MATH 425 Fall 2021 Berkolaiko Test 1.A

Must show work, write clearly with **dark pencil or pen**, each question on a separate page, submit in Gradescope, assign pages.

Q.1 (8 points) Plot the profit diagram for the spread consisting of one short (written) $E = 80$ call with premium 8.93 and one short $E = 70$ put with premium 8.16 (assume $r = 0$). Identify the region where this spread will generate profit.

(Not for credit) Assuming the spot (i.e. current) price of the underlying stock is 75, do you think the total delta for this spread is positive, negative or close to zero?

Q.2 (14 points) Price a European Call with the following parameters: $S_0 = 75$, $E = 80$, $r = 0.05$, $T = 3$ months (i.e. 3/12 of a year) using the binomial tree method with $L = 3$ levels and $u = 1.2$, $d = 0.8$.

At every node of the tree, a market maker will offer to sell the call at the calculated value plus 0.05 and will offer to buy the call at the calculated value minus 0.05. A speculator buys one call at time $t = 0$ from the market maker. The market maker hedges their position and the speculator doesn't. The stock goes up the next month and the speculator "takes the profit" by selling the call back to the market maker. Simulate hedging by the market maker (for 1 month only!) and find their profit or loss.

What is the profit or loss of the speculator? What is their "return on investment" (profit divided by the amount invested expressed as a percentage)?

Q.3 (8 points) The forward contract can be defined mathematically as follows: it is a claim with the payoff $S_T - F$ at the time $t = T$, where F is the number which makes the claim have zero value at the time $t = 0$ (S_T is the price of the underlying at $t = T$, F satisfying this condition is called the "price of the forward contract").

In the 1-level tree model we derived the time $t = 0$ value of a claim which pays V_u and V_d at the nodes with underlying prices S_u and S_d correspondingly. This value is given by the formulas

$$V_0 = e^{-rT}\left(V_u q + V_d(1-q)\right), \qquad \text{where} \quad q = \frac{e^{rT}S_0 - S_d}{S_u - S_d}.$$

Use the definition of the forward contract to derive the price F in the 1-level tree model.

Done with the test and bored? For 2 extra points derive the forward price in the L-level tree (use the shortcut formula overleaf).

Formulas that might be useful (do not detach)

- Call and put payoff

$$C_T(S_T, E) = \max(0, S_T - E), \qquad P_T(S_T, E) = \max(0, E - S_T).$$

- Put-call parity and price bounds

$$\begin{aligned} C_t(S, E) + Ee^{-r(T-t)} &= P_t(S, E) + S_t, \\ \max(S_t - Ee^{-r(T-t)}, 0) \ \le\ C_t(S_t, E) \ &\le\ S_t, \\ \max(Ee^{-r(T-t)} - S_t, 0) \ \le\ P_t(S_t, E) \ &\le\ Ee^{-r(T-t)}. \end{aligned}$$

- Price of the forward contract

$$F_t = S_t e^{r(T-t)}.$$

- Tree back-propagation formula

$$V^{\text{next}} = e^{-r\Delta t}\left(V_u^{\text{prev}} q + V_d^{\text{prev}}(1-q)\right), \qquad \text{where} \quad q = \frac{e^{r\Delta t} - d}{u - d}.$$

- Short-cut formula for European-type claims

$$V_0^0 = e^{-rT} \sum_{j=0}^{L} V_j^L \binom{L}{j} q^j (1-q)^{L-j}, \quad q = \frac{e^{r\Delta t} - d}{u - d},$$

where the price of the underlying is assumed to be $S_j^L = S_0 u^j d^{L-j}$.