simplicity.

2. For the American Call with the parameters $S_0 = 100$, $E = 95$, $r = 0.05$, expiration in 3 years, use the tree model with $\Delta t = 1$, $u = 1.2$, $d = 0.8$. Calculate the option price. At which nodes is the early exercise optimal?

3. Given two *independent* random variables X and Y with probability densities f_X and f_Y, the probability density of their sum $Z = X + Y$ is given by the convolution formula

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(z - y) f_Y(y) dy. \tag{1}$$

Show that the sum of two independent standard normal variables is a normal variable; identify its mean and variance.

Points: /30

Formulas that might be useful

- Call payoff

$$C_T(S_T, E) = \max(0, S_T - E)$$

- Put payoff

$$P_T(S_T, E) = \max(0, E - S_T)$$

- Put-call parity

$$C_t(S, E) + Ee^{-r(T-t)} = P_t(S, E) + S_t.$$

- 1 level tree model for call

$$C = (S_0 - e^{-rT} S_d)\Delta, \qquad \text{where } \Delta = \frac{S_u - E}{S_u - S_d}.$$

- Probability density function of random variable X is an integrable function $f_X(x)$ such that

$$F_X(x) := \mathbb{P}(X < x) = \int_{-\infty}^{x} f_X(s)ds.$$

In other words (when $f_X(x)$ is continuous)

$$f_X(x) = \frac{d}{dx} F_X(x).$$

- Probability density function of a normal $N(\mu, \sigma^2)$ variable is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \tag{2}$$

- "Risk neutral probability"

$$q = \frac{e^{r\delta t} - d}{u - d}$$

- Tree back-propagation formula

$$V^{next} = e^{-r\delta t} \left(V_{up}^{prev} q + V_{down}^{prev} (1 - q) \right).$$

- Tree Delta

$$\Delta = \frac{V_{up} - V_{down}}{S_{up} - S_{down}}.$$