TEXAS A&M UNIVERSITY
DEPARTMENT OF MATHEMATICS

MATH 425-500

Final version A, 4 May 2017

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this work.

Name (print):

No detailed analytical work — no points.

Each question is worth 10 points unless specified otherwise

1. From Options Industry Council: "A long call condor consists of four different call options of the same expiration. The strategy is constructed of 1 long in-the-money call, 1 short higher middle strike in-the-money call, 1 short middle out-of-money call, 1 long highest strike out-of-money call."

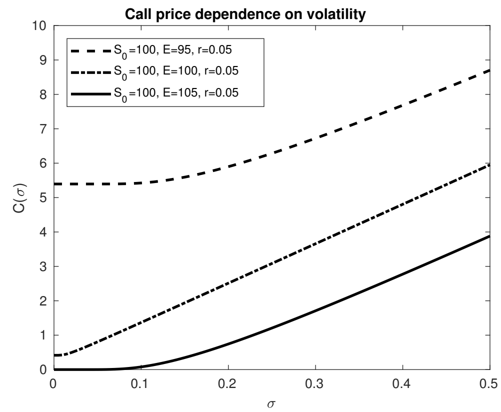
Assume the spot price is $S_0 = 60$. Consider the following spread:
1 long 50 call @ 10.45,
1 short 55 call @ 6.24,
1 short 65 call @ 1.31,
1 long 70 call @ 0.46.

1. Sketch the payoff and the profit diagrams.
2. Does buying the above spread represent a bet that the stock price will grow, fall or stay flat?
3. Create a spread with the identical payoff but using only puts.

2. Consider Black–Scholes–Merton formula for the call price $C(t, S, E, \sigma, r, T)$ as a function of volatility σ (see the back of the exam for formulas). In class we saw the following graph of $C(\sigma)$ for three different choices of the strike E. Calculate the limit

$$\lim_{\sigma \to 0+} C(t, S, E, \sigma, r, T)$$

and explain how it agrees with the plot (note that $r > 0$).



3. Assuming for simplicity that $r = 0$. The Sharpe Ratio of an asset whose rate of return has mean μ and standard deviation σ is defined as

$$R = \frac{\mu}{\sigma}.$$

It is used for comparing risk-adjusted returns of different assets.

1. Do higher values of R correspond to "better" or "worse" investment?
2. Assume that X_1 and X_2 are identically distributed *independent* variables and let $Y = X_1 + X_2$. Calculate the Sharpe Ratio R_Y in term of the Sharpe Ratio $R_X = R_{X_1} = R_{X_2}$.
3. Assume that X_1 and X_2 are *perfectly correlated*, namely $X_2 = X_1$; as before $Y = X_1 + X_2$. Calculate R_Y fin terms of R_X.
4. In an asset management company, should managers of two different portfolios coordinate their strategies or act independently? Argue your answer from the formulas you obtained.

4. (20 points) Price the following *American put option* using the tree model: stock price now is $S_0 = 100$, strike is $E = 120$, interest rate is 2%, time to expiration 3 months.

1. Construct the 3-level tree (1 level per month) if the stock price is expected to either go up by 10% or go down by 10%. Price the above option using this tree.
2. Describe the hedging procedure undertaken by a writer of the option who seeks to eliminate risk. You must assume that the holder of the option will do what is best for them (i.e. exercise early if optimal). Consider the price path: Up, Down, Up.

You may use 2 pages for your answer.

Points: /50

Formulas that might be useful (detach and take with you)

- Call / put payoff
$$C_T(S_T, E) = \max(0, S_T - E), \qquad P_T(S_T, E) = \max(0, E - S_T).$$
- Put-call parity
$$C_t(S, E) + Ee^{-r(T-t)} = P_t(S, E) + S_t.$$
- Probability density function of random variable X is integrable function $f_X(x)$ such that
$$F_X(x) := \mathbb{P}(X < x) = \int_{-\infty}^{x} f_X(s)ds.$$
In other words (when $f_X(x)$ is continuous)
$$f_X(x) = \frac{d}{dx}F_X(x).$$
- Standard normal distribution
$$f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}, \qquad N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-y^2/2}\,dy.$$
- Tree back-propagation formula
$$V^{next} = e^{-r\delta t}\left(V_{up}^{prev}q + V_{down}^{prev}(1-q)\right), \quad \text{where} \quad q = \frac{e^{r\delta t} - d}{u - d}.$$
- Tree Delta
$$\Delta = \frac{V_{up} - V_{down}}{S_{up} - S_{down}}.$$
- Black–Scholes–Merton model for call / put price
$$C(t, S, E) = SN(d_1) - Ee^{-r(T-t)}N(d_2), \qquad P(t, S, E) = Ee^{-r(T-t)}N(-d_2) - SN(-d_1),$$
where
$$d_{1,2} = \frac{\ln(S/E) + \left(r \pm \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}.$$
- useful idenitity for deriving the Greeks
$$SN'(d_1) - Ee^{-r(T-t)}N'(d_2) = 0.$$
- Greeks for the call / put

	Call	Put
$\Delta = \frac{\partial V}{\partial S}$	$N(d_1)$	$-N(-d_1)$
$\Gamma = \frac{\partial^2 V}{\partial S^2}$	$\frac{N'(d_1)}{S\sigma\sqrt{T-t}}$	$\frac{N'(d_1)}{S\sigma\sqrt{T-t}}$
vega$= \frac{\partial V}{\partial \sigma}$	$SN'(d_1)\sqrt{T-t}$	$SN'(d_1)\sqrt{T-t}$
$\theta = \frac{\partial V}{\partial t}$	$\frac{-S\sigma}{2\sqrt{T-t}}N'(d_1) - rEe^{-r(T-t)}N(d_2)$	$\frac{-S\sigma}{2\sqrt{T-t}}N'(d_1) + rEe^{-r(T-t)}N(d_2)$
$\rho = \frac{\partial V}{\partial r}$	$(T-t)Ee^{-r(T-t)}N(d_2)$	$-(T-t)Ee^{-r(T-t)}N(-d_2)$