TEXAS A&M UNIVERSITY
DEPARTMENT OF MATHEMATICS

MATH 425

Midterm test version A, 17 Oct 2019

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this work.

Name (print):

No detailed analytical work — no points.

Each question is worth 10 points unless noted otherwise

1. You are given the following prices for the 2020-01-17 options on JPM stock:

- $E = 105$ call 9.43
- $E = 115$ call 8.08
- $E = 125$ put 7.50
- $E = 135$ put 15.55

Spot (current) price on 2019-10-17 is $S_0 = 120$, interest rate $r = 0$. Investigate the following spread: long 105 call, short 115 call, short 125 put, long 135 put.

1. Plot the profit diagram of the spread.
2. What is is wrong with the diagram? (Give a 1-2 sentence explanation citing appropriate mathematical notions).
3. Assuming only one price given to you had a mistake, identify that price by showing it contradicts our theoretical results (see back for formulas).

You may use two pages for your answer.

2. Price an American Put option with the following parameters: $S_0 = 100$, $E = 110$, $r = 0.10$, $T = 3$ months (i.e. 3/12 of a year) using the binomial tree method with $L = 3$ levels and $u = 1.1$, $d = 0.9$.

Assume you sold the put for \$12.00 (which is above the fair price). Simulate hedging on the price path Up, Down, Down to verify that you get the keep the difference between \$12.00 and the fair price.

You may use two pages for your answer.

3.

1. Exponentially distributed random variable is a positive variable X with the density

$$p(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0, \\ 0 & x \leq 0. \end{cases}$$

Find the probability of X being in the interval $[-1, 1]$.

2. Let $X_1, \ldots, X_n$ be i.i.d. random variables with distribution $N(\mu, \sigma^2)$. Find the distribution of the random variable $Y = \frac{1}{n} \sum_{i=1}^{n} X_i$, and sketch its density function on the same graph with the density of X_i. Cite relevant facts to justify your steps.

Formulas that might be useful (better not detach)

- Call and put payoff

$$C_T(S_T, E) = \max(0, S_T - E), \qquad P_T(S_T, E) = \max(0, E - S_T).$$

- Put-call parity and price bounds

$$\begin{aligned}
C_t(S, E) + Ee^{-r(T-t)} &= P_t(S, E) + S_t, \\
\max(S_t - Ee^{-r(T-t)}, 0) \;\le\; C_t(S_t, E) &\;\le\; S_t, \\
\max(Ee^{-r(T-t)} - S_t, 0) \;\le\; P_t(S_t, E) &\;\le\; Ee^{-r(T-t)}.
\end{aligned}$$

- Tree back-propagation formula

$$V^{\text{next}} = e^{-r\Delta t}\left(V_u^{\text{prev}} q + V_d^{\text{prev}}(1-q)\right), \qquad \text{where} \quad q = \frac{e^{r\Delta t} - d}{u - d}.$$

- Tree Delta

$$\Delta = \frac{V_u - V_d}{S_u - S_d}.$$

- Expectation is linear:

$$\mathbb{E}(\alpha_1 X_1 + \alpha_2 X_2 + \beta) = \alpha_1 \mathbb{E}X_1 + \alpha_2 \mathbb{E}X_2 + \beta.$$

- Properties of the variance:

$$\begin{aligned}
&\operatorname{var}(\alpha X + \beta) = \alpha^2 \operatorname{var}(X), \\
&\operatorname{var}(X_1 + X_2) = \operatorname{var}(X_1) + \operatorname{var}(X_2) \qquad \text{if } X_1, X_2 \text{ are uncorrelated.}
\end{aligned}$$

- Probability density function of a normal $N(\mu, \sigma^2)$ variable is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

- If $Z \sim N(0,1)$ then $X = \sigma Z + \mu$ has distribution $N(\mu, \sigma^2)$. In particular, normal variables remain normal under linear transformations $X \mapsto \alpha X + \beta$.

- If X_1, X_2 are independent normal, $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$, then

$$X_1 + X_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2).$$

Points:	**/30**