TEXAS A&M UNIVERSITY
DEPARTMENT OF MATHEMATICS

MATH 425-500

Final version A, 5 May 2016

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this work.

Name (print):

No detailed analytical work — no points.

Each question is worth 10 points unless specified otherwise

1. Multiple choice: circle the correct answer and give a brief (one sentence or picture) reason. Note: "naked option" is an option for which no hedging is performed.

1. The buyer of a naked put hopes that the underlying asset's price (increases / decreases / doesn't matter).

2. The buyer of a put who wishes to hedge the option needs to (buy / short) the underlying asset.

3. The buyer of a put who is actively hedging it would like the price of the underlying to (increase / decrease / doesn't matter).

4. The buyer of a put who is actively hedging shoud expect to (buy / sell) asset as the underlying price increases.

5. The buyer of a put who is actively hedging it would like the volatility of the underlying to (increase / decrease / doesn't matter).

2. Let $P(S, E, \tau)$ be the price of put as given by the Black–Scholes–Merton formula

$$P(S, E, \tau) = Ee^{-r\tau}N(-d_2) - SN(-d_1),$$

where

$$d_{1,2} = \frac{\ln(S/E) + \left(r \pm \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}.$$

1. Derive the answer for vega
$$\frac{\partial P}{\partial \sigma} = S\sqrt{\tau}N'(d_1) \tag{1}$$
and prove that it is sign-definite (either always positive or always negative — determine which).
2. Explain your answer for the sign of vega from the financial perspective. Compare your answer with Question 1 part 5.

3. Use the portfolio lemma to derive the inequality

$$\max\left(Ee^{-rT} - S_0, 0\right) \leq P(S_0, T) \leq Ee^{-rT} \tag{2}$$

for the price P at time $t = 0$ of a put with strike E and expiration T.

4. For the stock price model
$$S_t = S_0 e^{(\mu-\sigma^2/2)t+\sigma\sqrt{t}Z}, \tag{3}$$
where Z is the standard normal random variable,

1. Define the *equivalent rate of return* to be the random variable R such than $S_t = S_0e^{Rt}$. Show that $R \sim N(\mu - \sigma^2/2, \sigma^2/t)$.
2. Derive the expectation
$$\mathbb{E}(S_t) = S_0e^{\mu t}. \tag{4}$$

5. Price the following *American put option* using the tree model: stock price now is $S_0 = 100$, strike is $E = 110$, interest rate is 8%, time to expiration 3 months.

1. Construct the 3-level tree (1 level per month) if the stock price is expected to either go up by 10% or go down by 10%. Price the above option using this tree.
2. Describe the hedging procedure undertaken by a writer of the option who seeks to eliminate risk. You must assume that the holder of the option will do what is best for them (i.e. exercise early if optimal). Consider the price path: Up, Down, Down

You may use 2 pages for your answer.

Points: /50